SOLUTION TO HW 3

#1 The function $f(x) = \begin{cases} x^2 + a^2 + 4, & \text{if } x \leq 0 \\ \dfrac{4\sin(ax)}{x}, & \text{if } x > 0 \end{cases}$ is

clearly continuous at all $x \neq 0$, since for either $x < 0$ or $x > 0$, it is a combination of continuous functions.

In order for $f(x)$ to be continuous at $x=0$, it must satisfy

$$f(0) = \lim_{x \to 0} f(x)$$

Note that $f(0) = 0^2 + a^2 + 4 = a^2 + 4$, and:

$$\lim_{x \to 0_+} f(x) = \lim_{x \to 0_+} \frac{4\sin(ax)}{x} = 4 \lim_{x \to 0_+} \overbrace{\frac{a\sin(ax)}{ax}}^{\to 1} = 4a$$

$$\lim_{x \to 0_-} f(x) = \lim_{x \to 0_-} x^2 + a^2 + 4 = a^2 + 4.$$

Thus, for $f(x)$ to be continuous at $x=0$, we need:

$$4a = a^2 + 4 \implies a^2 - 4a + 4 = 0$$

$$\implies (a-2)^2 = 0$$

$$\implies \boxed{a = 2}$$

Note: This is the only value of a for which $f(x)$ is cont. at $x=0$

#2

a) $$\lim_{x\to+\infty} \frac{x^2+3x-4}{8x-5} = \lim_{x\to\infty} \frac{x\,\cancel{x^2}\left(1+\frac{3}{x}-\frac{4}{x^2}\right)}{\cancel{x}\left(8-\frac{5}{x}\right)} = \boxed{+\infty}$$

b) $$\lim_{x\to-\infty} \frac{x^7+2x^4+8}{5x^3-12} = \lim_{x\to-\infty} \frac{x^4\,\cancel{x^7}\left(1+\frac{2}{x^3}+\frac{8}{x^7}\right)}{\cancel{x^3}\left(5-\frac{12}{x^3}\right)} = \boxed{+\infty}$$

c) $$\lim_{x\to+\infty} \frac{2x^6+3x^5-7x^2+9}{8x^6-3x^3+10} = \lim_{x\to+\infty} \frac{\cancel{x^6}\left(2+\frac{3}{x}-\frac{7}{x^4}+\frac{9}{x^6}\right)}{\cancel{x^6}\left(8-\frac{3}{x^3}+\frac{10}{x^6}\right)} = \boxed{\frac{1}{4}}$$

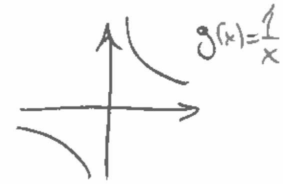
d) $\lim_{x\to1} \frac{x^2+2x+3}{x-1}$ does not exist because:

$$\lim_{x\to1_-} \frac{\overbrace{x^2+2x+3}^{\to 6_-}}{\underbrace{x-1}_{\to 0_-}} = -\infty \qquad \left(\begin{array}{l}\text{if } x\to1_-\text{, then } x-1<0, \\ \text{so } \frac{x^2+2x+3}{x-1} \to -\infty\end{array}\right)$$

$$\lim_{x\to1_+} \frac{\overbrace{x^2+2x+3}^{\to 6_+}}{\underbrace{x-1}_{\to 0_+}} = +\infty \qquad \left(\begin{array}{l}\text{if } x\to1_+\text{, then } x-1>0 \\ \text{so } \frac{x^2+2x+3}{x-1} \to +\infty\end{array}\right)$$

e) $$\lim_{x\to+\infty} \frac{4x^3+3x^2-1}{5x^{10}+4x^2+2} = \lim_{x\to+\infty} \frac{\cancel{x^3}\left(4+\frac{3}{x}-\frac{1}{x^3}\right)}{x^7\,\cancel{x^{10}}\left(5+\frac{4}{x^8}+\frac{2}{x^{10}}\right)} = \boxed{0}$$

f)



$$\lim_{x\to0_+} \frac{1}{x} = +\infty$$

$$\lim_{x\to0_-} \frac{1}{x} = -\infty$$

$$\lim_{x\to0} \frac{1}{x} \quad \text{D.N.E.}$$