

MEASUREMENT AND PLOTTING

February 27, 2014

SIGNIFICANT FIGURES

No physical measurement is ever completely precise. All measurements are subject to some uncertainty, and the determination of the magnitude of this uncertainty is an essential part of the analysis of the experiment. Experimental data include three components: 1) the value measured, 2) the uncertainty, and 3) the units. A typical result is

$$(3.6 \pm 0.2) \text{ m} \tag{1}$$

Determining uncertainties can be more difficult than that of averaged values. While important in experimental physics, uncertainties are not in the focus of this course. Below a general description of uncertainties is provided.

ERRORS, AVERAGE, AND UNCERTAINTIES

The accuracy of any measurement is limited by experimental errors and uncertainties. An *error* is a discrepancy between the measured value of some quantity and its true value. Errors in measurements arise from different sources:

a) A common type of error is blunders due to carelessness in making a measurement, such as in an incorrect reading of an instrument. Of course these kinds of mistakes should be avoided.

b) Errors also arise from defective or uncalibrated instruments. For example, if a balance does not read zero when there is no mass on it, then all of its readings will be in error, and we must either recalibrate it, or be careful to subtract the empty reading from all subsequent measurements.

Even after we have made every effort to eliminate this kind of error, the accuracy of our measurements is still limited, due to experimental uncertainties. Uncertainties reflect unpredictable random variations in the measurement process: variations in the experimental system, in the measuring apparatus, and in our own perception! Since these variations are random, they will tend to cancel out if the average over a set of repeated measurements. To measure a quantity in the laboratory, one should repeat the experiment many times. The average of all the results is the best estimate of the value of the quantity.

Besides the uncertainty introduced in a measurement due to random fluctuations, vibrations, etc, there is the so-called systematic or reading uncertainty which has to do with the limited accuracy of the measuring instruments we use. For example, if we use a meter stick to measure a length, we can, at best, estimate the length to about 1/10 of a millimeter. Beyond that we have no knowledge. It is important to realize that this kind of uncertainty persists, even if we obtain identical readings on repeated trials.

To summarize all measurements have an uncertainty (random and systematic). Often we refer to this uncertainty as an error, but it should be emphasized that an error reflects the limit of knowledge of the true value we want to measure.

Below we shall describe a method for calculating a) the maximum uncertainty in the measured quantities and b) the maximum uncertainty in the final quantity, which is a function of measured quantities.

In order to minimize the effect of random errors, one should perform several independent measurements of the same quantity and then take an average of all these readings. In taking the average, the random fluctuations tend to cancel out. In fact, the larger the number of measurements taken, the smaller will be the effect of random errors. So when we have a set of n measurements, x_1, x_2, \dots, x_n of a quantity x , we take, as our *best estimate* for the measured value x , the average value.

$$\text{Average} \quad \bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} \quad (2)$$

(Important note: when making repeated measurements of a quantity, one should pay attention to the consistency of the results. If one of the numbers is substantially different from the others, it is likely that a blunder has been made; this values should be excluded when analyzing the results.)

Quite often in these laboratory exercises one has to compare the value obtained as a result of measurement, with the standard, or generally accepted value. One then has to compute the **percentage error** defined as one hundred times the ratio of the difference between the average incurred value and the accepted value.

$$\text{percentage error} = \frac{\text{measured average value} - \text{accepted value}}{\text{accepted value}} \times 100\%. \quad (3)$$

Sometimes when there is no accepted value, one may perform different kind of experiments and end up with different average values for some quantity. One can then compare measurements by computing their **percentage discrepancy**, defined as one hundred times the ratio of the difference between the two measurements to their average.

$$\text{percentage discrepancy} = \frac{\text{value 1} - \text{value 2}}{\text{average value 1 and value 2}} \times 100\%. \quad (4)$$

Also one can use the simplified formula

$$\text{percentage discrepancy} = \frac{\text{value 1} - \text{value 2}}{\text{value 1 (or 2)}} \times 100\%. \quad (5)$$

SIGNIFICANT FIGURES

A number expressing the result of a measurement or of computations based on measurements should be written with proper number of **significant figures**. What we mean by that is the number of reliably known digits in a number. The number of significant figures is independent of the position of the decimal point, for example 2.163 cm, 21.63 mm or 0.02163 m have the same number of significant figures.

In computing, all digits which are not significant, should be dropped. (It is better to round off instead of truncating). A result which is obtained by multiplication or division has the same number of significant figures as the measured quantity with the least number of significant figures. Suppose that we want to calculate the area of a rectangular plate whose measured length is 11.3 cm and measured width is 6.8 cm. The area is then calculated with a pocket calculator and found to be

$$\text{Area} = 11.3 \text{ cm} \times 6.8 \text{ cm} = 76.84 \text{ cm}^2$$

Since the width has only two significant figures we conclude that the area is 77 cm².

The last significant figure should not exceed that of the uncertainty, if the latter is known. For instance, it would be incorrect to write $(3.64 \pm 0.2) \text{ m}$ since the last significant figure 4 is not precisely known because of the uncertainties.

If uncertainties are not calculated, as is the actual case in this course, then two important things remain: 1) consider measurement results critically and exclude blunders; 2) keep 3-4 (better 4) significant figures everywhere. Keeping less significant figures will show that you don't care for the accuracy. Keeping more significant figures will show that you do not understand that the accuracy of your measurements is limited.

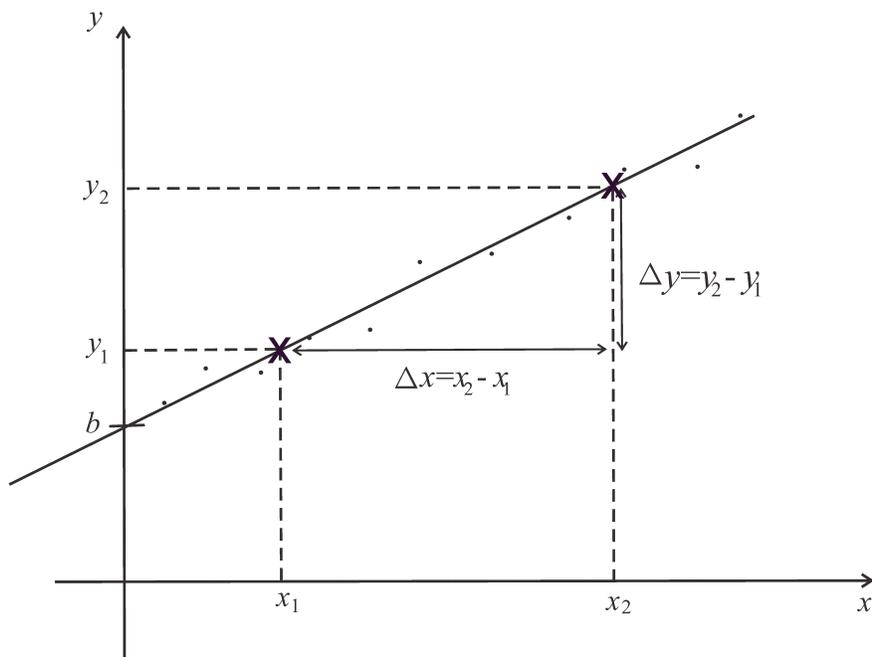
GRAPHICAL REPRESENTATION OF DATA

In almost every laboratory exercise you will have to plot a graph based on the data taken. A graph lets us visualize the relation between two physical quantities. In plotting a graph follow the following steps:

1. Arrange the data to be plotted in tabular form.
2. Graphs have two perpendicular axes, the X-axis and the Y-axis. Decide which of the two quantities is to be plotted along each axis. It is conventional to plot the independent quantity along the X-axis and the dependent one along the Y-axis.
3. Choose the scale of units for each axis of the graph. The scale determines how many squares correspond to one unit of the quantity being plotted along a particular axis. The scale should be chosen so that the final curve spreads over all the graph paper.
4. Mark the main divisions along each axis and next to it indicate what quantity is plotted and the corresponding units.

5. Each data point should be clearly visible on the graph.
6. Draw a smooth curve through the data points. The curve should come as close as possible to most of data points. The fact that not all points lie along the fitted curve, just indicates that measurements are subject to some uncertainty.

FITTING DATA TO A STRAIGHT LINE



If the experimental data can be fitted by a straight line, this straight line can be found by the eye and drawn on the plot with the help of a ruler. An important condition for choosing the best straight line is that there are nearly equal numbers of points above and below the line. Do not force the line to go exactly through the points or through the origin of the coordinate system. The equation for the straight line is given by

$$y = ax + b, \tag{6}$$

where a is the slope of the line and b is the intercept, that is, the value of y at $x = 0$.

The next task is to extract the parameters in Eq. (6). To do this, one can choose two so-called basis points on the line (not necessarily the experimental points!) far away from each other. For these points one has

$$y_1 = ax_1 + b, \quad y_2 = ax_2 + b.$$

With the basis points (x_1, y_1) and (x_2, y_2) read out from the plot, one can solve this system of equations for a and b . In particular, for the slope one obtains

$$a = \frac{y_2 - y_1}{x_2 - x_1} \equiv \frac{\Delta y}{\Delta x}. \quad (7)$$

The geometrical interpretation of the slope is seen in the figure.

FITTING DATA WITH SOFTWARE

(The instructions below have been written for Windows PC. On Mac the procedure can slightly differ from this.)

Obtaining the slope of a straight line fitting experimental points graphically is a rather inaccurate and labor-consuming method. One can instead input the data into plotting/fitting software and let it do the job. This way one can fit any curves, not only straight lines. Popular software that can be used is, for instance, Excel of the Microsoft Office, as well as the similar software Calc from the free Open Office developed by Sun Microsystems. The data sheets contain columns of data labeled by A, B, C, etc. One of these columns, usually A, represents the x values while another column, say, C, represents the y values.

To plot the data, mark columns A and C, leaving column B unmarked. Then click on the “Chart Wizard or Diagram Assistant” icon and choose presentation of the data by points (scatter). Do not choose pies and bar diagrams. Add the needed formatting such as axes labels.

To fit the data, rightclick on one of the points on the plot and choose “Add trendline” from the pop-up menu. Another menu, “Format trendline”, will appear in which you will have to choose the fitting function (Trendline type). Choose “linear” and click OK. Now you will see the fitting straight line on your plot.

To obtain the slope, rightclick or doubleclick on the fitting line and enter the “Format trendline” again. In the “Options” tab, mark “Display equation on chart”, then click OK. You will see Eq. (6) on the plot with the numerical values of a and b .