Mathematical Physics – PHY307 Fall 2018

Professor Dmitry Garanin Assignment 2

1. Indefinite integrals. Use substitutions to calculate the integrals

a) $\int dv\, 12v(7+6v^2)^9$
b) $\int dz\, 7z^2(14+8z^3)^{-5}$
c) $\int dv\, (v-2v^3)\cos(v^2-v^4)$
d) $\int dt\, \frac{1}{t}\sin[1-\ln t]$

2. Definite integrals. Use differentiation over a parameter to calculate the integrals

a) $\int_0^1 dx\, \frac{x^2-1}{\ln x}$; consider $I(t)=\int_0^1 dx\, \frac{x^t-1}{\ln x}$;
b) $\int_0^\infty dx\, \frac{\sin x}{x}$; consider $I(t)=\int_0^\infty dx\, \frac{\sin x}{x}e^{-tx}$. Using the result, calculate $\int_0^1 \frac{1-\cos x}{x^2}$
c) $A=\int_0^\infty dx\, e^{-x^2}$; consider $I(t)=\int_0^\infty dx\, \frac{e^{-x^2}}{1+\left(\frac{x}{t}\right)^2}$ so that $A=\lim_{t\to\infty} I(t)$.

Then $I(t)=t\int_0^\infty dy\, \frac{e^{-t^2y^2}}{1+y^2}$.

Integrate the relation $\frac{d}{dt}\frac{e^{-t^2}I(t)}{t}=-2t\int_0^\infty dy\; e^{-t^2(1+y^2)}=-2e^{-t^2}A$, etc.
d) $\int_0^\infty dx\, x^2e^{-x^2}$; consider $I(t)=\int_0^\infty dx\, e^{-tx^2}=\frac{1}{2}\sqrt{\frac{\pi}{t}}$; Alternatively use integration by parts.

3. Definite integrals. Calculate using functions of a complex variable

a) $\int_0^{2\pi} \frac{d\theta}{10+6\cos\theta}$
b) $\int_0^\infty dx\, \frac{\sin x}{\sqrt{x}}$ and $\int_0^\infty dx\, \frac{\cos x}{\sqrt{x}}$. Kill the square root and express these integrals via Re and Im of $\int_0^\infty dx\, e^{ix^2}$. Express this via the integral over a straight path in the complex plane at $\omega=\pi/2$ to the real axis.
c) $\int_0^\infty dx\, \frac{\cos x}{a^2+x^2}$; also $\int_0^\infty dx\, \frac{\cos x}{(a^2+x^2)^2}$ by complex variables and differentiation over parameter
d) $\int_0^\infty dx\, \frac{\sqrt{x}}{a^2+x^2}$; also $\int_0^\infty dx\, \frac{\sqrt{x}}{(a^2+x^2)^2}$ by complex variables and differentiation over parameter

4. Multiple integrals. For the pyramid enclosed by the coordinate planes and the plane $x+y+z=1$

a) Find its volume
b) Find the coordinates of the centroid (center of mass for the uniform density)
c) If the density is $z$, find $M$ and $\bar{z}$

5. Multiple integrals.

a) $I=\int_{-\infty}^\infty dx\, e^{-x^2}$ by considering $I^2$ as a product of the integrals over $x$ and $y$, considering it as a double integral and calculating the latter using polar coordinates.
b) $\int d^2k\, \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{a^2+k^2}$

c) $\int d^3k \, \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{a^2+k^2}$

d) $\int_0^\infty \int_0^\infty dxdy \frac{x^2+y^2}{1+(x^2-y^2)^2} e^{-2xy}$ by change of variables $u = x^2 - y^2, v = 2xy$.