

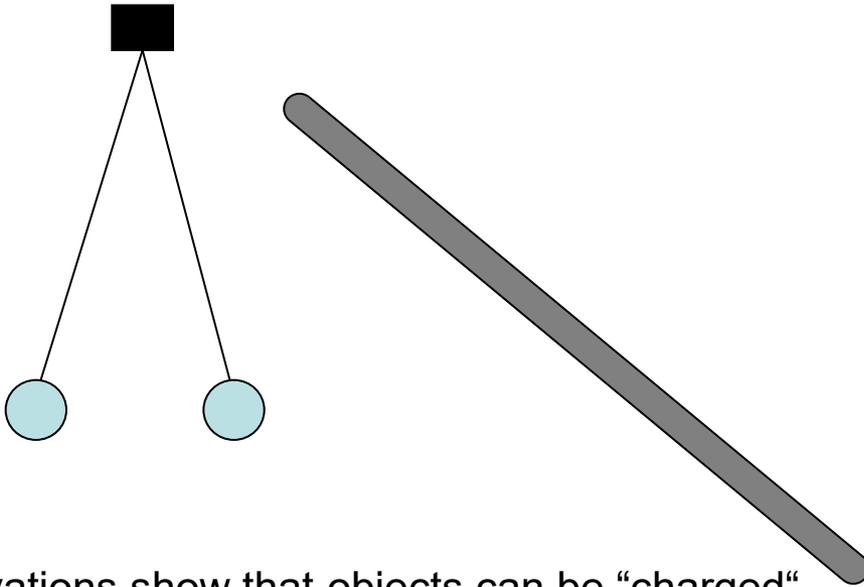
1 –Electrostatics

Electricity & Magnetism includes a vast variety of phenomena that can be divided in four groups:

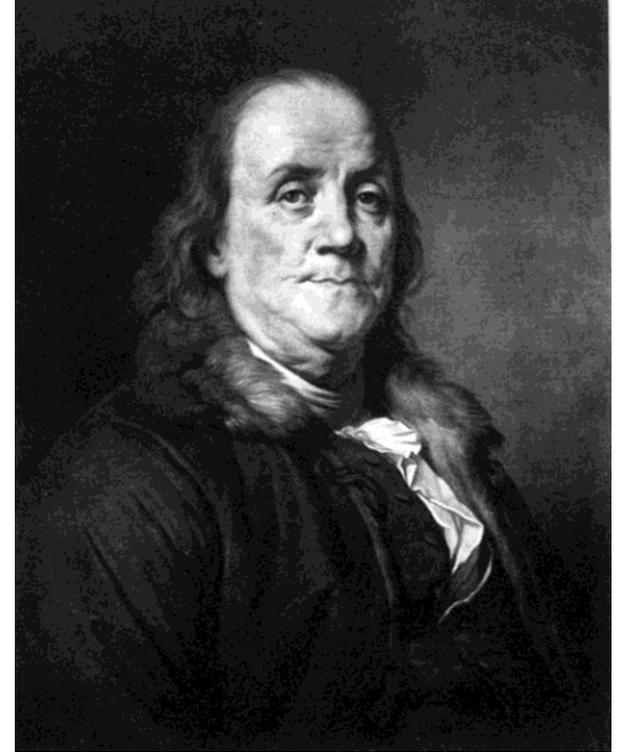
- **Electrostatics** (electric charges at rest, standing hair)
- **Electric currents & circuits** (moving electric charges)
- **Magnetism** (permanent magnets, magnetic effects due to electric currents)
- **Electromagnetism & Electrodynamics** (time-dependent phenomena in electrical machines, electromagnetic waves)

Electricity (and less Magnetism) penetrates modern life, whereas in the past people could only observe a few electrostatic phenomena such as sparks. That was the beginning of the investigation of electricity.

First experiments on electrostatics



Observations show that objects can be “charged” by rubbing and by contact between charged and uncharged objects. Objects charged in the same way (such as two light balls in the Figure that have been charged by a contact with a stab that was previously charged by rubbing with fur or silk) repel from each other. Objects charged in different ways (such as stabs of different materials rubbed with different materials: Plastic rubbed with fur or glass rubbed with silk) may also attract each other. Thus one can speak of two kinds of electric charges that can be arbitrarily called “positive” and “negative”.



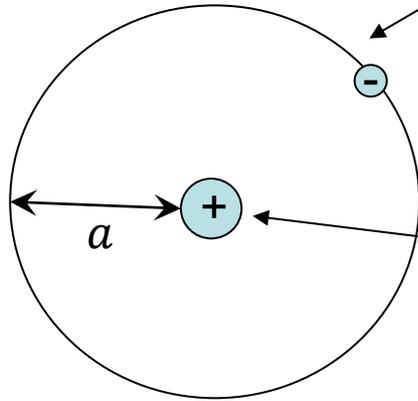
Benjamin Franklin (1706-1790)

Proposed single-fluid model of electrostatics, close to modern views: Surplus of the electric fluid makes the object charged “positively” while shortage of this fluid makes it charged “negatively”.

Now we know that Franklin’s “electric fluid” is nothing else than electrons that have assigned negative charge and can be transferred from one ² object to the other

Atomic nature of electricity

Atom



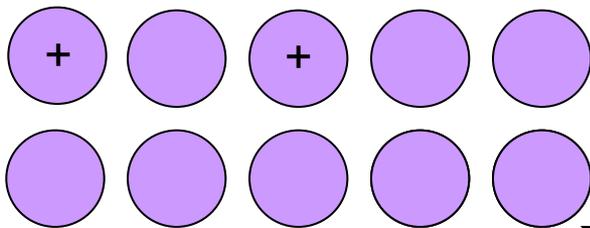
$$a \sim 0.5 \times 10^{-10} \text{ m}$$

Electron is a small and light ($m_e = 0.911 \times 10^{-30} \text{ kg}$) negatively charged particle ($q_e = e = -1.60 \times 10^{-19} \text{ C}$) that rotates around the core. Typical atom contains many electrons that can be closer to or further from the core. Those further from the core are loosely bound and they can be removed by rubbing or otherwise

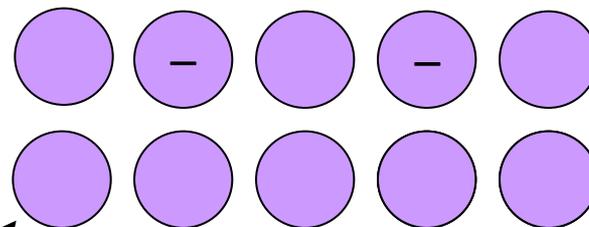
Atom's core contains practically the whole mass of the atom. It consists of protons and neutrons. The mass of proton and neutron is $m_p = m_n = 1.67 \times 10^{-27} \text{ kg}$ (much greater than the electron mass). Neutron is not charged. Proton is charged positively, $q_p = +e = 1.60 \times 10^{-19} \text{ C}$.

Normally atoms are not charged as the charge of their electrons balances the charge of their core. Removing electrons creates a positively charged ion, and placing additional electrons on the atom creates a negatively charged ion. Jumping of electrons from one body to the other can cause sparks.

Positively charged material:
Some electrons are removed



Negatively charged material:
There are excessive electrons



Atoms

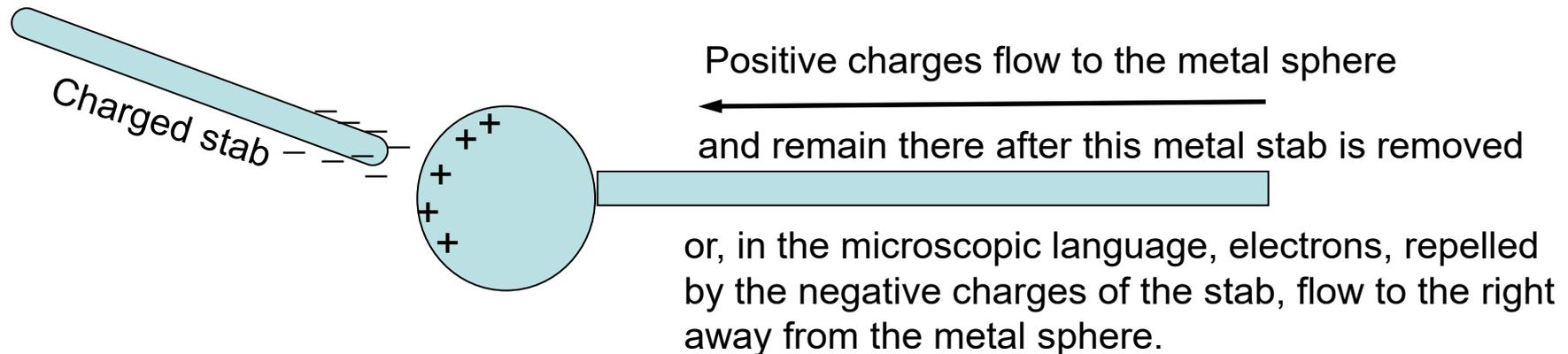
Insulators and conductors

In insulators (also called dielectrics) particular electrons are bound to particular atomic cores. Sometimes they jump between atoms but these are rare events. Charges put at a particular place of an insulator remain there for a long time.

In conductors (these are mostly metals) electrons that are far from the cores can freely move from one atom to the other and thus over the whole body, so that they belong not to particular atoms but to the whole specimen.

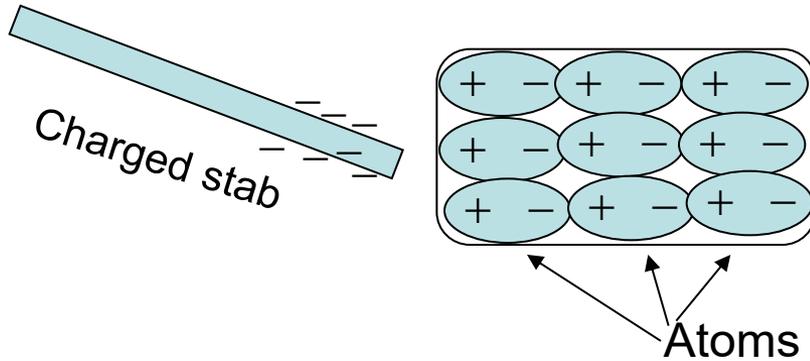
There are materials intermediate between conductors and insulators such as semiconductors, water, moist materials, etc. Water cannot be used in applications as conductor (unlike metals) but, on the other hand, it does not prevent charges from moving (unlike good insulators).

Conductors and bad insulators can be charged by induction:



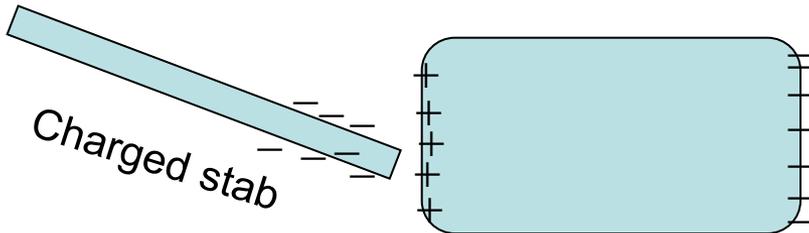
Polarization of insulators and conductors

Insulators:



Individual atoms are polarized by the external charge (stab). As a result the polarized piece of the insulator attracts to the charged stab because its positive charges are closer to the stab than its negative charges. This effect is small

Conductors:



Conductors in this example polarize via travelling of positive charges to the left and negative charges to the right. In the atomic language, electrons move to the right thus creating uncompensated positive charge of atomic cores on the left side. There is attraction between the charged stab and the polarized piece of conductor, similarly to the insulator. This effect is big.

Coulomb's law

$$F_{12} = k \frac{q_1 q_2}{r^2}$$

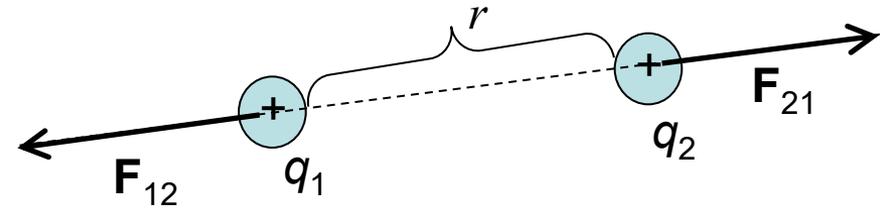
In SI the unit of charge is Coulomb (C) and the Coulomb constant k is given by

$$k = \frac{1}{4\pi\epsilon_0} = 0.9 \times 10^{10} \text{ N m}^2 / \text{C}^2$$

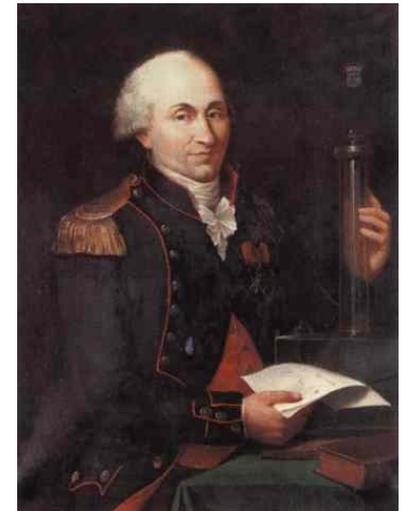
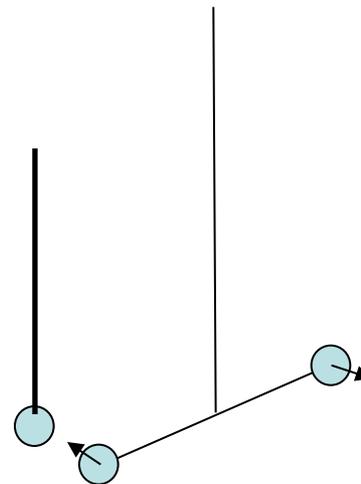
where the so-called vacuum permittivity ϵ_0 has the value $\epsilon_0 = 0.885 \times 10^{-11}$ in SI.

Both k and ϵ_0 are related to the choice of the system of units and have no physical meaning.

The formula for the Coulomb force above does not define the force completely as any force is a vector. This definition is complemented by the sketch on the right. If the force formula yields a positive value, charges repel each other. If it yields a negative value, the charges attract each other.



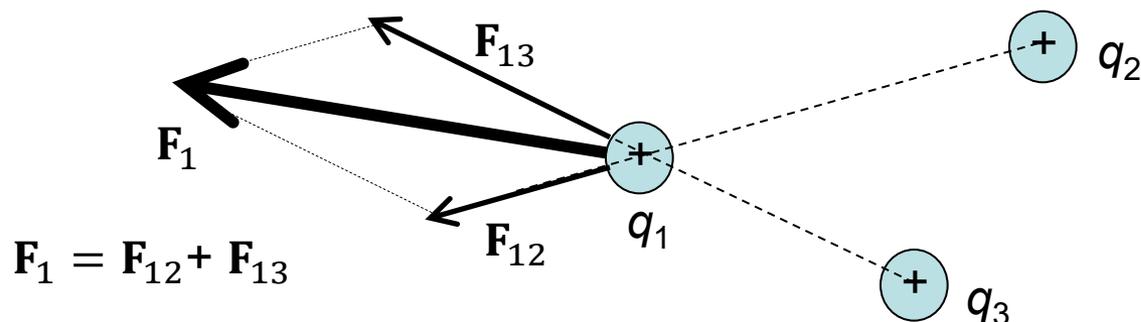
Coulomb's torsion balance (schematically)



Charles Coulomb (1736-1806)

Coulomb law resembles the gravity law, with important differences!

If there are several charges, the force acting on each charge is the sum of all Coulomb forces acting on the charge from the other charges.



Example: An electron and a proton have charges of an equal magnitude but opposite sign of 1.6×10^{-19} C. If the electron and proton in a hydrogen atom are separated by a distance of 0.5×10^{-10} m, what is the electrostatic force exerted on the electron by the proton? Is it attraction or repulsion?

Formulation: $q_e \equiv e = -1.6 \times 10^{-19}$ C, $q_p \equiv -e = 1.6 \times 10^{-19}$ C, $r = 5 \times 10^{-11}$ m, F -?

Solution: Use the Coulomb's law:

$$F_{12} = k \frac{q_1 q_2}{r^2} = -k \frac{e^2}{r^2} = -0.9 \times 10^{10} \left(\frac{1.6 \times 10^{-19}}{5 \times 10^{-11}} \right)^2 = -0.92 \times 10^{-9} \text{ N}$$

The result is negative, and this means the charges are attracting each other

Electric field

If there are many electric charges in space, q_1, q_2 , etc. then the Coulomb force acting on a charge q_0 at any point \mathbf{r} is the (vector!) sum of all Coulomb forces acting between q_0 and q_1, q_0 and q_2 , etc. This force is evidently proportional to q_0 . If we divide this force by q_0 we obtain the electric field $\mathbf{E}(\mathbf{r}) = \mathbf{F}/q_0$ that characterizes the distribution of charges in space and is independent of the „test“ charge q_0 . Since Coulomb forces are acting at a distance, one can ascribe the following deep meaning to the field. Charges placed in space somehow modify the space at all its points, not only at those points where they are placed. The result of this modification is that when a test charge q_0 is put at any point \mathbf{r} then the force $\mathbf{F} = q_0\mathbf{E}(\mathbf{r})$ is acting on it.

Electric field of a point charge q follows from the Coulomb's law and it is given by

$$E = k \frac{q}{r^2}$$

- simplified!

Unit of electric field:
1 N/C

where r is distance from the charge q . Vector \mathbf{E} is directed from the point charge for $q > 0$ and to the point charge for $q < 0$.

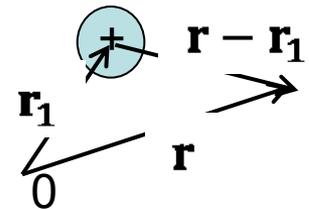
Full vector form of the electric field is

$$\mathbf{E} = k \frac{q}{r^2} \frac{\mathbf{r}}{r} = kq \frac{\mathbf{r}}{r^3}$$

where the vector \mathbf{r} goes from the charge q to the observation point.

More general

$$\mathbf{E} = kq_1 \frac{\mathbf{r} - \mathbf{r}_1}{|\mathbf{r} - \mathbf{r}_1|^3}$$



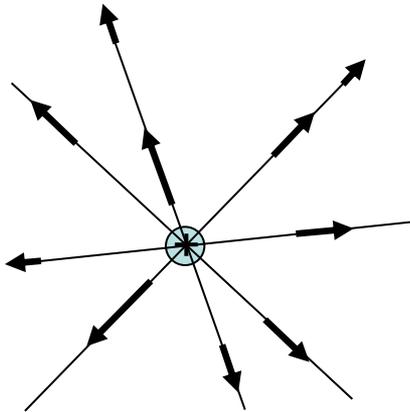
where charge q_1 is at \mathbf{r}_1 and observation point is at \mathbf{r} .

Electric field lines

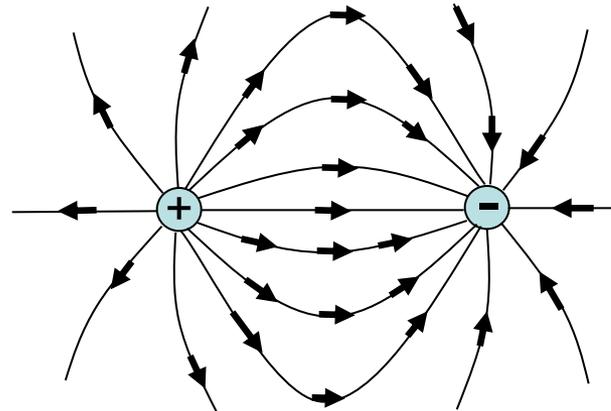
Electric field is an example of the mathematical object called “field”. In mathematics, *field* is any function of the position vector \mathbf{r} . That is, at any point in space, the value of the function is defined. There are scalar fields (the value of the function is scalar) and vector fields (the value of the function is vector). Electric field is a vector field.

Electric field lines are lines drawn in such a way that they go parallel to the vector \mathbf{E} at any point. Examples of the electric field lines are shown below.

One point charge



Two point charges of equal magnitude



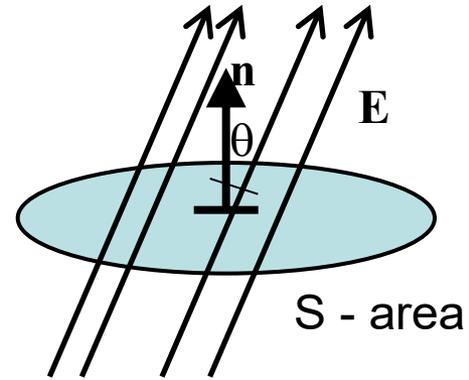
Electric field lines are similar to the lines of flow of a fluid. Positive charges are sources of \mathbf{E} while negative charges are sinks of \mathbf{E} .

Electric flux and Gauss law

Electric flux through a small surface S is defined by

$$\Phi = \mathbf{E} \cdot \mathbf{n}S = E_{\perp}S = ES \cos \theta$$

where the unit vector \mathbf{n} ($|\mathbf{n}|=1$) is called normal vector and is perpendicular to the surface. (Note that all kinds of fluxes are defined in this way, for instance, flux of water through a cross-section of a pipe.)



Calculating electric flux through a large surface requires calculus, in general. One has to split up the surface into many small surfaces ΔS and sum the fluxes over them all:

$$\Phi = \sum_{\text{Surface}} E_{\perp} \Delta S$$

In some cases it can be done easily. For instance, the electric flux from a point charge Q over a sphere of radius R with the center on the charge:

$$\Phi = ES = k \frac{Q}{R^2} 4\pi R^2 = 4\pi kQ = 4\pi \frac{1}{4\pi\epsilon_0} Q = \frac{Q}{\epsilon_0}$$

One can show that this result is also valid if the charge Q is away from the center of the sphere (but still inside the sphere!). Also one can replace the sphere by any closed surface, this does not change the flux. This is the Gauss law: The electric flux through a closed surface is proportional to the total charge within the surface

$$\Phi = \sum_{\text{Closed Surface}} E_{\perp} \Delta S = \frac{1}{\epsilon_0} \sum_{\text{Inside the surface}} Q$$

Charges and electric field in conductors

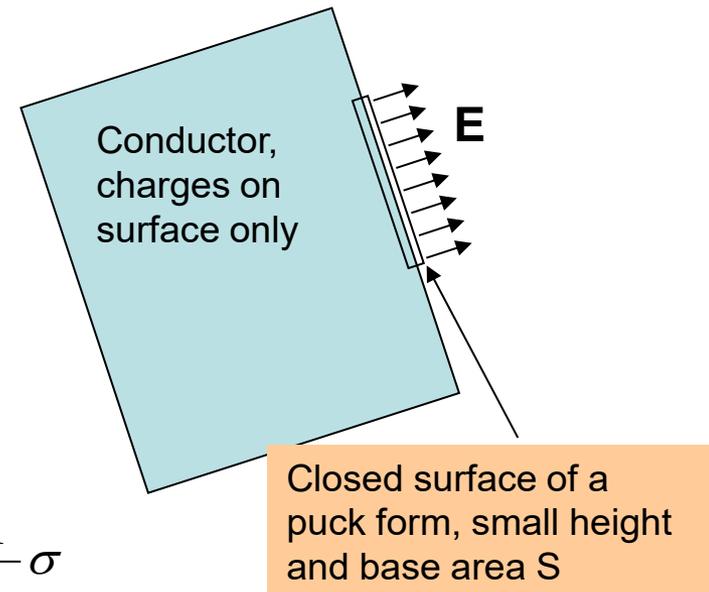
1. Electric field everywhere inside a charged conductor at equilibrium is zero (otherwise electric charges would move);
2. Electric field at the outside surface of a conductor is perpendicular to the surface (otherwise the charges would move along the surface);
3. There is no charges inside the conductor (consequence of the Gauss law); all charges are on the surface;
4. Electric field at the outside surface of a conductor is related to the surface density of charges: $E = \sigma/\epsilon_0$

Volume density of charge: $\rho = \frac{Q}{V}$

Surface density of charge: $\sigma = \frac{Q}{S}$

From the Gauss law follows:

$$\Phi = ES = \frac{1}{\epsilon_0} Q \Rightarrow E = \frac{1}{\epsilon_0} \frac{Q}{S} = \frac{1}{\epsilon_0} \sigma$$



The field is directed outside for $\sigma > 0$