

# SOLUTION TO HW 7

$$f(x) = \frac{x^2}{2} - \frac{5}{2} \ln(x^2 + 1)$$

$$f'(x) = x - \frac{5}{2} \cdot \frac{1}{x^2 + 1} \cdot 2x = x - \frac{5x}{x^2 + 1}.$$

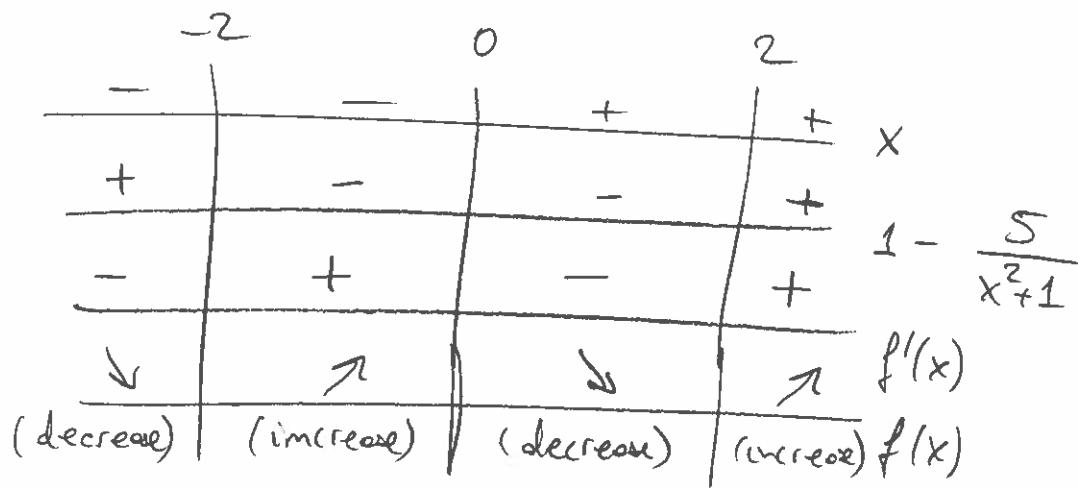
a) Since  $f'(x)$  exists for all  $x$  in  $I = [-3, 3]$ , the critical points are those where  $f'(x) = 0$ .

$$f'(x) = x \left( 1 - \frac{5}{x^2 + 1} \right) = 0 \iff x = 0 \text{ or } 1 - \frac{5}{x^2 + 1} = 0$$

Note:  $1 - \frac{5}{x^2 + 1} = 0 \iff x^2 + 1 - 5 = 0 \iff x^2 = 4 \iff x = \pm 2$

So the critical points are  $x = -2, x = 0$  and  $x = 2$ .

b) Let's analyze the sign of  $f'(x)$ :



(b)  $f(x)$  is increasing if and only if  
 $\underline{-2 < x < 0}$  or  $\underline{2 < x < 3}$

(c)  $f(x)$  is decreasing if and only if  
 $\underline{-3 < x < -2}$  or  $\underline{0 < x < 2}$

(d) By the first derivative test:

$x = -2$  is a local min

$x = 0$  is a local max

$x = 2$  is a local min

(e)  $f(-3) = f(3) = \frac{9}{2} - \frac{5}{2} \ln 10 < 0$  because  $\ln 10 > \frac{9}{5}$

$f(-2) = f(2) = 2 - \frac{5}{2} \ln 5 < 0$  because  $\ln 5 > \frac{4}{5}$

$f(0) = 0$ . (because  $5 > e^{4/5} \approx e$ )

(Clearly, the largest of the above numbers is  $f(0) = 0$ , and the smallest is  $f(-2) = f(2) = 2 - \frac{5}{2} \ln 5$  so:

$\min I f(x) = 2 - \frac{5}{2} \ln 5 = f(-2) = f(2)$

$\max I f(x) = 0 = f(0)$

