SOLUTION TO HW 7

$$f(x) = \frac{x^2}{2} - \frac{5}{2}\ln(x^2+1)$$

$$f'(x) = x - \frac{5}{2}\cdot\frac{1}{x^2+1}\cdot 2x = x - \frac{5x}{x^2+1}.$$

a) Since $f'(x)$ exists for all x in $I=[-3,3]$, the critical points are those where $f'(x)=0$.

$$f'(x) = x\left(1-\frac{5}{x^2+1}\right) = 0 \iff x=0 \text{ or } 1-\frac{5}{x^2+1}=0$$

Note: $1-\dfrac{5}{x^2+1}=0 \iff x^2+1-5=0 \iff x^2=4$

$\iff x=\pm 2.$

So the critical points are $x=-2$, $x=0$ and $x=2$.

b) Let's analyze the sign of $f'(x)$:

$-2 \qquad 0 \qquad 2$

−	−	+	+	x
+	−	−	+	$1-\frac{5}{x^2+1}$
−	+	−	+	$f'(x)$
↘	↗	↘	↗	
(decrease)	(increase)	(decrease)	(increase)	$f(x)$

(b) $f(x)$ is <u>increasing</u> if and only if

$-2 < x < 0$ or $2 < x < 3$

(c) $f(x)$ is <u>decreasing</u> if and only if

$-3 < x < -2$ or $0 < x < 2$

(d) By the first derivative test:

$x = -2$ is a <u>local min</u>

$x = 0$ is a <u>local max</u>

$x = 2$ is a <u>local min</u>

(e) $f(-3) = f(3) = \frac{9}{2} - \frac{5}{2} \ln 10 < 0$ because $\ln 10 > \frac{9}{5}$

(because $10 > e^{9/5} \approx e$)

$f(-2) = f(2) = 2 - \frac{5}{2} \ln 5 < 0$ because $\ln 5 > \frac{4}{5}$

$f(0) = 0.$ (because $5 > e^{4/5} \approx e$)

Clearly, the largest of the above numbers is $f(0) = 0$, and the smallest is $f(-2) = f(2) = 2 - \frac{5}{2} \log 5$

so,

$$\min_I f(x) = 2 - \frac{5}{2} \ln 5 = f(-2) = f(2)$$

$$\max_I f(x) = 0 = f(0)$$