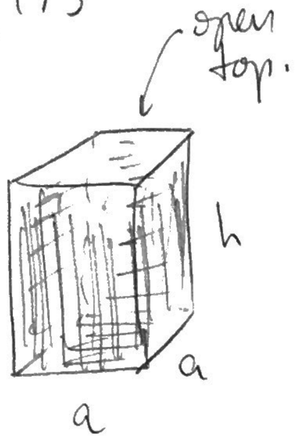
MAT 175 Solution to HW10



Volume of box (target function) $V = a^2 h$

Surface Area of box (constraint) $A = 4ah + a^2 = 48$

$$4ah + a^2 = 48 \Rightarrow h = \frac{48 - a^2}{4a} = \frac{12}{a} - \frac{a}{4}$$

So $V(a) = a^2 \cdot \left(\frac{12}{a} - \frac{a}{4}\right) = 12a - \frac{a^3}{4}$

$$V : [0, 4\sqrt{3}] \to \mathbb{R}$$

$$V'(a) = 12 - \frac{3a^2}{4} = 0 \Leftrightarrow 3a^2 = 48 \Leftrightarrow a^2 = 16$$

$$\Leftrightarrow a = 4$$ ← only critical point.

Endpoints:

$V(0) = 0$

$V(4\sqrt{3}) = 0$

Crit. pt.

$V(4) = 48 - 16 = \underline{\underline{32}}\ ft^3$ ← Global maximum of $V : [0, 4\sqrt{3}] \to \mathbb{R}$

A: The max. volume possible for such a box is $32\ ft^3$, attained w/ dimensions $\underline{\underline{4 \times 4 \times 2}}$ $(a = 4, h = 2)$