PROBLEMS 3

(1)

To find the acceleration, we use [redacted] with $v_0 = 0$:

$$\Delta x = v_0 t + \tfrac{1}{2}a_x t^2 = 0 + \tfrac{1}{2}a_x t^2$$

$$a_x = \frac{2\Delta x}{t^2} = \frac{2(2.25\ \text{m})}{(3.0\ \text{s})^2} = 0.50\ \text{m/s}^2$$

$$\vec{a} = a_x\hat{i} = 0.50\ \text{m/s}^2\hat{i}$$

(2)

(i) 1. Write the general equation for the position vector $\vec{r}$ as a function of time t for constant acceleration $\vec{a}$ in terms of $\vec{r}_0$, $\vec{v}_0$, and $\vec{a}$, and substitute $\vec{r}_0 = \vec{v}_0 = 0$.

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2 = 0 + 0 + \tfrac{1}{2}\vec{a}t^2 = \tfrac{1}{2}\vec{a}t^2$$

2. Use $\Sigma\vec{F} = m\vec{a}$ to write the acceleration $\vec{a}$ in terms of the resultant force $\Sigma\vec{F}$ and the mass m.

$$\vec{a} = \frac{\Sigma\vec{F}}{m}$$

3. Compute $\Sigma\vec{F}$ from the given forces.

$$\begin{aligned}\Sigma\vec{F} &= \vec{F}_1 + \vec{F}_2\\ &= (-2.00\ \text{N}\ \hat{i} - 4.00\ \text{N}\ \hat{j}) + (-2.60\ \text{N}\ \hat{i} + 5.00\ \text{N}\ \hat{j})\\ &= -4.60\ \text{N}\ \hat{i} + 1.00\ \text{N}\ \hat{j}\end{aligned}$$

4. Find the acceleration $\vec{a}$.

$$\vec{a} = \frac{\Sigma\vec{F}}{m} = -11.5\ \text{m/s}^2\ \hat{i} + 2.50\ \text{m/s}^2\ \hat{j}$$

5. Find the position $\vec{r}$ for a general time t.

$$\vec{r} = \tfrac{1}{2}\vec{a}t^2 = \tfrac{1}{2}a_x t^2\hat{i} + \tfrac{1}{2}a_y t^2\hat{j} = (-5.75\ \text{m/s}^2\ \hat{i} + 1.25\ \text{m/s}^2\ \hat{j})t^2$$

6. Find $\vec{r}$ at $t = 1.60\,\text{s}$.

$$\vec{r} = \boxed{-14.7\ \text{m}\ \hat{i} + 3.20\ \text{m}\ \hat{j}}$$

(ii) Write the velocity $\vec{v}$ by taking the time derivative of the step-5 result. Evaluate the velocity at $t = 1.6\,\text{s}$.

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = 2(-5.75\ \text{m/s}^2\ \hat{i} + 1.25\ \text{m/s}^2\ \hat{j})t$$

$$\vec{v}(1.6\ \text{s}) = \boxed{-18.4\ \text{m/s}\ \hat{i} + 4.00\ \text{m/s}\ \hat{j}}$$

(3)

1. The free-fall acceleration is given by $a = F_g/m$:

$$a = \frac{F_g}{m} = \frac{GmM_E/r^2}{m} = \frac{GM_E}{r^2}$$

2. The distance r is related to the radius of Earth R_E and the altitude h:

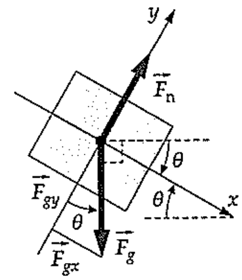
$$r = R_E + h = 6370\ \text{km} + 400\ \text{km} = 6770\ \text{km}$$

3. The acceleration is then:

$$a = \frac{GM_E}{r^2} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\ \text{kg})}{(6.77\times10^{6}\ \text{m})^2} = \boxed{8.70\ \text{m/s}^2}$$

④

1. First we draw a free-body diagram. Two forces act on the package, the gravitational force and the normal force. We choose the direction of the acceleration, down the ramp, as the $+x$ direction. *Note:* The angle between $\vec{F}_g$ and the $-y$ direction is the same as the angle between the horizontal and the incline as we see from the free-body diagram. We can also see that $F_{gx} = F_g \sin\theta$.



2. To find a_x we apply Newton's second law ($\Sigma F_x = ma_x$) to the package. (*Note:* $\vec{F}_n$ is perpendicular to the x axis and $F_g = mg$.) $\quad F_{nx} + F_{gx} = ma_x$ where $F_{nx} = 0$ and $F_{gx} = F_g \sin\theta = mg\sin\theta$

3. Substituting and solving for the acceleration gives: $\quad 0 + mg\sin\theta = ma_x$ so $a_x = g\sin\theta$

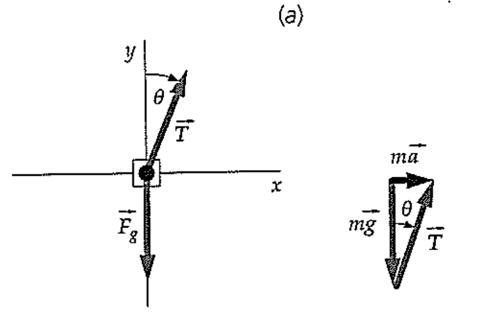
4. Relate the downward component of the velocity of the box to its velocity component v_x in the x direction: $\quad v_d = v_x \sin\theta$

5. The velocity component v_x is related to the displacement Δx along the ramp by the kinematic equation: $\quad v_x^2 = v_{0,x}^2 + 2a_x\,\Delta x$

6. Substituting for a_x in the kinematic equation (step 5) and setting $v_{0,x}$ to zero gives: $\quad v_x^2 = 2g\sin\theta\,\Delta x$

⑤

(a)



(b) (c)

(*a*) 1. Draw a free-body diagram for the yo-yo. Choose the $+x$ direction to be the direction of the yo-yo's acceleration vector.

2. Apply $\Sigma F_x = ma_x$ to the yo-yo. Then simplify using trigonometry:

$$T_x + F_{gx} = ma_x$$
$$T\sin\theta + 0 = ma_x$$
or
$$T\sin\theta = ma_x$$

3. Apply $\Sigma F_y = ma_y$ to the yo-yo. Then, simplify using trigonometry and $F_g = mg$. Since the acceleration is in the $+x$ direction, $a_y = 0$:

$$T_y + F_{gy} = ma_y$$
$$T\cos\theta - mg = 0$$
or
$$T\cos\theta = mg$$

4. Divide the step-2 result by the step 3 result and solve for the acceleration. Because the acceleration vector is in the $+x$ direction, $a = a_x$:

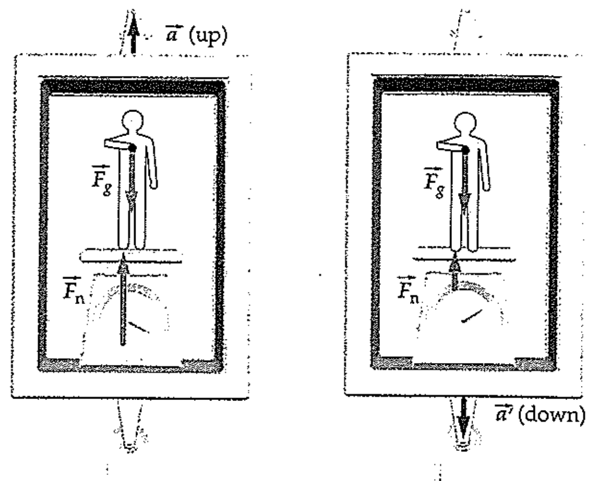
$$\frac{T\sin\theta}{T\cos\theta} = \frac{ma_x}{mg} \quad \text{so} \quad \tan\theta = \frac{a_x}{g} \quad \text{and}$$
$$a_x = g\tan\theta = (9.81\ \text{m/s}^2)\tan 22.0° = \boxed{3.96\ \text{m/s}^2}$$

(*b*) Using the step-3 result, solve for the tension:

$$T = \frac{mg}{\cos\theta} = \frac{(0.0400\ \text{kg})(9.81\ \text{m/s}^2)}{\cos 22.0°} = \boxed{0.423\ \text{N}}$$

6

A



(a) (b)

(a) 1. Draw a free-body diagram of yourself ~~(Figure 4-22)~~:

2. Apply $\Sigma F_y = ma_y$: $\quad F_{ny} + F_{gy} = ma_y$

$F_n - mg = ma_y$

3. Solve for F_n. This is the reading on the scale (your apparent weight): $\quad F_n = mg + ma_y = m(g + a_y)$

4. $a_y = +a$: $\quad F_n = \boxed{m(g + a)}$

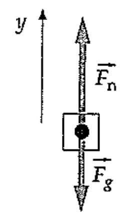
~~Figure 4-22~~

(b) $a_y = -a'$. Substitute for a_y in the Part-(a), step-3 result: $\quad F_n = m(g + a_y) = \boxed{m(g - a')}$

(c) The velocity is positive but decreasing, so the acceleration is negative. Thus, $a_y = -8.0\,\text{m/s}^2$. Substitute into the Part-(a), step-3 result:

$F_n = m(g + a_y) = (80\text{ kg})(9.81\text{ m/s}^2 - 8.0\text{ m/s}^2)$

$= 144.8\text{ N} = \boxed{1.40 \times 10^2\text{ N}}$



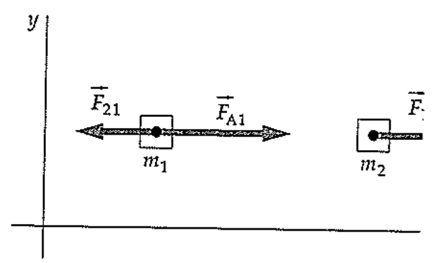
7

(a) 1. Draw free-body diagrams for the two boxes ~~(Figure 4-32)~~.

2. Apply $\Sigma \vec{F} = m\vec{a}$ to box 1. $\quad F_{A1} - F_{21} = m_1 a_{1x}$

3. Apply $\Sigma \vec{F} = m\vec{a}$ to box 2. $\quad F_{12} = m_2 a_{2x}$

4. Express both the relation between the two accelerations and the relation between the magnitudes of the forces the blocks exert on each other. The accelerations are equal because the speeds are equal at all times, so the rate of change of the speeds are equal. The forces are equal in magnitude because the forces constitute a N3L force pair: $\quad a_{2x} = a_{1x} = a_x$, $\quad F_{21} = F_{12} = F$

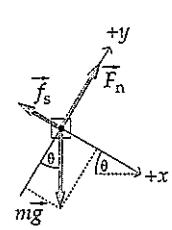


5. Substitute these back into the step-2 and step-3 results and solve for a_x. $\quad a_x = \boxed{\dfrac{F_{A1}}{m_1 + m_2}}$

(b) Substitute your expression for a_x into either the step-2 or the step-3 result and solve for F. $\quad F = \boxed{\dfrac{m_2}{m_1 + m_2} F_{A1}}$

8

. Draw a free-body diagram for the coin when the book cover inclined at angle θ, where $\theta \leq \theta_{max}$ (Figure 5-7). Draw the y axis normal to the book cover:



.. The coefficient of static friction relates the frictional and normal forces: $\quad f_s \leq \mu_s F_n$

i. We apply $\Sigma F_y = ma_y$ to the coin and solve for the normal force: $\quad \Sigma F_y = ma_y$

$F_n - mg\cos\theta = 0 \Rightarrow F_n = mg\cos\theta$

l. Substitute for F_n in $f_s \leq \mu_s F_n$ (Equation 5-1): $\quad f_s \leq \mu_s F_n \Rightarrow f_s \leq \mu_s mg\cos\theta$

5. Apply $\Sigma F_x = ma_x$ to the coin. Then solve for the friction force: $\quad \Sigma F_x = ma_x$

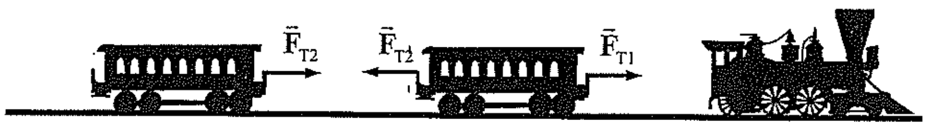
$-f_s + mg\sin\theta = 0 \Rightarrow f_s = mg\sin\theta$

5. Substituting $mg\sin\theta$ for f_s in the step-4 result gives: $\quad mg\sin\theta \leq \mu_s mg\cos\theta \Rightarrow \tan\theta \leq \mu_s$

7. θ_{max}, the largest angle satisfying the condition $\tan\theta \leq \mu_s$, is the largest angle such that the coin does not slide: $\quad \boxed{\mu_s = \tan\theta_{max}}$

9

Since all forces of interest in this problem are horizontal, draw the free-body diagram showing only the horizontal forces. $\vec{\mathbf{F}}_{T1}$ is the tension in the coupling between the locomotive and the first car, and it pulls to the right on the first car. $\vec{\mathbf{F}}_{T2}$ is the tension in the coupling between the first car an the second car. It pulls to the right on car 2, labeled $\vec{\mathbf{F}}_{T2R}$ and to the left on car 1, labeled $\vec{\mathbf{F}}_{T2L}$. Both cars have the same mass m and the same acceleration a. Note that $\left|\vec{\mathbf{F}}_{T2R}\right| = \left|\vec{\mathbf{F}}_{T2L}\right| = F_{T2}$ by Newton's 3rd law.



Write a Newton's 2nd law expression for each car.

$$\sum F_1 = F_{T1} - F_{T2} = ma \qquad \sum F_2 = F_{T2} = ma$$

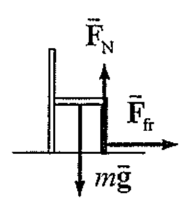
Substitute the expression for ma from the second expression into the first one.

$$F_{T1} - F_{T2} = ma = F_{T2} \rightarrow F_{T1} = 2F_{T2} \rightarrow \boxed{F_{T1}/F_{T2} = 2}$$

This can also be discussed in the sense that the tension between the locomotive and the first car is pulling 2 cars, while the tension between the cars is only pulling one car.

10

(a) Assume that the earthquake is moving the Earth to the right. If an object is to "hold its place", then the object must also be accelerating to the right with the Earth. The force that will accelerate that object will be the static frictional force, which would also have to be to the right. If the force were not large enough, the Earth would move out from under the chair somewhat, giving the appearance that the chair were being "thrown" to the left. Consider the free-body diagram shown for a chair on the floor. It is apparent that the normal force is equal to the weight since there is no motion in the vertical direction. Newton's 2nd law says that $F_{fr} = ma$. We also assume that the chair is just on the verge of slipping, which means that the static frictional force has its maximum value of $F_{fr} = \mu_s F_N = \mu_s mg$. Equate the two expressions for the frictional force to find the coefficient of friction.



$$ma = \mu_s mg \rightarrow \boxed{\mu_s = a/g}$$

If the static coefficient is larger than this, then there will be a larger maximum frictional force, and the static frictional force will be more than sufficient to hold the chair in place on the floor.

(b) For the 1989 quake, $\dfrac{a}{g} = \dfrac{4.0\,\mathrm{m/s^2}}{9.8\,\mathrm{m/s^2}} = 0.41$. Since $\mu_s = 0.25$, $\boxed{\text{the chair would slide}}$.