MAT 175

Solution to HW 2

a) $\lim\limits_{x\to 7} \dfrac{x^2-8x+7}{x-7} = \lim\limits_{x\to 7} \dfrac{\cancel{(x-7)}(x-1)}{\cancel{x-7}} = \lim\limits_{x\to 7} x-1 = \boxed{6}$

b) $\lim\limits_{\theta\to 0} \theta^4 \sin\left(\dfrac{\theta^3}{\theta^2+2}\right) = 0$ by the squeeze theorem,

since $\quad -1 \le \sin\left(\dfrac{\theta^3}{\theta^2+2}\right) \le 1$

hence $\quad -\theta^4 \le \theta^4 \sin\left(\dfrac{\theta^3}{\theta^2+2}\right) \le \theta^4$

so $\lim\limits_{\theta\to 0} -\theta^4 = 0 = \lim\limits_{\theta\to 0} \theta^4$ implies that

also $\lim\limits_{\theta\to 0} \underset{\text{(goes to zero)}}{\theta^4} \underbrace{\sin\left(\dfrac{\theta^3}{\theta^2+2}\right)}_{\text{(bounded)}} = 0$

(Alternatively, and more simply, note that 0 is in the domain of $f(\theta) = \theta^4 \cdot \sin\left(\dfrac{\theta^3}{\theta^2+2}\right)$, which is continuous everywhere, so $\lim\limits_{\theta\to 0} \theta^4 \sin\left(\dfrac{\theta^3}{\theta+2}\right) = f(0) = 0$.)

c) $\lim\limits_{x\to -2} \dfrac{\sqrt{2-x}-2}{2x+4} = \lim\limits_{x\to -2} \dfrac{(\sqrt{2-x}-2)(\sqrt{2-x}+2)}{(2x+4)(\sqrt{2-x}+2)} =$

$= \lim\limits_{x\to -2} \dfrac{2-x-4}{(2x+4)(\sqrt{2-x}+2)} = \lim\limits_{x\to -2} \dfrac{-\cancel{(x+2)}}{2\cancel{(x+2)}(\sqrt{2-x}+2)}$

$= \lim\limits_{x\to -2} -\dfrac{1}{2(\sqrt{2-x}+2)} = -\dfrac{1}{2(\sqrt{4}+2)} = \boxed{-\dfrac{1}{8}}$

d) $\lim_{t \to 0} \frac{3 \sin(6t) \cos(7t)}{8t} = \lim_{t \to 0} 3 \cdot 6 \cdot \frac{\sin 6t}{6t} \frac{\cos 7t}{8}$

$= \frac{18}{8} \underbrace{\lim_{t \to 0} \frac{\sin(6t)}{6t}}_{1} \cdot \underbrace{\lim_{t \to 0} \cos 7t}_{1} = \boxed{\frac{9}{4}}$