10 – Temperature & Kinetic theory

Definition of temperature

Temperature (T) is the measure of physical objects to be hot, warm and cold.

Experience shows that when most objects become warmer, they increase their length. This property of thermal expansion is used to build thermometers. The height of a column of mercury or colored alcohol in a tube measures the temperature.

To exactly define the temperature scale, one needs two anchor points. Both Farenheit (F) and Celcius (C) scales use freezing and boiling points of water.

- Freezing point of water: 32°F or 0°C
- Boiling point of water: 212°F or 100°C

- Differences: 180°F or 100°C – F-Degree is finer

Relations between Farenheit and Celcius temperatures:

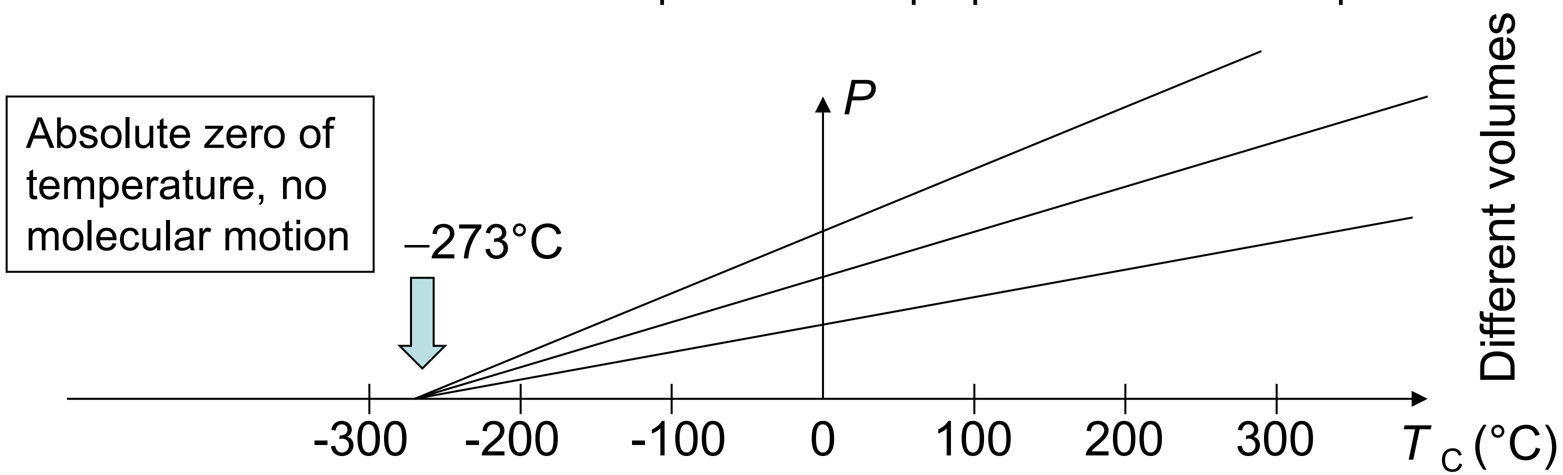
$$T_C = \frac{5}{9}\left(T_F - 32\right)$$

$$T_F = \frac{9}{5}T_C + 32$$

Molecular nature of temperature, Kelvin scale

Physically, temperature measures kinetic energy of molecules and atoms in bodies. As there is a huge quantity of the latter and they are all moving randomly in different directions that change because of interactions and collisions, heated bodies do not move as the whole. This form of kinetic energy is not mechanical, from the macroscopic point of view, and it is a part of the so-called internal energy.

For an ideal gas (very weak interaction between molecules), pressure at a constant volume or volume at a constant pressure are proportional to the temperature



Kelvin temperature scale: uses the same intervals as the Celsius scale but is shifted:

$T = T_C + 273.2, \qquad T_C = T - 273.2$ Unit: K(elvin), like T = 273 K

Physicists use the Kelvin scale in which for a ideal gas is $PV = RT, \quad R = \text{const}$

Thermal expansion

Change of temperature *T* causes change of the length *L* of physical bodies

$$\Delta L = \alpha L_0 \Delta T \quad \text{or} \quad L = L_0(1+\alpha\Delta T) \quad \text{if} \quad \Delta L = L - L_0 \,, \quad \Delta T \equiv T - T_0$$

Here α is the thermal expansion coefficient.
For most of solids α is positive and $\alpha \sim 10^{-5}\ \mathrm{K}^{-1}$ – very small but important in the engineering.

Example: Bridge expansion. The steel bed of a suspension bridge is 200 m long at 20°C. If the extremes of temperature to which it might be exposed are -30°C to 40°C, how much will it contract and expand? For the steel $\alpha = 1.2\times10^{-5}\ \mathrm{K}^{-1}$

Solution: The difference in temperatures (the working temperature interval) is ΔT=70°C=70 K.
One obtains

$$\Delta L = \alpha L_0 \Delta T = 12\times10^{-6}\times200\times70 = 0.168\ \mathrm{m} \approx 17\ \mathrm{cm}$$

- a substantial change of length!

Volume thermal expansion is considered similarly to the linear thermal expansion:

$$\Delta V = \beta V_0 \Delta T \quad \text{or} \quad V = V_0(1+\beta\Delta T) \quad \text{if} \quad \Delta V = V - V_0$$

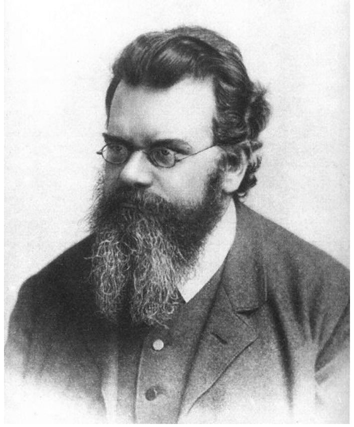
If linear expansion in all three directions is the same, one obtains from $L = L_0(1+\alpha\Delta T)$

$$V = V_0(1+\alpha\Delta T)^3 \cong V_0(1+3\alpha\Delta T) = V_0(1+\beta\Delta T) \quad \text{where} \quad \boxed{\beta = 3\alpha}$$

Equation of state of the ideal gas

$$PV = Nk_BT$$

N – number of molecules in the gas
k_B – Boltzmann constant, $k_B = 1.38 \times 10^{-23}$ J/K



Ludwig Boltzmann
(1844-1906), Austria.
The father of kinetic theory.

Example: How many molecules are on average in 1 liter air at the athmospheric pressure and room temperature?

Formulation: Room temperature, $T = 300$ K; $V = 1$ liter $= 1000$ cm$^3 = 10^{-3}$ m^3, $P = 10^5$ N/m^2. N=?

Solution:

$$N = \frac{PV}{k_BT} = \frac{10^5 \times 10^{-3}}{1.38 \times 10^{-23} \times 300} = 2.4 \times 10^{22}$$

- A huge number!

There is another, "chemical", form of the equation of state of the ideal gas

$$PV = \nu RT$$

where ν is the number of moles of the gas and R is the "universal gas constant". We are NOT using this formula in our course.

Density of the ideal gas

Density of an ideal gas (*M* – mass of the whole gas, *m* – mass of one molecule)

$$\rho = \frac{M}{V} = \frac{Nm}{V} = \frac{\frac{PV}{k_B T} m}{V} = \frac{Pm}{k_B T}$$ - does not depend on the amount of gas

Example: The density of air is 1.29 kg/m^3 at atmospheric pressure and T = 0°C = 273.2 K. What is the average mass of the molecule of air?

Solution:

$$m = \frac{\rho k_B T}{P} = \frac{1.29 \times 1.38 \times 10^{-23} \times 273.2}{10^5} = 4.86 \times 10^{-26} \mathrm{kg}$$

Check: Air consists mostly of Nitrogen that forms molecules consisting of <u>two</u> atoms. Each Nitrogen atom has a nucleous that consists of 14 protons and neutrons, combined. The mass of proton and neutron is 1.67 $\times$ 10^{-27} kg. Thus the mass of a Nitrogen molecule is

$$m = 2 \times 14 \times 1.67 \times 10^{-27} = 4.68 \times 10^{-26} \ \mathrm{kg}$$

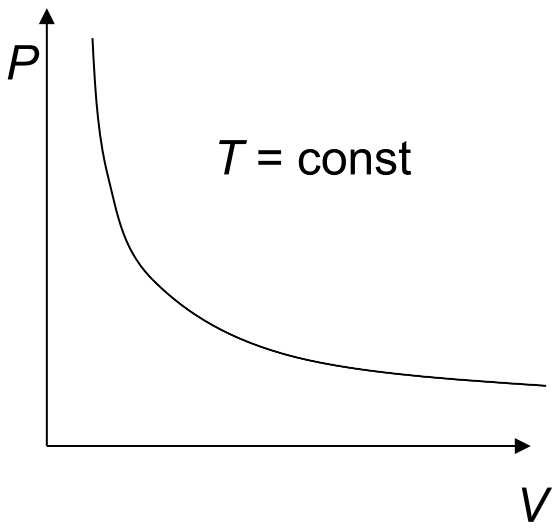
The mass of an average air molecule is slightly higher because the air also contains Oxigen whose molecule contains 2 $\times$ 16 = 32 protons and neutrons, combined.

Processes of the ideal gas

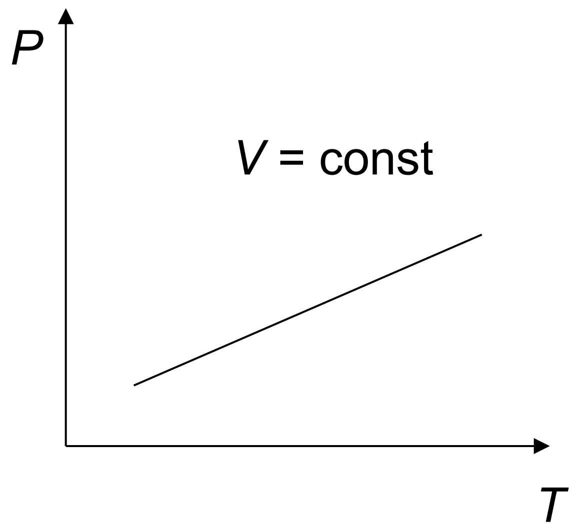
$$PV = Nk_BT$$

Three main types of processes: Isothermic, Isochoric, and Isobaric

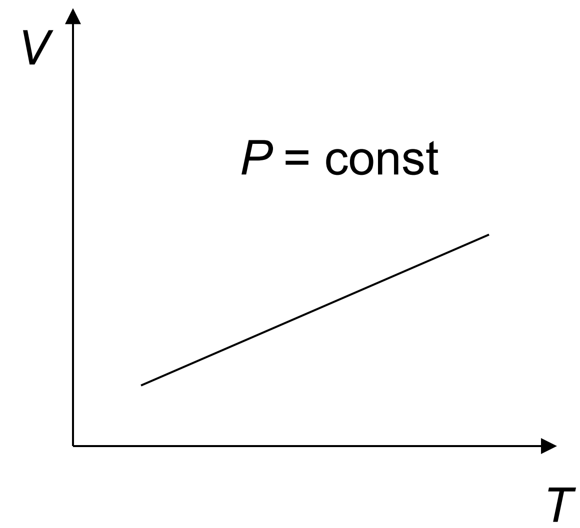
1. Isothermic process



2. Isochoric process



3. Isobaric process



Volume thermal expansion coefficient at constant pressure:

$$\Delta V = \frac{Nk_B}{P}\Delta T = \beta V \Delta T \quad \Rightarrow \quad \beta = \frac{Nk_B}{PV} = \frac{Nk_B}{Nk_BT} = \frac{1}{T}$$

Kinetic interpretation of the temperature

The microscopic origin of the pressure of the gas on the walls of the container surrounding it is the mechanical impact of molecules that hit the walls and rebound. Calculating the momentum transferred from molecules to the wall allows to calculate the pressure on the walls. The result is

$$PV = \frac{2}{3} N \langle \varepsilon_K \rangle$$ where $$\langle \varepsilon_K \rangle = \left\langle \frac{mv^2}{2} \right\rangle$$ is the average translational kinetic energy of the molecule

Comparing this result with $PV = Nk_B T$ yields the relation

$$\boxed{\langle \varepsilon_K \rangle = \frac{3}{2} k_B T}$$

between the temperature of the gas and kinetic energy of its molecules

Example: Speed of air molecules. We calculate the root-mean-square speed $v_{\text{rms}} = \sqrt{\langle v^2 \rangle} = \sqrt{\dfrac{3k_B T}{m}}$

For Nitrogen the molecular mass is $m = 4.68 \times 10^{-26}\ \text{kg}$ thus

$$v_{\text{rms}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 300}{4.68 \times 10^{-26}}} = 515\ \text{m/s}$$

For Oxigen the molecular mass is somewhat larger and thus v_{rms} is somewhat smaller