PROBLEMS №11

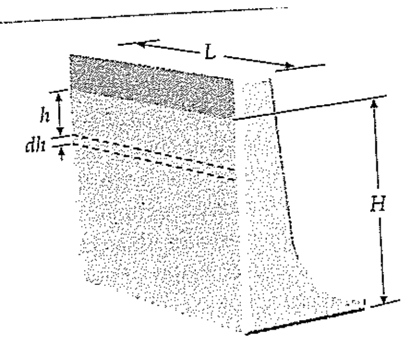
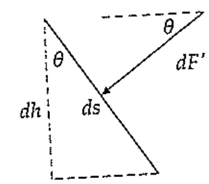


FIGURE 13-2

(1)

1. Express the force dF of the water on the element of length L and height dh in terms of the pressure $P_{at} + \rho hg$ on the dam by the water: $dF = P\,dA = (P_{at} + \rho gh)L\,dh$

2. Integrate from $h = 0$ to $h = H$ to find the horizontal component of the force of the water on the dam:
$$F = \int_{h=0}^{h=H} dF = \int_0^H (P_{at} + \rho gh)L\,dh = P_{at}LH + \frac{1}{2}\rho gLH^2$$



3. The downstream surface of the dam is not vertical. Sketch an edge-on view (Figure 13-3) of a horizontal strip across the downstream side of the surface, a strip of length L and width ds. Let dh be the height of the strip:

FIGURE 13-3

4. Relate the force dF' exerted on this strip by the air to the pressure of the air and the area of the strip: $dF' = P_{at}\,dA = P_{at}L\,ds$

5. Express the horizontal component of dF'_x in terms of dh: $dF'_x = dF \cos\theta = P_{at}L\,ds\cos\theta = P_{at}L\,dh$

6. Integrate from $h = 0$ to $h = H$ to find the horizontal component of the force of the air on the downstream side of the dam:
$$F' = \int_{h=0}^{h=H} dF' = \int_0^H P_{at}L\,dh = P_{at}LH$$

7. The net horizontal force on the dam is $F - F'_x$:
$$F - F'_x = (P_{at}LH + \tfrac{1}{2}\rho gLH^2) - P_{at}LH = \tfrac{1}{2}\rho gLH^2$$
$$= \tfrac{1}{2}(1000\text{ kg/m}^3)(9.81\text{ N/kg})(30\text{ m})(25\text{ m})^2$$
$$= 9.20 \times 10^7\text{ N} = \boxed{9.2 \times 10^7\text{ N}}$$

(2)

1. Using Equations 13-2 and 13-3, find the ratio of your body's density to the density of water:
$$\frac{\rho}{\rho_{water}} = \frac{F_g}{F_g - F_{g\,app}} = \frac{F_g}{F_g - 0.05F_g} = 1.05$$

2. Your total body volume equals the volume of fat plus the volume of lean tissue: $V_{tot} = V_{fat} + V_{lean}$

3. Because mass equals density times volume, volume equals mass divided by density. Substitute the corresponding mass-to-density ratio for each volume in the step-2 result:
$$\frac{m_{tot}}{\rho} = \frac{m_{fat}}{\rho_{fat}} + \frac{m_{lean}}{\rho_{lean}}$$

4. The mass of fat is $f_{fat}m_{tot}$, where f_{fat} is the fraction of fat, and the mass of lean is $f_{lean}m_{tot}$, where f_{lean} is the fraction of lean. Substitute for m_{fat} and m_{lean} in the step-3 result:
$$\frac{m_{tot}}{\rho} = \frac{f_{fat}m_{tot}}{\rho_{fat}} + \frac{f_{lean}m_{tot}}{\rho_{lean}}$$

5. The fraction of fat plus the fraction of lean tissue equals 1: $f_{fat} + f_{lean} = 1$

6. Divide both sides of the step-4 result by m_{tot} and substitute $1 - f_{fat}$ for f_{lean}:
$$\frac{1}{\rho} = \frac{f_{fat}}{\rho_{fat}} + \frac{(1 - f_{fat})}{\rho_{lean}}$$

7. Solve the step-6 result for f_{fat}:
$$f_{fat} = \frac{1 - (\rho_{lean}/\rho)}{1 - (\rho_{lean}/\rho_{fat})}$$

8. Using the step-1 result, substitute for ρ in the step-7 result and solve for f_{fat}:
$$f_{fat} = \frac{1 - (\rho_{lean}/1.05\rho_{water})}{1 - (\rho_{lean}/\rho_{fat})} = \frac{1 - (1.1/1.05)}{1 - (1.1/0.90)} = 0.21$$

9. Convert to a percentage: $100\% \times f_{fat} = \boxed{21\%}$

(3)

1. Because the iceberg is in equilibrium, the buoyant force equals its weight:
$$F_g = B$$
$$\rho_{IB}Vg = \rho_{SW}V_{sub}g$$

2. Solve for V_{sub}/V:
$$f = \frac{V_{sub}}{V} = \frac{\rho_{IB}}{\rho_{SW}} = \frac{0.92 \times 10^3\text{ kg/m}^3}{1.025 \times 10^3\text{ kg/m}^3} = 0.898 = \boxed{0.90}$$

4)

The pressure at points a and b are equal since they are the same height in the same fluid. If they were unequal, the fluid would flow. Calculate the pressure at both a and b, starting with atmospheric pressure at the top surface of each liquid, and then equate those pressures.

$$P_a = P_b \rightarrow P_0 + \rho_{oil} g h_{oil} = P_0 + \rho_{water} g h_{water} \rightarrow \rho_{oil} h_{oil} = \rho_{water} h_{water} \rightarrow$$

$$\rho_{oil} = \frac{\rho_{water} h_{water}}{h_{oil}} = \frac{\left(1.00\times10^3\ \mathrm{kg/m^3}\right)\left(0.272\ \mathrm{m} - 0.0941\ \mathrm{m}\right)}{\left(0.272\ \mathrm{m}\right)} = \boxed{6.54\times10^2\ \mathrm{kg/m^3}}$$

5

Use the equation of continuity (Eq. 10-4) to relate the volume flow of water at the two locations, and use Bernoulli's equation (Eq. 10-5) to relate the conditions at the street to those at the top floor. Express the pressures as atmospheric pressure plus gauge pressure.

$$A_{street} v_{street} = A_{top} v_{top} \rightarrow$$

$$v_{top} = v_{street} \frac{A_{street}}{A_{top}} = \left(0.60\ \mathrm{m/s}\right) \frac{\pi\left(5.0\times10^{-2}\ \mathrm{m}\right)^2}{\pi\left(2.6\times10^{-2}\ \mathrm{m}\right)^2} = 2.219\ \mathrm{m/s} \approx \boxed{2.2\ \mathrm{m/s}}$$

$$P_0 + P_{\substack{gauge \\ street}} + \tfrac{1}{2}\rho v_{street}^2 + \rho g y_{street} = P_0 + P_{\substack{gauge \\ top}} + \tfrac{1}{2}\rho v_{top}^2 + \rho g y_{top} \rightarrow$$

$$P_{\substack{gauge \\ top}} = P_{\substack{gauge \\ street}} + \tfrac{1}{2}\rho\left(v_{street}^2 - v_{top}^2\right) + \rho g y\left(y_{street} - y_{top}\right)$$

$$= \left(3.8\ \mathrm{atm}\right)\left(\frac{1.013\times10^5\ \mathrm{Pa}}{\mathrm{atm}}\right) + \tfrac{1}{2}\left(1.00\times10^3\ \mathrm{kg/m^3}\right)\left[\left(0.60\ \mathrm{m/s}\right)^2 - \left(2.219\ \mathrm{m/s}\right)^2\right]^2$$

$$+ \left(1.00\times10^3\ \mathrm{kg/m^3}\right)\left(9.8\ \mathrm{m/s^2}\right)\left(-18\ \mathrm{m}\right)$$

$$= 2.063\times10^5\ \mathrm{Pa}\left(\frac{1\ \mathrm{atm}}{1.013\times10^5\ \mathrm{Pa}}\right) \approx \boxed{2.0\ \mathrm{atm}}$$

6 See Slides

7 See Slides

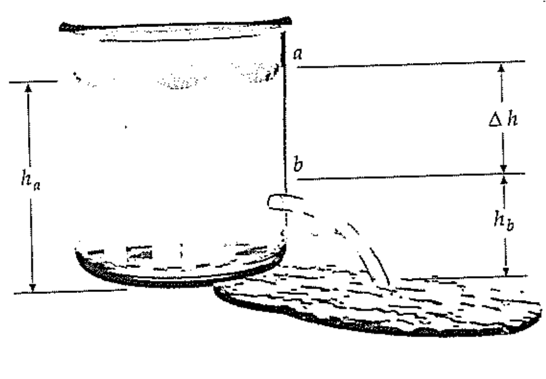


FIGURE 13-18

8

1. The Bernoulli equation with $v_a = 0$ gives: $P_a + \rho g h_a + 0 = P_b + \rho g h_b + \frac{1}{2}\rho v_b^2$
2. The pressure at point a and at point b is the same, P_{at}, because both points are open to the atmosphere: $P_a = P_{at}$ and $P_b = P_{at}$, so $P_{at} + \rho g h_a + 0 = P_{at} + \rho g h_b + \frac{1}{2}\rho v_b^2$
3. Solve the step-2 result for the speed v_b of the water flowing from the hole: $v_b^2 = 2g(h_a - h_b) = 2g\,\Delta h$, so $v_b = \boxed{\sqrt{2g\,\Delta h}}$

9 See Slides

10

The change in pressure with height is given by Eq. 10-3b.

$$\Delta P = \rho g \Delta h \rightarrow \frac{\Delta P}{P_0} = \frac{\rho g \Delta h}{P_0} = \frac{(1.05\times10^3\,\text{kg/m}^3)(9.80\,\text{m/s}^2)(6\,\text{m})}{1.013\times10^5\,\text{Pa}} = 0.609 \rightarrow$$

$$\boxed{\Delta P = 0.6\,\text{atm}}$$

11

The fluid pressure must be 18 torr higher than air pressure as it exits the needle, so that the blood will enter the vein. The pressure at the entrance to the needle must be higher than 18 torr, due to the viscosity of the blood. To produce that excess pressure, the blood reservoir is placed above the level of the needle. Use Poiseuille's equation to calculate the excess pressure needed due to the viscosity, and then use Eq. 10-3b to find the height of the blood reservoir necessary to produce that excess pressure.

$$Q = \frac{\pi R^4 (P_2 - P_1)}{8\eta_{\text{blood}} L} \rightarrow P_2 = P_1 + \frac{8\eta_{\text{blood}} L Q}{\pi R^4} = \rho_{\text{blood}} g \Delta h \rightarrow$$

$$\Delta h = \frac{1}{\rho_{\text{blood}} g}\left(P_1 + \frac{8\eta_{\text{blood}} L Q}{\pi R^4}\right)$$

$$= \frac{1}{\left(1.05\times10^3\,\frac{\text{kg}}{\text{m}^3}\right)(9.80\,\text{m/s}^2)}\left(\begin{array}{c}(18\,\text{mm-Hg})\left(\frac{133\,\text{N/m}^2}{1\,\text{mm-Hg}}\right) + \\ \frac{8(4\times10^{-3}\,\text{Pa}\cdot\text{s})(4.0\times10^{-2}\,\text{m})\left(\frac{4.0\times10^{-6}\,\text{m}^3}{60\,\text{s}}\right)}{\pi(0.20\times10^{-3}\,\text{m})^4}\end{array}\right)$$

$$= \boxed{1.8\,\text{m}}$$

12

The buoyant force, equal to the weight of mantle displaced, must be equal to the weight of the continent. Let h represent the full height of the continent, and y represent the height of the continent above the surrounding rock.

$$W_{\text{continent}} = W_{\substack{\text{displaced}\\\text{mantle}}} \rightarrow Ah\rho_{\text{continent}} g = A(h-y)\rho_{\text{mantle}} g \rightarrow$$

$$y = h\left(1 - \frac{\rho_{\text{continent}}}{\rho_{\text{mantle}}}\right) = (35\,\text{km})\left(1 - \frac{2800\,\text{kg/m}^3}{3300\,\text{kg/m}^3}\right) = \boxed{5.3\,\text{km}}$$

13

The upward force due to air pressure on the bottom of the wing must be equal to the weight of the airplane plus the downward force due to air pressure on the top of the wing. Bernoulli's equation can be used to relate the forces due to air pressure. We assume that there is no appreciable height difference between the top and the bottom of the wing.

$$P_{\text{top}} A + mg = P_{\text{bottom}} A \rightarrow (P_{\text{bottom}} - P_{\text{top}}) = \frac{mg}{A}$$

$$P_0 + P_{\text{bottom}} + \tfrac{1}{2}\rho v_{\text{bottom}}^2 + \rho g y_{\text{bottom}} = P_0 + P_{\text{top}} + \tfrac{1}{2}\rho v_{\text{top}}^2 + \rho g y_{\text{top}}$$

$$v_{\text{top}}^2 = \frac{2(P_{\text{bottom}} - P_{\text{top}})}{\rho} + v_{\text{bottom}}^2 \rightarrow$$

$$v_{\text{top}} = \sqrt{\frac{2(P_{\text{bottom}} - P_{\text{top}})}{\rho} + v_{\text{bottom}}^2} = \sqrt{\frac{2mg}{\rho A} + v_{\text{bottom}}^2} = \sqrt{\frac{2(2.0\times10^6\,\text{kg})(9.80\,\text{m/s}^2)}{(1.29\,\text{kg/m}^3)(1200\,\text{m}^2)} + (95\,\text{m/s})^2}$$

$$= 185.3\,\text{m/s} \approx \boxed{190\,\text{m/s}}$$

14

(a) The gauge pressure is given by Eq. 10-3a. The height is the height from the bottom of the hill to the top of the water tank.

$$P_G = \rho g h = \left(1.00\times10^3\ \mathrm{kg/m^3}\right)\left(9.80\ \mathrm{m/s^2}\right)\left[5.0\ \mathrm{m}+\left(110\ \mathrm{m}\right)\sin 58^\circ\right] = \boxed{9.6\times10^5\ \mathrm{N/m^2}}$$

(b) The water would be able to shoot up to the top of the tank (ignoring any friction).

$$h = 5.0\ \mathrm{m}+\left(110\ \mathrm{m}\right)\sin 58^\circ = \boxed{98\ \mathrm{m}}$$

15

Write Equation 13-26 for the Reynolds number, expressing each quantity in SI units:

$$N_R = \frac{2r\rho}{\eta} = \frac{2(0.010\ \mathrm{m})(1060\ \mathrm{kg/m^3})(0.30\ \mathrm{m/s})}{4.0\times10^{-3}\ \mathrm{Pa\cdot s}}$$

$$= 1590 = \boxed{1.6\times10^3}$$