MAT175, Fall 2018

Practice Problems for Midterm 1

1. Compute the following limits, or explain why they do not exist:

   a) $\lim_{x\to 0} \dfrac{2\sin(4x)}{3x}$

   b) $\lim_{x\to \pi/2} \dfrac{2\sin(4x)}{3x}$

   c) $\lim_{x\to 2} \dfrac{x^2+3x-10}{x-2}$

   d) $\lim_{x\to 2} \dfrac{x^2-4x+8}{x-2}$

   e) $\lim_{a\to 3} \dfrac{\sqrt{a+1}-2}{a+3}$

   f) $\lim_{a\to 3} \dfrac{\sqrt{a+1}-2}{a-3}$

   g) $\lim_{t\to 0} t^2 \cos\left(\dfrac{\pi}{t}\right)$

2. Compute the following limits at infinity, or explain why they do not exist:

   a) $\lim_{x\to +\infty} \dfrac{\sin(x)}{x}$

   b) $\lim_{x\to +\infty} \dfrac{x^3+2x+1}{x^2-1}$

   c) $\lim_{x\to -\infty} \dfrac{x^3+2x+1}{x^2-1}$

   d) $\lim_{x\to +\infty} \dfrac{3x^5+x^3+x+1}{4x^5-x^2-8}$

   e) $\lim_{x\to -\infty} \dfrac{3x^5+x^3+x+1}{4x^5-x^2-8}$

   f) $\lim_{x\to +\infty} \dfrac{x^3+2x^2+1}{-2x^7+3x^4+x^2}$

   g) $\lim_{x\to -\infty} \dfrac{x^3+2x^2+1}{-2x^7+3x^4+x^2}$

3. Sketch the graph of the function $f(x)$ defined by

$$f(x) = \begin{cases} x^2 & \text{if } x \le 2 \\ 2-x & \text{if } x > 2 \end{cases}$$

   Is $f(x)$ continuous at all points?

4. Sketch the graph of the function $f(x)$ defined by

$$f(x) = \begin{cases} e^x & \text{if } x \leq 0 \\ \cos(x) & \text{if } x > 0 \end{cases}$$

Is $f(x)$ continuous at all points?

5. What is the value of a that makes the following function continuous at all points?

$$f(x) = \begin{cases} \dfrac{2\sin(ax)}{x} & \text{if } x \leq 0 \\ x + a^2 + 1 & \text{if } x > 0 \end{cases}$$

6. Show that the derivative of $f(x) = 2x + 3$ is equal to $f'(x) = 2$ **using the definition as a limit** of a difference quotient.

7. Show that the derivative of $g(x) = 5x^2 - x$ is equal to $f'(x) = 10x - 1$ **using the definition as a limit** of a difference quotient.

8. Compute the first derivative of the following functions:

   a) $f(x) = 1 + 3\pi x^4 - 2x + e^x$

   b) $F(x) = \dfrac{2}{x} - \sqrt{5}x - 5\sqrt{x} + 5\cos(x-1)$

   c) $g(t) = \dfrac{1}{\sqrt{2t+2}} - te^{2t+1} - \tan(4t^2 + \frac{\pi}{4})$

   d) $A(\theta) = 3\sin\theta\cos\theta + e^\theta\cos(5\theta)$

   e) $q(s) = \dfrac{e^{s^2+1} - \sin(\sqrt{s})}{s^2+1}$

9. Find the tangent line to the graph of the functions below at the given point:

   a) $f(x) = 1 + 3\pi x^4 - 2x + e^x$, at the point $(0, 2)$

   b) $F(x) = \dfrac{2}{x} - \sqrt{5}x - 5\sqrt{x} + 5\cos(x-1)$, at the point $(1, 2 - \sqrt{5})$

   c) $g(t) = \dfrac{1}{\sqrt{2t+2}} - te^{2t+1} - \tan(4t^2 + \frac{\pi}{4})$, at the point $(0, \frac{1}{\sqrt{2}} - 1)$

   d) $A(\theta) = 3\sin\theta\cos\theta + e^\theta\cos(5\theta)$, at the point $(0, 1)$

   e) $q(s) = \dfrac{e^{s^2+1} - \sin(\sqrt{s})}{s^2+1}$, at the point $(0, e)$

10. Suppose a particle moves along a straight line in such a way that its position (measured as distance from the origin) is given by $s(t) = t^2 - 2\sqrt{t^2+1} + e^t$ at time t. Find the velocity and acceleration of the particle when $t = 1$.

11. If the position (measured as height from the ground) of an object thrown straight up from an initial height of 32 feet is given by $s(t) = -16t^2 + 16t + 32$ at time t, find both the velocity and acceleration at the moment the object hits the ground.

12. The equation $x^2 + y^3 + y = 1$ implicitly defines a function $y = y(x)$ near the point $(1, 0)$. Find the equation of the tangent line to the curve $x^2 + y^3 + y = 1$ at the point $(1, 0)$.

13. Let $f(x) = x^3 + x - 2$ and $g(x)$ be its inverse function. Compute $g'(0)$ and $g''(0)$.