Mathematical Physics - PHY307 Fall 2010

Prof. D. Garanin Assignment 6

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1. Simpson integration rule obtained by approximating the integrand by parabolas over small intervals has the form

$$\int_a^b f(x)dx = \left(\frac{1}{3}f(x_0) + \frac{4}{3}f(x_1) + \frac{2}{3}f(x_2) + \frac{4}{3}f(x_3) + \ldots + \frac{4}{3}f(x_{N-1}) + \frac{1}{3}f(x_N)\right)\Delta x,$$

where N is even. Implement this formula in Mathematica. Demonstrate numerically that the Simpson rule yields exact results for third-order polynomials. Investigate convergence of the Simpson rule for the integral

$$\int_0^{\pi/2} \cos(x)dx = 1$$

as it was done in the lecture notes and compare it with the convergence of the rectangular, trapezoidal, and Durand rules (including the log-log plot). Do the same comparison for the integral

$$\frac{3}{\pi}\int_0^{\sqrt{3}} \frac{dx}{1+x^2} = 1.$$

Is there a difference in the accuracy of the Simpson rule for the two integrals?

2. Calculate analytically the integral

$$P(G) = \frac{1}{(2\pi)^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \frac{dxdy}{1 - G\lambda(x,y)}$$

where

$$\lambda(x,y) = \frac{1}{2}(\cos x + \cos y)$$

and $0 \le G < 1$. Do the calculation in two ways. First, calculate the integral in one step as a double integral. Second, integrate in two steps, first over y and then over x. Are results the same? How $P(G)$ behaves at small G and at G close to 1?

3. Assuming that $a > 0$ and $m \ge 0$, evaluate

$$\int_0^{\infty} \frac{\cos^2(mx)}{a^2 + x^2}dx.$$

Check the result in the limit of large m.