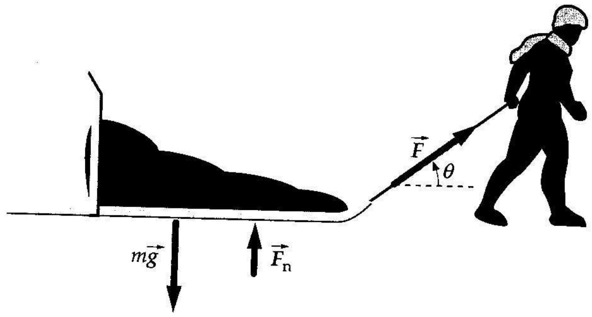
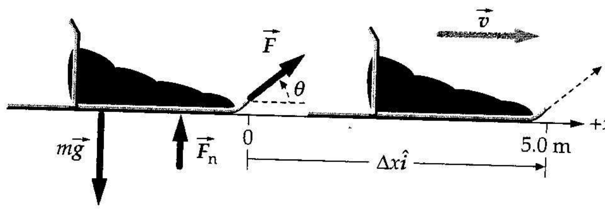
1



[redacted]



[redacted]

(*a*) 1. Sketch the sled both in its initial position and in its position after moving the 5.0 m. Draw the x axis in the direction of the motion [redacted].

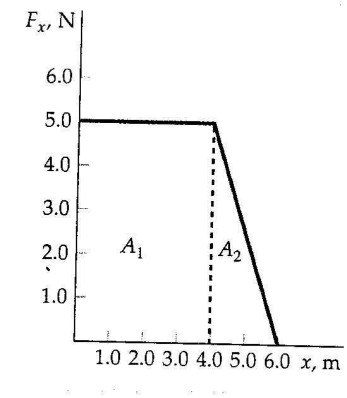
2. The work done by you on the sled is $F_x\,\Delta x$. This is the total work done on the sled. The other two forces each act perpendicular to the x direction [redacted], so they do zero work:

$$W_{\text{total}} = W_{\text{you}} = F_x\,\Delta x = F\cos\theta\,\Delta x$$
$$= (180\text{ N})(\cos 40°)(5.0\text{ m}) = 689\text{ J}$$
$$= \boxed{6.9 \times 10^2\text{ J}}$$

(*b*) Apply the work–kinetic-energy theorem to the sled and solve for the final speed:

$$W_{\text{total}} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$
$$v_f^2 = v_i^2 + \frac{2W_{\text{total}}}{m}$$
$$= 0 + \frac{2(689\text{ J})}{80\text{ kg}} = 17.2\text{ m}^2/\text{s}^2$$
$$v_f = \sqrt{17.2\text{ m}^2/\text{s}^2} = 4.151\text{ m/s} = \boxed{4.2\text{ m/s}}$$

2



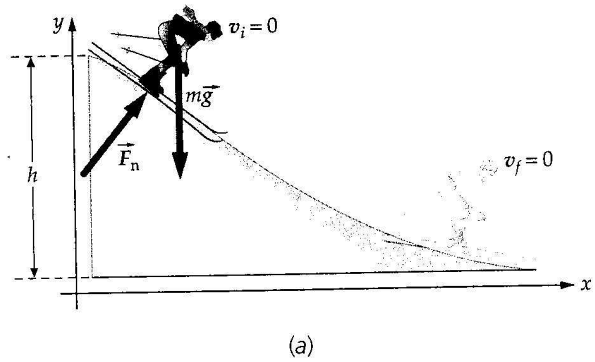
[redacted]

1. We find the work done by calculating the area under the F_x-versus-x curve:

$$W = A_{\text{total}}$$

2. This area is the sum of the two areas shown. The area of a triangle is one half the altitude times the base:

$$W = A_{\text{total}} = A_1 + A_2$$
$$= (5.0\text{ N})(4.0\text{ m}) + \tfrac{1}{2}(5.0\text{ N})(2.0\text{ m})$$
$$= 20\text{ J} + 5.0\text{ J} = \boxed{25\text{ J}}$$

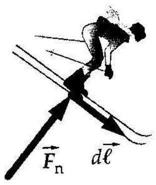


(a)

1. Make a sketch of yourself and draw the two force vectors on the sketch (Figure 6-23a). Also include coordinate axes. The work–kinetic-energy theorem, with $v_1 = 0$, relates the final speed v_f to the total work.

2. The final speed is related to the final kinetic energy, which in turn is related to the total work by the work–kinetic-energy theorem:
   $W_{total} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

3. For each of you, the total work is the work done by the normal force plus the work done by the gravitational force:
   $W_{total} = W_n + W_g$



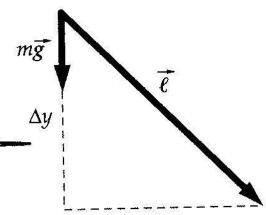
(b) [redacted]

4. The force $m\vec{g}$ on you is constant, but the force $\vec{F}_n$ is not constant. First we calculate the work done by $\vec{F}_n$. Calculate the work dW_n done on you by $\vec{F}_n$ for an infinitesimal displacement $d\vec{\ell}$ [redacted] at an arbitrary location along the run:
   $dW_n = \vec{F}_n \cdot d\vec{\ell} = F_n \cos\phi \, d\ell$

5. Find the angle ϕ between the directions of $\vec{F}_n$ and $d\vec{\ell}$. The displacement $d\vec{\ell}$ is tangent to the slope:
   $\phi = 90°$

6. Calculate the work done by $\vec{F}_n$ for the entire run:
   $W_n = \int F_n \cos 90° \, d\ell = \int (0)\, d\ell = 0$

7. The force of gravity $\vec{F}_g$ is constant, so the work done by gravity is $W_g = \vec{F}_g \cdot \vec{\ell}$, where $\vec{\ell}$ [redacted] is the net displacement from the top to the bottom of the lift:
   $W_g = m\vec{g} \cdot \vec{\ell} = -mg\hat{j} \cdot (\Delta x\hat{i} + \Delta y\hat{j})$
   $= -mg\,\Delta y$



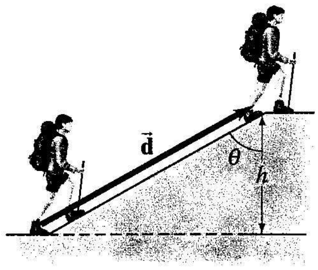
[redacted]

8. The skier is descending the hill, so Δy is negative. From Figure 6-23a, we see that $\Delta y = -h$:
   $\Delta y = -h$

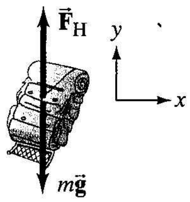
9. Substituting gives:
   $W_g = mgh$

10. Apply the work–kinetic-energy theorem to find v_f:
    $W_n + W_g = \Delta K$

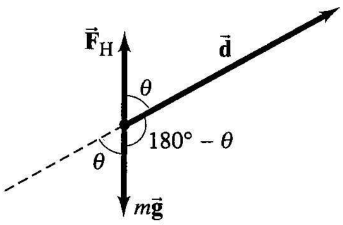
11. The final speed depends only on h, which is the same for both runs. Both of you will have the same final speeds.
    $0 + mgh = \frac{1}{2}mv_f^2 - 0$ so $v_f = \sqrt{2gh}$
    YOU WIN! (The bet was that she would not be going faster than you.)



(a)



(b)



(c)

1. **Draw a free-body diagram.** The forces on the backpack are shown in Fig. 6–4b: the force of gravity, $m\vec{\mathbf{g}}$, acting downward; and $\vec{\mathbf{F}}_H$, the force the hiker must exert upward to support the backpack. Since we assume there is negligible acceleration, horizontal forces on the backpack are negligible.
2. **Choose a coordinate system.** We are interested in the vertical motion of the backpack, so we choose the y coordinate as positive vertically upward.
3. **Apply Newton's laws.** Newton's second law applied in the vertical direction to the backpack gives

$$\Sigma F_y = ma_y$$
$$F_H - mg = 0.$$

Hence,

$$F_H = mg = (15.0\,\text{kg})(9.80\,\text{m/s}^2) = 147\,\text{N}.$$

4. **Find the work done by a specific force.** (*a*) To calculate the work done by the hiker on the backpack, we write Eq. 6–1 as

$$W_H = F_H(d\cos\theta),$$

and we note from Fig. 6–4a that $d\cos\theta = h$. So the work done by the hiker is

$$W_H = F_H(d\cos\theta) = F_H h = mgh$$
$$= (147\,\text{N})(10.0\,\text{m}) = 1470\,\text{J}.$$

Note that the work done depends only on the change in elevation and not on the angle of the hill, θ. The hiker would do the same work to lift the pack vertically the same height h.

(*b*) The work done by gravity on the backpack is (from Eq. 6–1 and Fig. 6–4c)

$$W_G = F_G d\cos(180° - \theta).$$

Since $\cos(180° - \theta) = -\cos\theta$, we have

$$W_G = F_G d(-\cos\theta) = mg(-d\cos\theta)$$
$$= -mgh$$
$$= -(15.0\,\text{kg})(9.80\,\text{m/s}^2)(10.0\,\text{m}) = -1470\,\text{J}.$$

NOTE The work done by gravity (which is negative here) doesn't depend on the angle of the incline, only on the vertical height h of the hill. This is because gravity acts vertically, so only the vertical component of displacement contributes to work done.

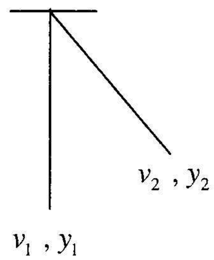
5. **Find the net work done.** (*a*) The *net* work done on the backpack is $W_{net} = 0$, since the net force on the backpack is zero (it is assumed not to accelerate significantly). We can also determine the net work done by adding the work done by each force:

$$W_{net} = W_G + W_H = -1470\,\text{J} + 1470\,\text{J} = 0.$$

NOTE Even though the *net* work done by all the forces on the backpack is zero, the hiker does do work on the backpack equal to 1470 J.

(5)

The only forces acting on Jane are gravity and the vine tension. The tension pulls in a centripetal direction, and so can do no work– the tension force is perpendicular at all times to her motion. So Jane's mechanical energy is conserved. Subscript 1 represents Jane at the point where she grabs the vine, and subscript 2 represents Jane at the highest point of her swing. The ground is the zero location for PE $(y=0)$. We have $v_1 = 5.3\,\text{m/s}$, $y_1 = 0$, and $v_2 = 0$ (top of swing). Solve for y_2, the height of her swing.



$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \;\rightarrow\; \tfrac{1}{2}mv_1^2 + 0 = 0 + mgy_2 \;\rightarrow$$

$$y_2 = \frac{v_1^2}{2g} = \frac{(5.3\,\text{m/s})^2}{2(9.8\,\text{m/s}^2)} = \boxed{1.4\text{ m}}$$

No, the length of the vine does not enter into the calculation, unless the vine is less than 0.7 m long. If that were the case, she could not rise 1.4 m high. Instead she would wrap the vine around the tree branch.

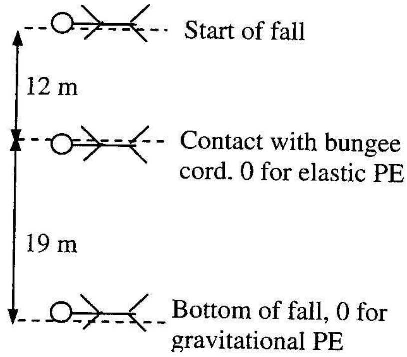
(6)

The block-spring combination is assumed to initially be at equilibrium, so the spring is neither stretched nor unstretched. At the release point, the speed of the mass is 0, and so the initial energy is all PE, given by $\tfrac{1}{2}kx_0^2$. That is the total energy of the system. Thus the energy of the system when the block is at a general location with some non-zero speed will still have this same total energy value. This is expressed by $E_{\text{total}} = \boxed{\tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kx_0^2}$.

(7)

Consider this diagram for the jumper's fall.

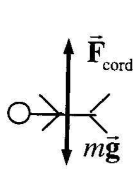
(*a*) The mechanical energy of the jumper is conserved. Use y for the distance from the 0 of gravitational PE and x for the amount of bungee cord "stretch" from its unstretched length. Subscript 1 represents the jumper at the start of the fall, and subscript 2 represents the jumper at the lowest point of the fall. The bottom of the fall is the zero location for gravitational PE $(y=0)$, and the location where the bungee cord just starts to be stretched is the zero location for elastic PE $(x=0)$. We have $v_1 = 0$, $y_1 = 31\text{ m}$, $x_1 = 0$, $v_2 = 0$, $y_2 = 0$, and $x_2 = 19\text{ m}$. Apply conservation of energy.



$$E_1 = E_2 \;\rightarrow\; \tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}kx_2^2 \;\rightarrow\; mgy_1 = \tfrac{1}{2}kx_2^2 \;\rightarrow$$

$$k = \frac{2mgy_1}{x_2^2} = \frac{2(62\text{ kg})(9.8\,\text{m/s}^2)(31\text{ m})}{(19\text{ m})^2} = 104.4\,\text{N/m} \approx \boxed{1.0\times10^2\,\text{N/m}}$$

(*b*) The maximum acceleration occurs at the location of the maximum force, which occurs when the bungee cord has its maximum stretch, at the bottom of the fall. Write Newton's 2nd law for the force on the jumper, with upward as positive.

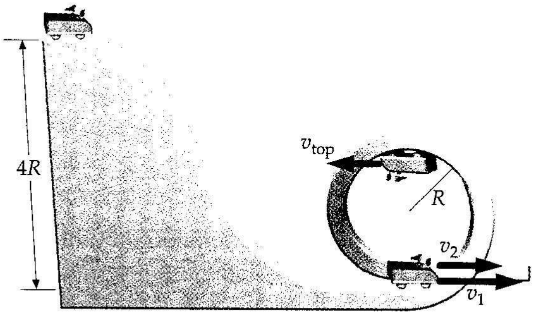


$$F_{\text{net}} = F_{\text{cord}} - mg = kx_2 - mg = ma \;\rightarrow$$

$$a = \frac{kx_2}{m} - g = \frac{(104.4\,\text{N/m})(19\text{ m})}{(62\text{ kg})} - 9.8\,\text{m/s}^2 = 22.2\,\text{m/s}^2 \approx \boxed{22\,\text{m/s}^2}$$

For (c) and (d)
See slides –

8



[redacted]

1. Choose the system to be the car, its contents, the track, and Earth. Draw a picture of the car and track, with the car at the starting point, at the bottom of the track, and again at the top of the loop [redacted]

2. Apply Newton's second law to relate the speed at the top of the loop to the normal force:
$$F_n + mg = m\frac{v_{top}^2}{R}$$

3. Apply the work–energy theorem to the interval prior to impact. There are no external forces and no internal nonconservative forces do work. Find the speed just prior to impact. Measuring heights from the bottom of the loop, the initial height of $4R$, where R is the radius of the loop, is two times the height of the top of the loop:
$$W_{ext} = \Delta E_{mech} - W_{nc}$$
$$0 = \Delta E_{mech} - 0$$
$$\therefore E_{mech\,f} = E_{mech\,i}$$
$$U_0 + K_0 = U_1 + K_1$$
$$mg\,4R + 0 = 0 + \tfrac{1}{2}mv_1^2$$
$$\text{so} \quad v_1 = \sqrt{8Rg}$$

4. The impact with the sandbag results in a 25 percent decrease in speed. Find the speed after impact:
$$v_2 = 0.75\,v_1 = 0.75\sqrt{8Rg}$$

5. Apply the work–energy theorem to the interval following impact. Find the speed at the top of the loop-the-loop:
$$U_{top} + K_{top} = U_2 + K_2$$
$$mg\,2R + \tfrac{1}{2}mv_{top}^2 = 0 + \tfrac{1}{2}m(0.75^2 \cdot 8Rg)$$
$$\text{so} \quad v_{top}^2 = (0.75^2 \cdot 8 - 4)\,Rg = 0.5Rg$$

6. Substituting for v_{top}^2 in the step-2 result gives:
$$F_n + mg = m\frac{0.5Rg}{R}$$
$$F_n + mg = 0.5mg$$

7. Solve for F_n:
$$F_n = -0.5mg$$

8. F_n is the magnitude of the normal force. It cannot be negative:

Oops! The car has left the track.

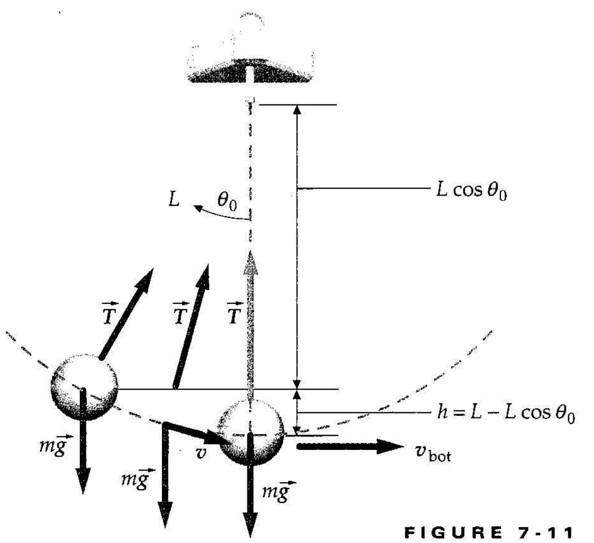


FIGURE 7-11

(*a*) 1. Make a sketch of the system in its initial and final configurations (Figure 7-11). We choose $y = 0$ at the bottom of the swing and $y = h$ at the initial position:

2. The external work done on the system equals the change in its mechanical energy minus the work done by internal nonconservative forces (Equation 7-10):

$$W_{ext} = \Delta E_{mech} - W_{nc}$$

3. There are no external forces acting on the system. The tension force is an internal nonconservative force:

$$W_{ext} = 0$$

$$W_{nc} = \int_1^2 \vec{T}\cdot d\vec{\ell}$$

4. The displacement increment $d\vec{\ell}$ equals the velocity times the time increment dt. Substitute into the step-3 result. The tension is perpendicular to the velocity, so $\vec{T}\cdot\vec{v} = 0$:

$$d\vec{\ell} = \vec{v}\,dt$$

$$\text{so}\quad W_{nc} = \int_1^2 \vec{T}\cdot d\vec{\ell} = \int_1^2 \vec{T}\cdot\vec{v}\,dt = 0$$

5. Substitute for W_{ext} and W_{nc} in the step-2 result. The bob initially is at rest:

$$W_{ext} = \Delta E_{mech} - W_{nc}$$

$$0 = \Delta E_{mech} - 0$$

$$\Delta E_{mech} = 0$$

6. Apply conservation of mechanical energy. The bob initially is at rest:

$$E_{mech\,f} = E_{mech\,i}$$

$$\tfrac{1}{2}mv_f^2 + mgy_f = \tfrac{1}{2}mv_i^2 + mgy_i$$

$$\tfrac{1}{2}mv_{bot}^2 + 0 = 0 + mgh$$

7. Conservation of mechanical energy thus relates the speed v_{bot} to the initial height $y_i = h$:

$$\tfrac{1}{2}mv_{bot}^2 = mgh$$

8. Solve for the speed v_{bot}:

$$v_{bot} = \sqrt{2gh}$$

9. To express speed in terms of the initial angle θ_0, we need to relate h to θ_0. This relation is illustrated in Figure 7-11:

$$L = L\cos\theta_0 + h$$

$$\text{so}\quad h = L - L\cos\theta_0 = L(1 - \cos\theta_0)$$

10. Substitute this value for h to express the speed at the bottom in terms of θ_0:

$$v_{bot} = \boxed{\sqrt{2gL(1 - \cos\theta_0)}}$$

(*b*) 1. When the bob is at the bottom of the circle, the forces on it are $m\vec{g}$ and $\vec{T}$. Apply $\Sigma F_y = ma_y$:

$$T - mg = ma_y$$

2. At the bottom, the bob has an acceleration v_{bot}^2/L in the centripetal direction (toward the center of the circle), which is upward:

$$a_y = \frac{v_{bot}^2}{L} = \frac{2gL(1 - \cos\theta_0)}{L} = 2g(1 - \cos\theta_0)$$

3. Substitute for a_y in the Part-(*b*), step-1 result and solve for T:

$$T = mg + ma_y = m(g + a_y) = m[g + 2g(1 - \cos\theta_0)]$$

$$= \boxed{(3 - 2\cos\theta_0)mg}$$

10

If the original spring is stretched a distance x from equilibrium, then the potential energy stored is $PE_{\text{full}} = \frac{1}{2}kx^2$. Alternatively, think of the original spring as being made up of the two halves of the spring, connected from end to end. Each half of the spring has a spring constant k', to be determined. As the spring is stretched a distance x, each half-spring is stretched a distance $x/2$. Each half-spring will have an amount of potential energy stored of $PE_{\text{half}} = \frac{1}{2}k'(x/2)^2$. The amount of energy in the two half-springs must equal the amount of energy in the full spring.

$$PE_{\text{full}} = 2PE_{\text{half}} \rightarrow \frac{1}{2}kx^2 = 2\left[\frac{1}{2}k'(x/2)^2\right] \rightarrow k' = \boxed{2k}$$

11

(a) The tension in the cord is perpendicular to the path at all times, and so the tension in the cord does not do any work on the ball. Thus the mechanical energy of the ball is conserved. Subscript 1 represents the ball when it is horizontal, and subscript 2 represents the ball at the lowest point on its path. The lowest point on the path is the zero location for PE $(y = 0)$. We have $v_1 = 0$, $y_1 = L$, and $y_2 = 0$. Solve for v_2.

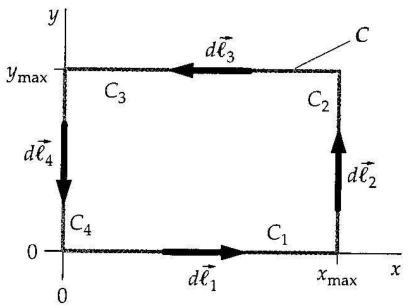
$$E_1 = E_2 \rightarrow \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \rightarrow mgL = \frac{1}{2}mv_2^2 \rightarrow v_2 = \boxed{\sqrt{2gL}}$$

(b) Use conservation of energy, to relate points 2 and 3. Point 2 is as described above. Subscript 3 represents the ball at the top of its circular path around the peg. The lowest point on the path is the zero location for PE $(y = 0)$. We have $v_1 = \sqrt{2gL}$, $y_1 = 0$, and $y_2 = 2(L-h) = 2(L - 0.80L) = 0.40L$. Solve for v_2.

$$E_2 = E_3 \rightarrow \frac{1}{2}mv_2^2 + mgy_2 = \frac{1}{2}mv_3^2 + mgy_3 \rightarrow \frac{1}{2}m(2gL) = \frac{1}{2}mv_2^2 + mg(0.40L) \rightarrow$$

$$\boxed{v_2 = \sqrt{1.2gL}}$$

12



[redacted]

1. The integral around C is equal to the sum of the integrals along the segments that make up C:

$$\oint_C \vec{F}\cdot d\vec{\ell} = \int_{C_1} \vec{F}\cdot d\vec{\ell}_1 + \int_{C_2} \vec{F}\cdot d\vec{\ell}_2 + \int_{C_3} \vec{F}\cdot d\vec{\ell}_3 + \int_{C_4} \vec{F}\cdot d\vec{\ell}_4$$

2. On C_1, $dy = 0$, so $d\vec{\ell}_1 = dx\hat{i}$:

$$\int_{C_1} \vec{F}\cdot d\vec{\ell}_1 = \int_0^{x_{max}} Ax\hat{i}\cdot dx\hat{i} = A\int_0^{x_{max}} x\,dx = \frac{1}{2}Ax_{max}^2$$

3. On C_2, $dx = 0$ and $x = x_{max}$, so $d\vec{\ell}_2 = dy\hat{j}$ and $\vec{F} = Ax_{max}\hat{i}$:

$$\int_{C_2} \vec{F}\cdot d\vec{\ell}_2 = \int_0^{y_{max}} Ax_{max}\hat{i}\cdot dy\hat{j} = Ax_{max}\int_0^{y_{max}} \hat{i}\cdot\hat{j}\,dy = 0$$

($\hat{i}\cdot\hat{j} = 0$ because $\hat{i}$ and $\hat{j}$ are perpendicular.)

4. On C_3, $dy = 0$, so $d\vec{\ell}_3 = dx\hat{i}$:

$$\int_{C_3} \vec{F}\cdot d\vec{\ell}_3 = \int_{x_{max}}^0 Ax\hat{i}\cdot dx\hat{i} = -A\int_0^{x_{max}} x\,dx = -\frac{1}{2}Ax_{max}^2$$

5. On C_4, $dx = 0$ and $x = 0$, so $d\vec{\ell}_4 = dy\hat{j}$ and $\vec{F} = 0$:

$$\int_{C_4} \vec{F}\cdot d\vec{\ell}_4 = \int_{y_{max}}^0 0\hat{i}\cdot dy\hat{j} = 0$$

6. Add the step-2, -3, -4, and -5 results:

$$\oint_C \vec{F}\cdot d\vec{\ell} = \frac{1}{2}Ax_{max}^2 + 0 - \frac{1}{2}Ax_{max}^2 + 0 = \boxed{0}$$