2 – Motion in one dimension

In Lecture 1 we have seen that each vector (say, position vector **r**) in a three-dimensional space can be represented by its three components (r_x , r_y , r_z) that can be considered independently of each other. In this Lecture we will concentrate on the behavior of one of these components, say $r_x \equiv x$. Other components either do not exist (as is the case for the motion in one dimension) or are ignored.

Let us repeat the definitions for the x-components of the velocity and acceleration

$$v_x = \frac{\Delta x}{\Delta t}, \qquad \Delta x = x_2 - x_1, \qquad \Delta t = t_2 - t_1$$

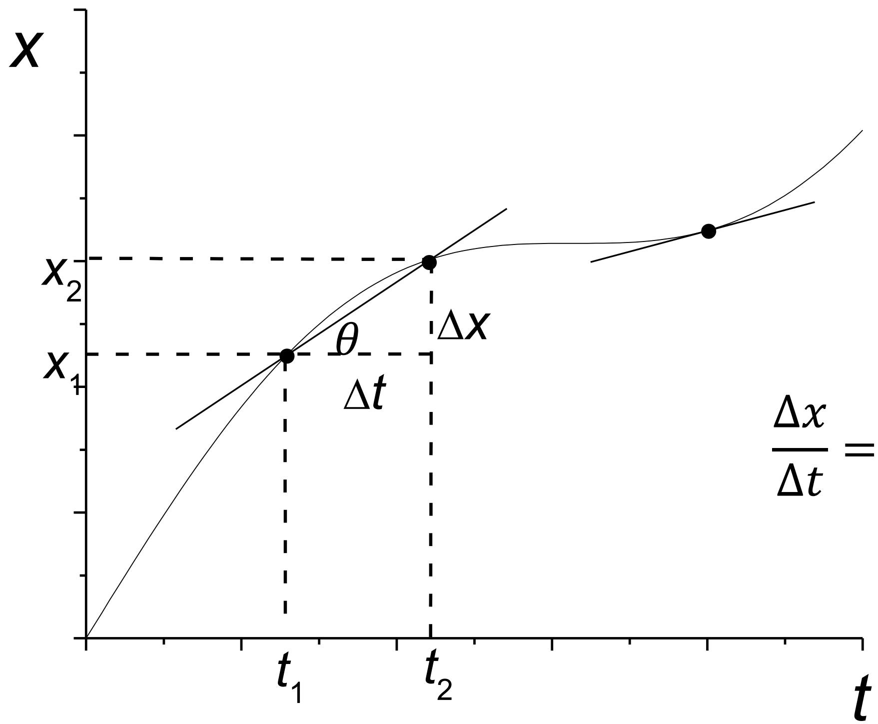
$$a_x = \frac{\Delta v_x}{\Delta t}$$

Here 1 is the initial state and 2 is a final state. Note that v_x and a_x can be both positive and negative

Graphical representation of motion

Coordinate *x* depending on time *t*, that is, *x*(*t*) can be represented graphically. Velocity v_x can be interpreted geometrically as the slope of the curve *x*(*t*).

The average velocity is defined geometrically with the help of the secant (cutting) straight line that cuts the *x(t)* curve in two points, 1 and 2.



$$\frac{\Delta x}{\Delta t} = \tan\theta = \text{slope}$$

The instantaneous velocity defines a straight line that is tangential to the curve *x*(*t*) at a given point (shown on the right of the graph).

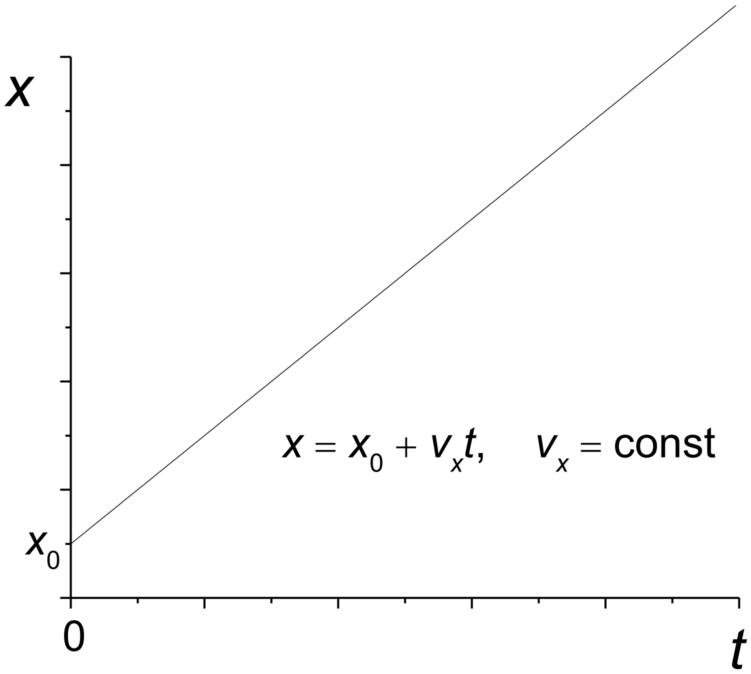
Motion with constant velocity

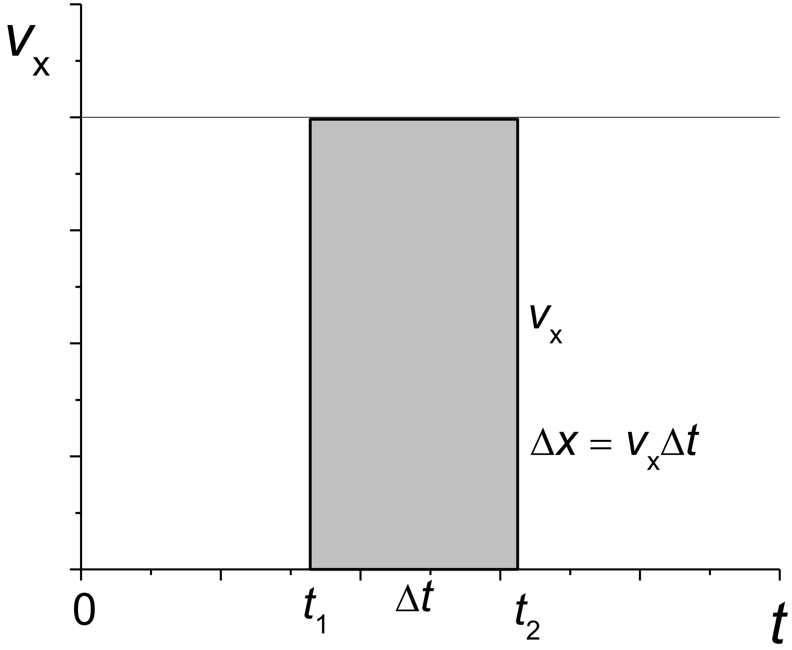
Motion with a constant velocity v_x is geometrically described as a straight line (see the graph on the left). Its analytical representation is

$$x = x_0 + v_x t, \qquad v_x = \text{const}$$

Considering two points, 1 and 2, one can write

$$x_1 = x_0 + v_x t_1, \quad x_2 = x_0 + v_x t_2$$
$$\Delta x = x_2 - x_1 = v_x(t_2 - t_1) = v_x \Delta t$$

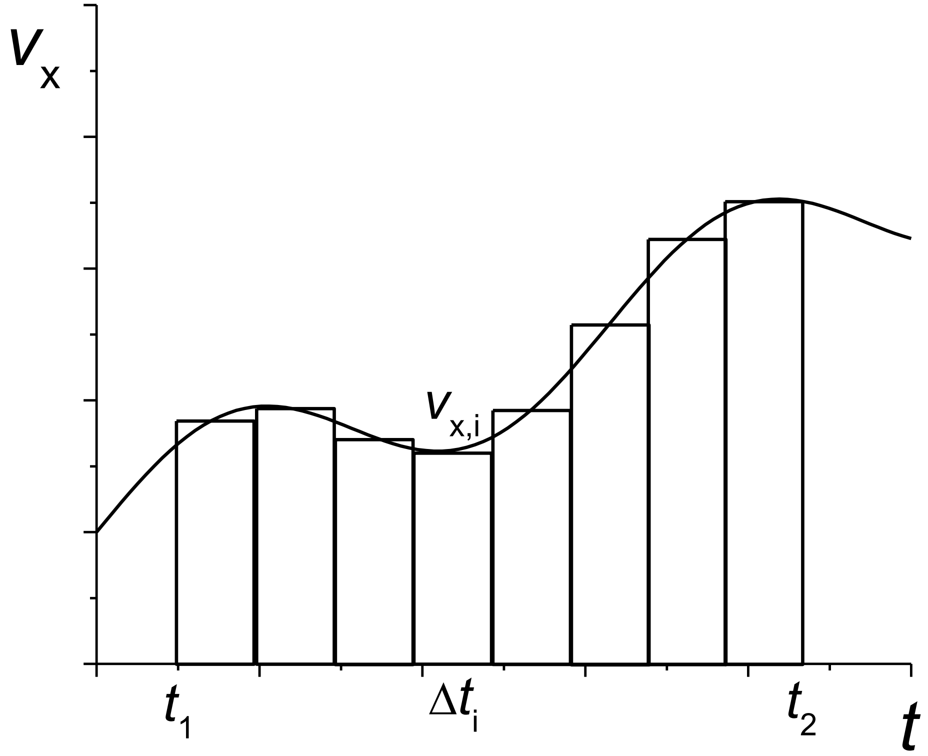




Constant velocity plotted as a function of time is obviously a horizontal line (see the graph on the right). One can see that the change of the coordinate *x* during the elapsed time Δt is given by the area under the velocity curve: $\Delta x = v_x \Delta t$. <u>Note</u>: If $v_x < 0$, the straight line representing v_x on the plot goes below the *t*-axis. In this and similar cases the area under the curve is defined as <u>negative</u>.

Displacement for the motion with a variable velocity

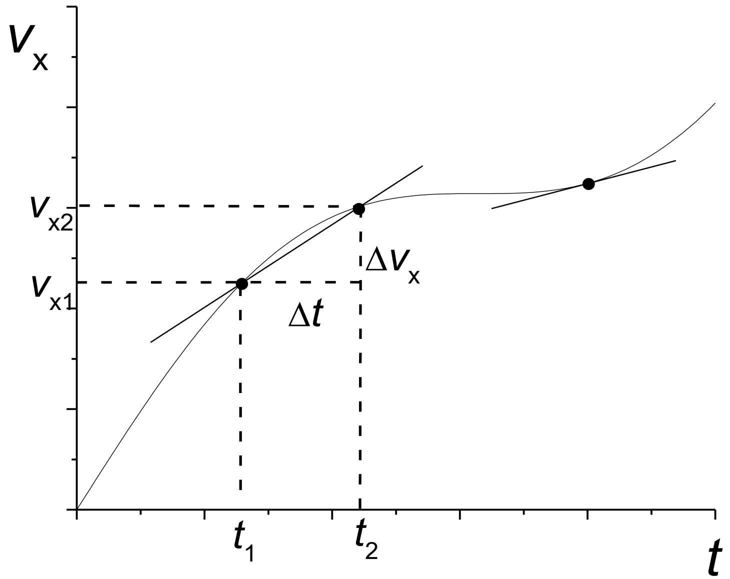
In the general case when the velocity v_x changes with time t, one can split the time interval t_2-t_1 into many small subintervals Δt_i and define the displacement x_2-x_1 as the area under the curve representing $v_x(t)$:



Practical application of the above requires calculus!

Graphical representation of acceleration

Acceleration a_x can be geometrically defined as the slope of the curve $v_x(t)$, similarly to the definition of the velocity v_x from the graph $x(t)$.



In turn, the change of the velocity v_x during a time interval can be represented as the area under the curve $a_x(t)$.

Motion with constant acceleration

This important kind of motion is represented by the formula

$$v_x = v_{x0} + a_x t, \qquad a_x = \text{const}$$

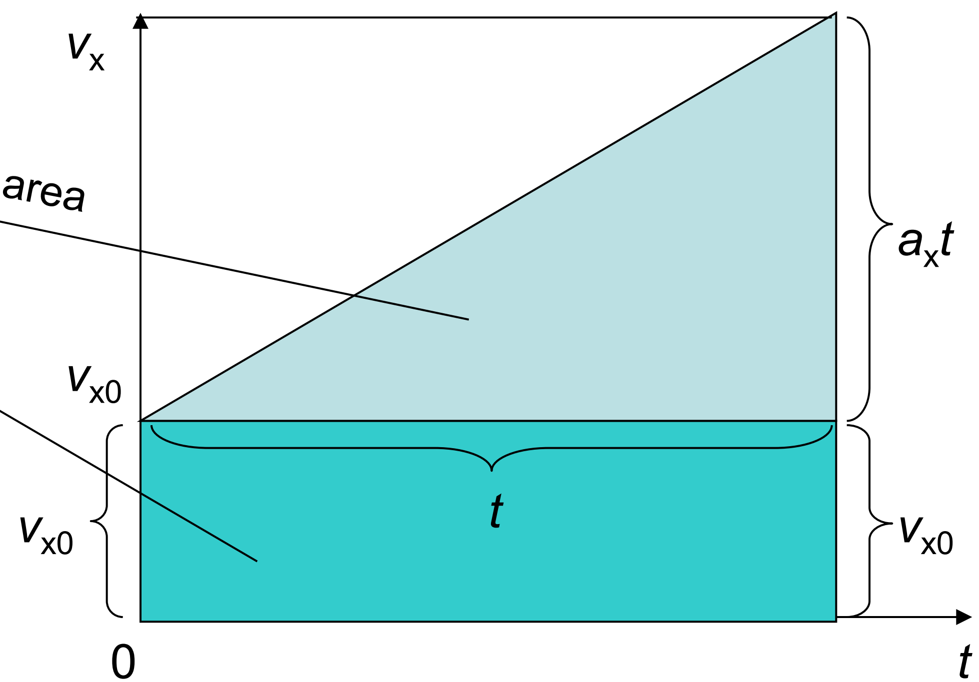
where v_x is a shortcut for the function $v_x(t)$ and the constant v_{x0} is the velocity at zero time, $v_{x0} = v_x(0)$. The time dependence of the x coordinate $x(t)$ can be found as the area under the „curve“ $v_x(t)$. This leads to

$$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$$

Initial condition

area

area



½ in the formula above appears because the area of the triangle is half of the area of the corresponding rectangle

If the motion begins at some moment of time t_0, the formulas for the motion with constant acceleration become

$$v_x = v_{x0} + a(t - t_0)$$

$$x = x_0 + v_{x0}\,(t - t_0) + \frac{1}{2}a(t - t_0)^2$$

Finally, there is a formula for the motion with constant acceleration is in terms of changes:

$$\Delta v = a\Delta t, \qquad \Delta x = v_0\Delta t + \frac{1}{2}a(\Delta t)^2.$$

Here $\Delta t \equiv t - t_0$ is the time elapsed since the initial moment of time t_0,

$\Delta v \equiv v - v_0$ is the change of the velocity, and

$\Delta x \equiv x - x_0$ is the displacement or distance covered.