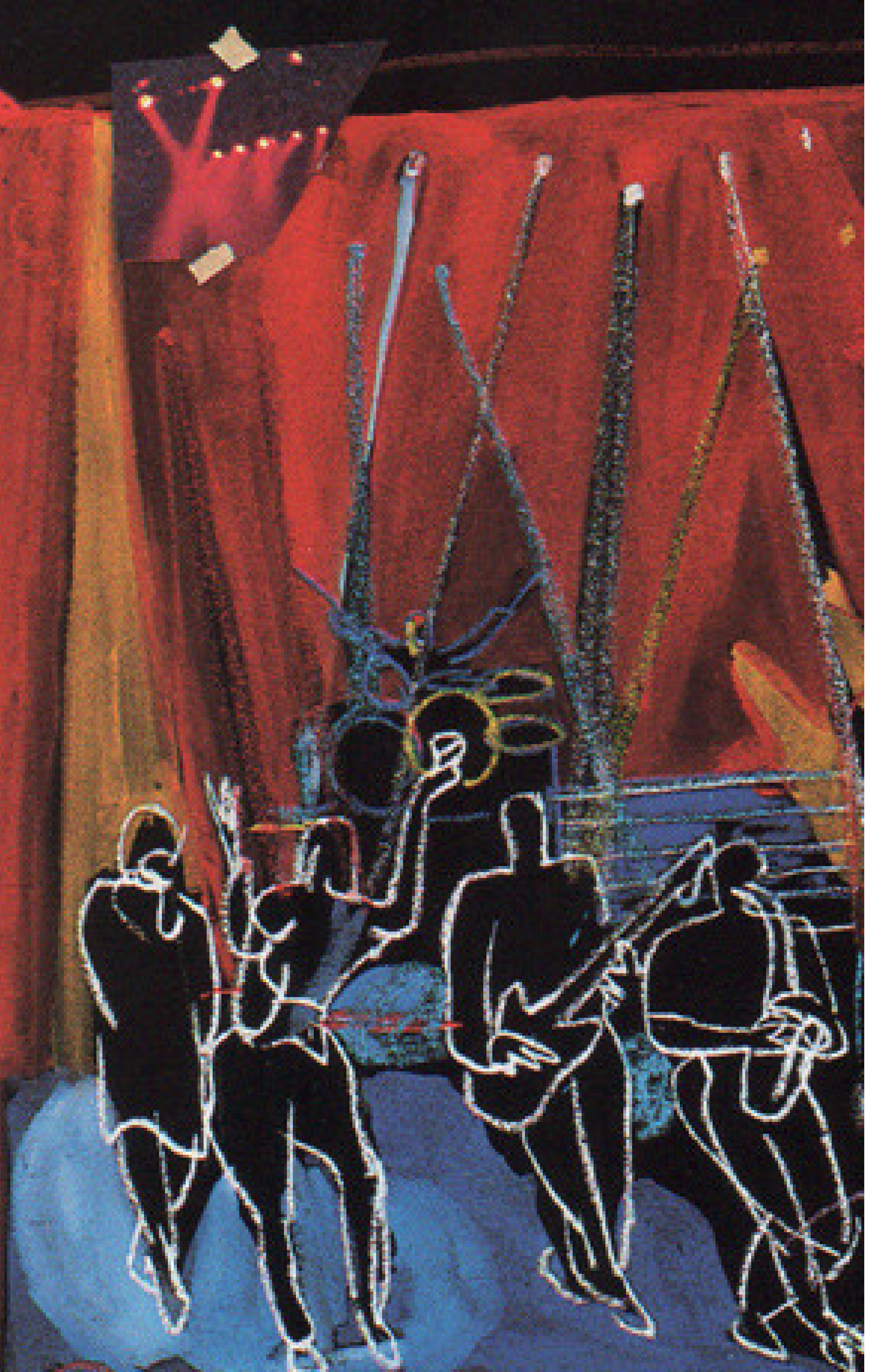


PHY-141

Standing Waves

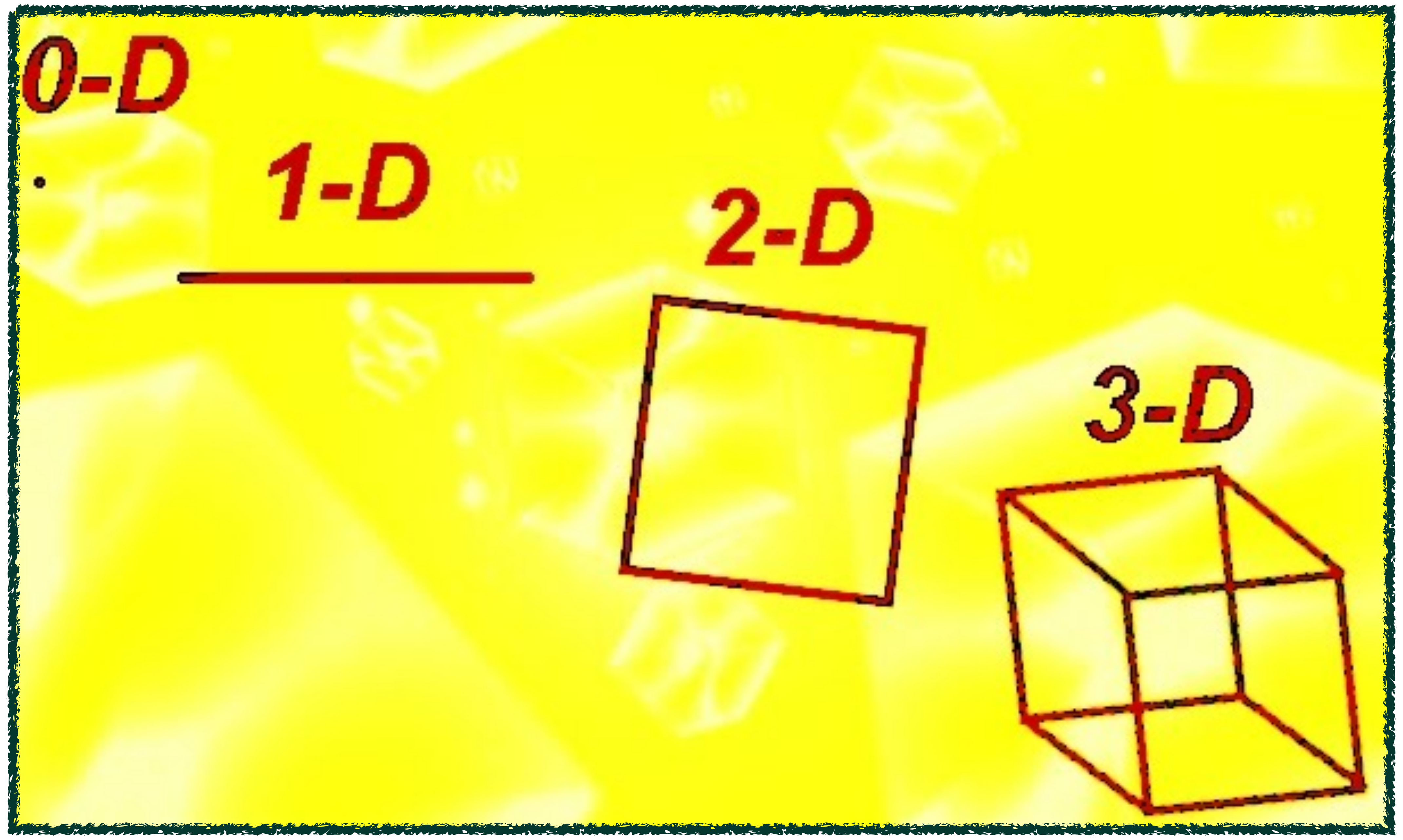
Luis Anchordoqui

Outline

- Up to now, we have been studying waves that propagate continuously through a medium, but we have not discussed what happens when waves encounter the boundary of the medium
- Waves do interact with boundaries of the medium, and all or part of the wave can be reflected
- For example, when you stand some distance from a rigid cliff face and yell, you can hear the sound waves reflect off the rigid surface as an echo
- The result of successive reflections is the formation of stationary (or standing) waves
- In today's class, we explore wave behavior at boundaries and the key characteristics of standing waves

From 3D to 1D

- Building on our previous mathematical models, we will now explore physical systems that behave as one-dimensional (1D) environments
- While everything in our world is technically 3D, we can simplify our description of objects that are significantly smaller in certain dimensions—much like treating a thin sheet of paper as 2D or a strand of hair as 1D



Strings and Cavities

We will focus on two primary examples to study wave propagation

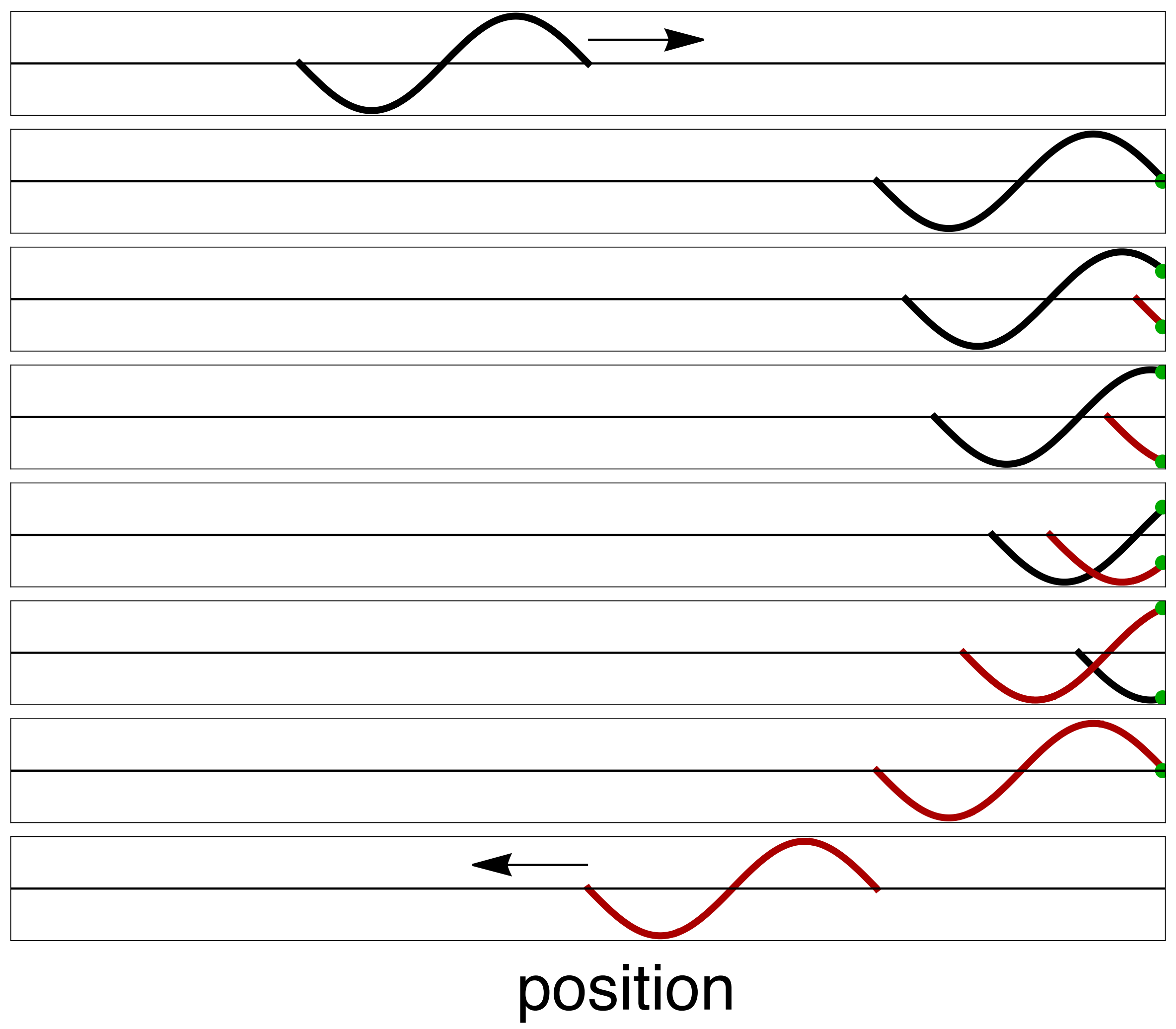
- Strings (solids) ☛ Long, thin systems like guitar strings
- Cavities (gases) ☛ Narrow enclosures filled with air, such as a flute

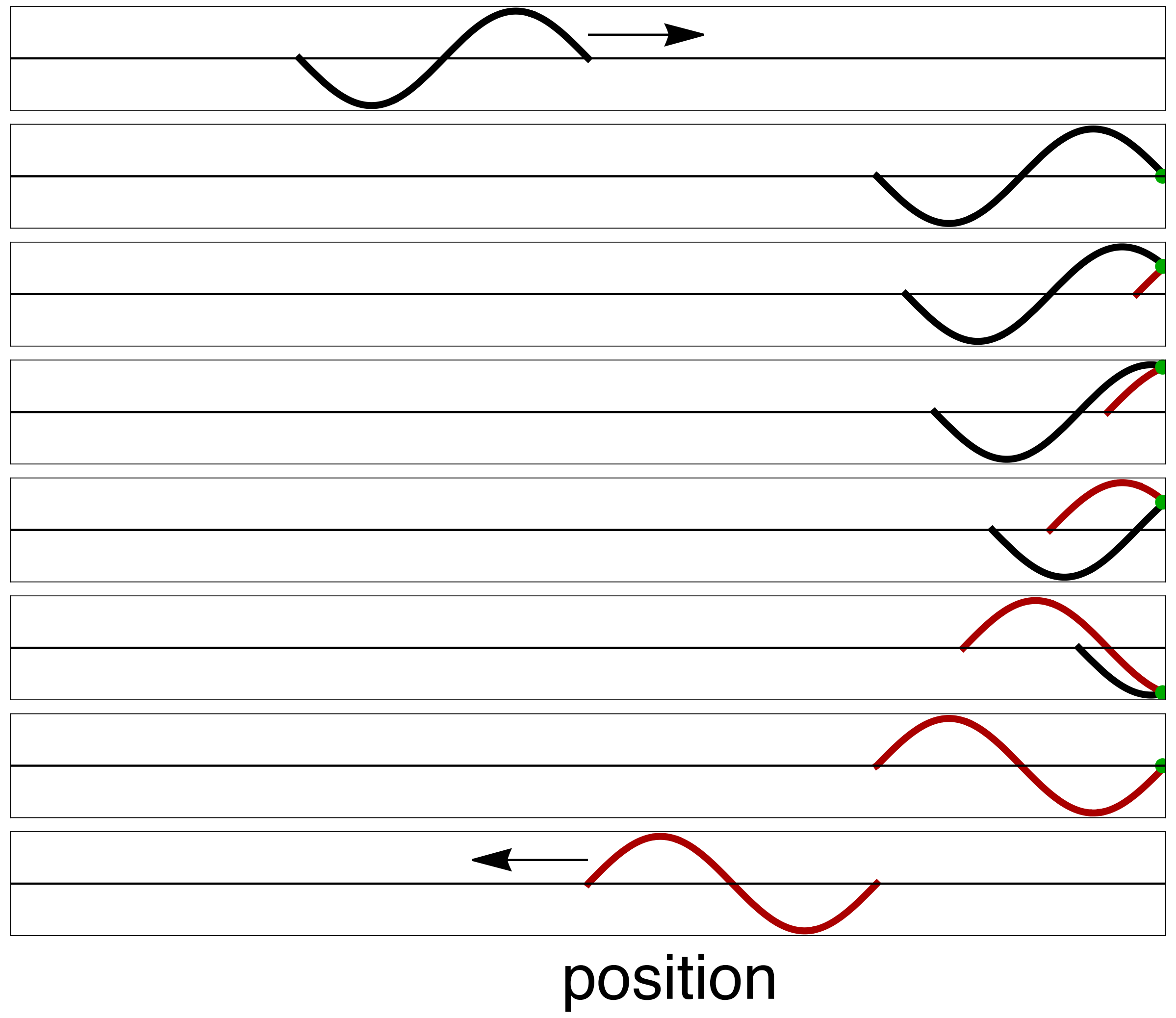
Flute as a 1D Object

- While the 1D approximation is slightly less perfect for cavities than for strings, both provide an excellent framework for understanding standing waves and how sound is produced in physical structures

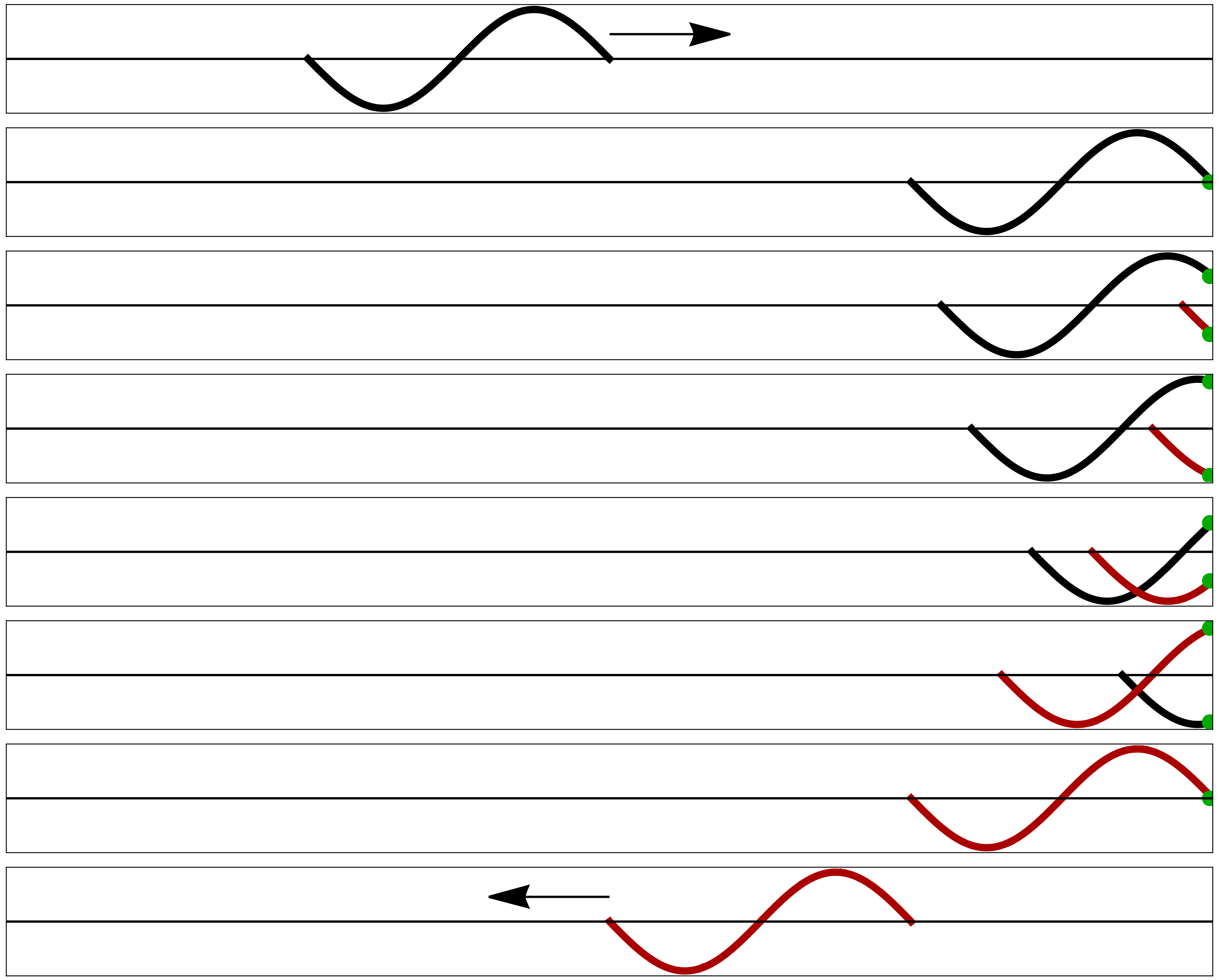
Phase Reversal

- When a pressure wave encounters a boundary, such as a wall or the open end of a tube, it reflects, traveling back in the opposite direction
- Beyond simply reversing direction, the reflected wave can differ from the initial wave depending on the boundary conditions
- To visualize this, imagine a short pulse consisting of a single compression followed by a rarefaction
- As the compression strikes a reflective surface, the local density increases, creating a new, reverse-traveling wave
- The critical question is whether this reflected pulse begins with the same compression or with a *phase-reversed* rarefaction



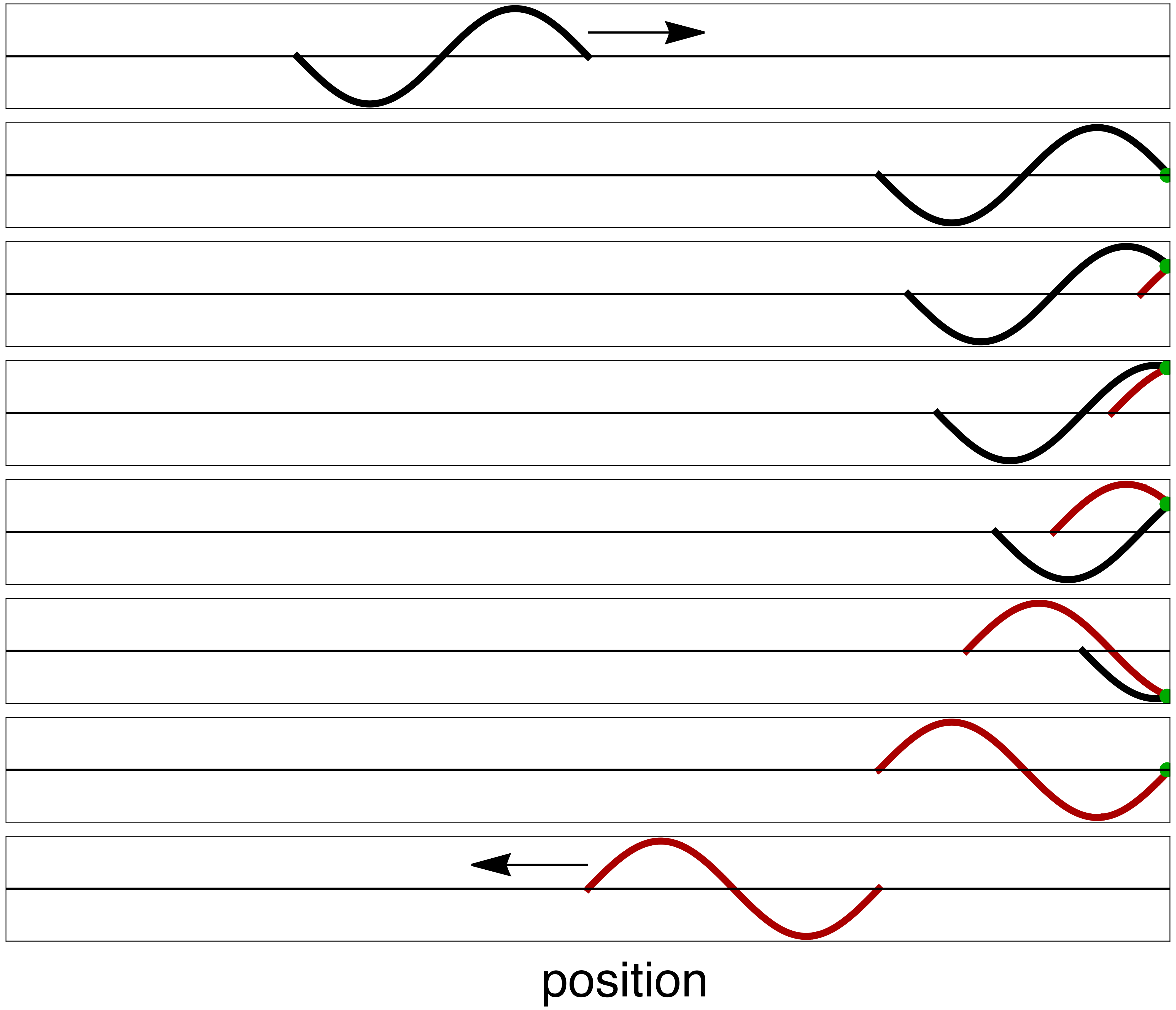


- The phenomenon in which a compressed wave reflects as a rarefied wave is known as phase reversal



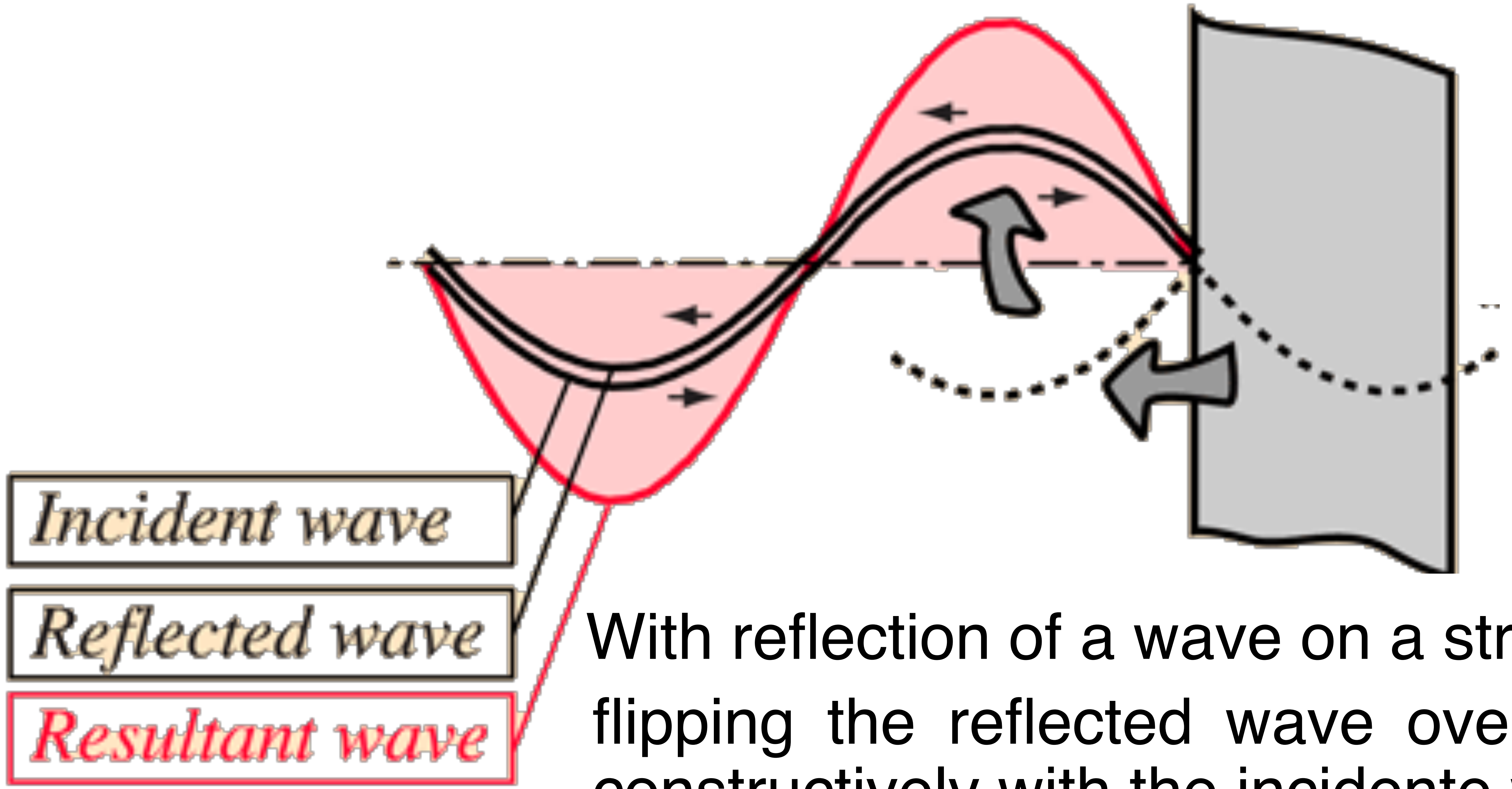
position

- Conversely, a reflection without phase reversal means the wave returns without any alteration to its value



Standing Waves Require Confinement within Finite-Sized Boundaries

- Standing waves require a resonant system where waves can continuously reflect back and forth
- Consequently, the strings or cavities described here cannot be infinite; they require a specific length with boundaries on both sides, creating a confined space for waves to oscillate



Instead of continuing past the barrier,

the wave is reflected

With reflection of a wave on a string, there is a 180 change in phase, flipping the reflected wave over so that it interferes constructively with the incidente wave

Boundary Conditions for Strings and Cavities

- ➢ Whether a string or cavity end induces phase reversal depends on its boundary conditions
- ➢ The ends can be categorized into two types

❋ for strings, ☛ *fixed end* (where the string is attached to a rigid support)

☛ *loose end* (where the end is free to move, such as floating in the air)

❋ for cavities, ends are either closed (closed by a rigid wall) or open to the atmosphere

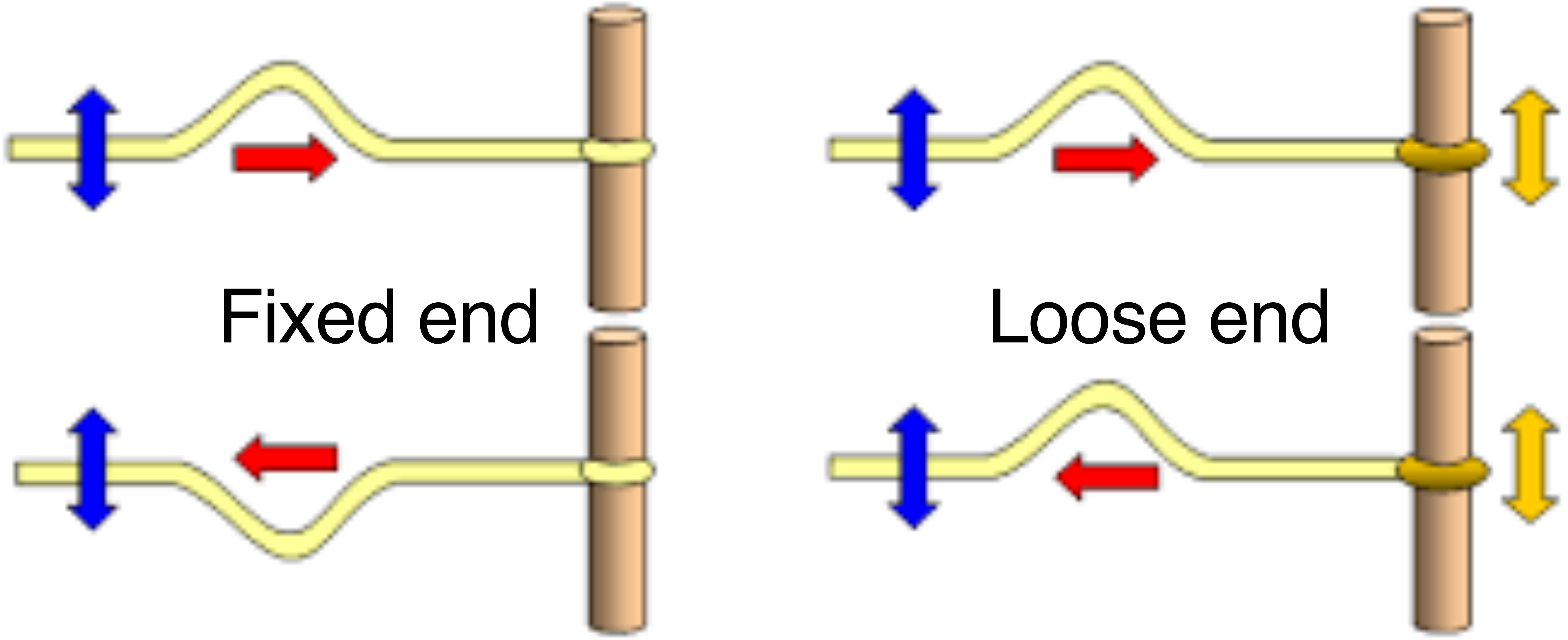
Boundary Condition Effects ☛ Fixed vs. Loose Ends

➢ While the physics behind why specific ends cause or avoid phase reversal requires deeper analysis, we can take the following behaviors as given

STRINGS

A. fixed end → phase is reversed

B. loose end → phase is not reversed



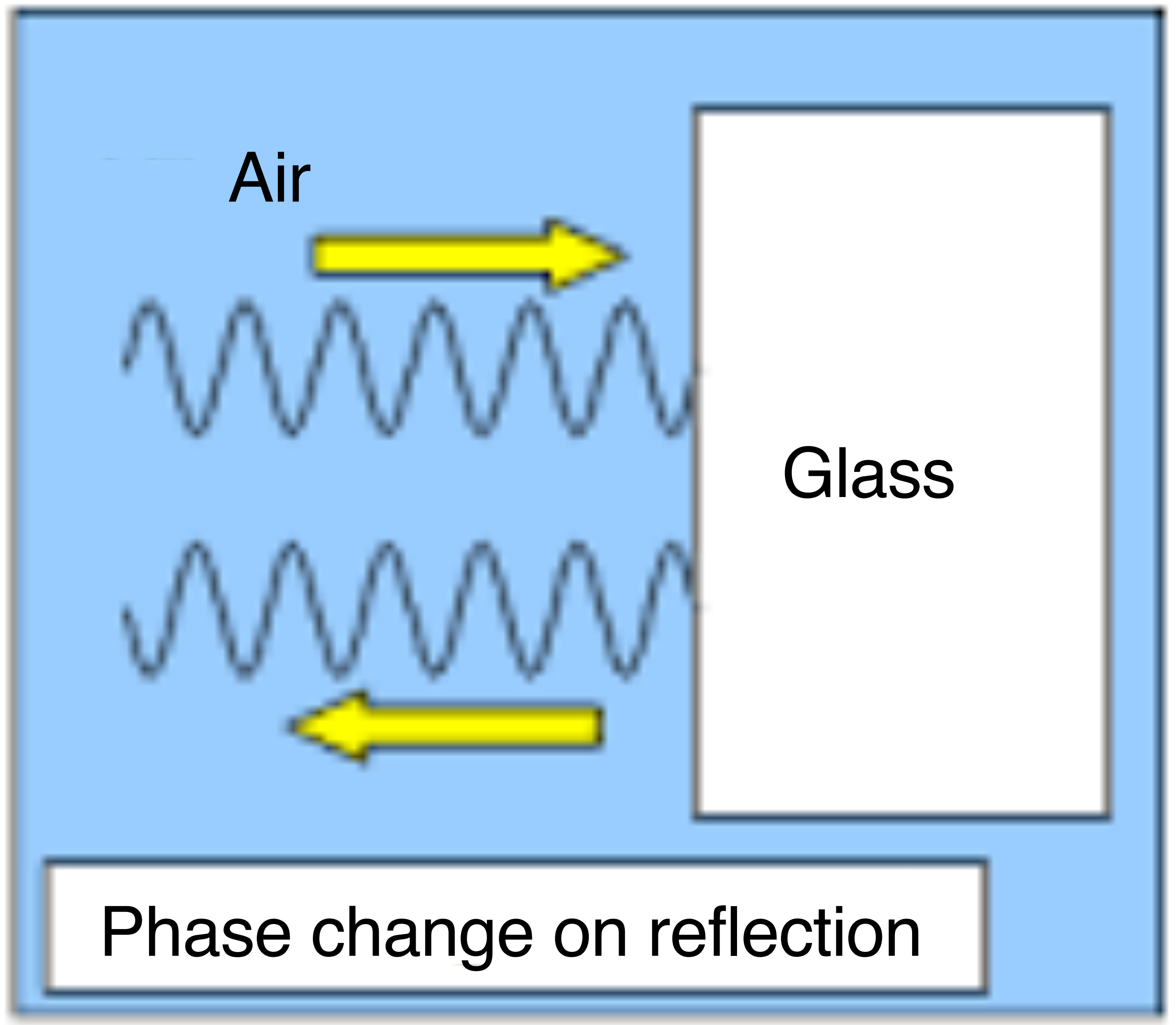
Boundary Condition Effects ☛ Open vs. Closed Ends

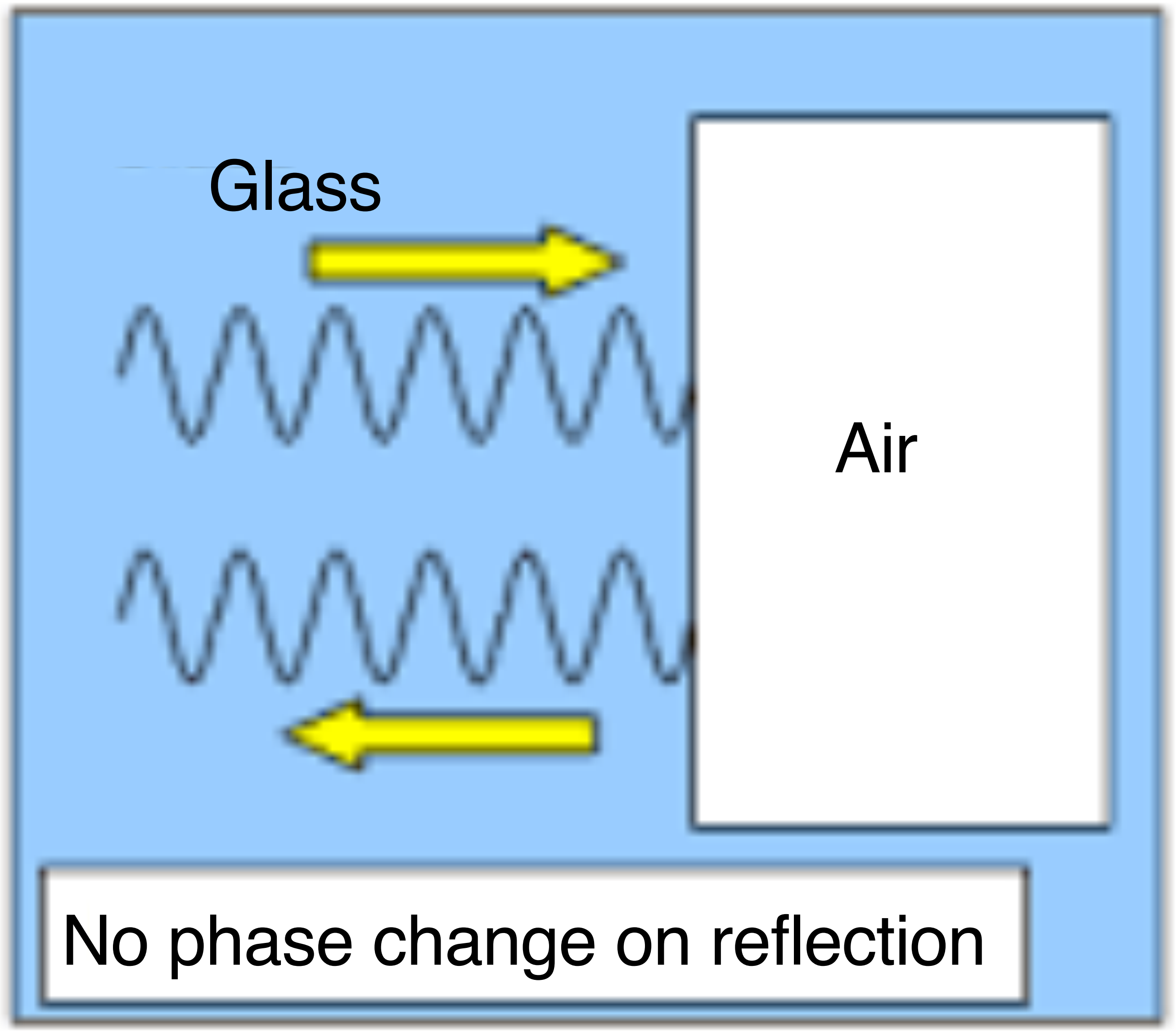
- While the physics behind why specific ends cause or avoid phase reversal requires deeper analysis, we can take the following behaviors as given

CAVITIES

A. open end → phase is reversed

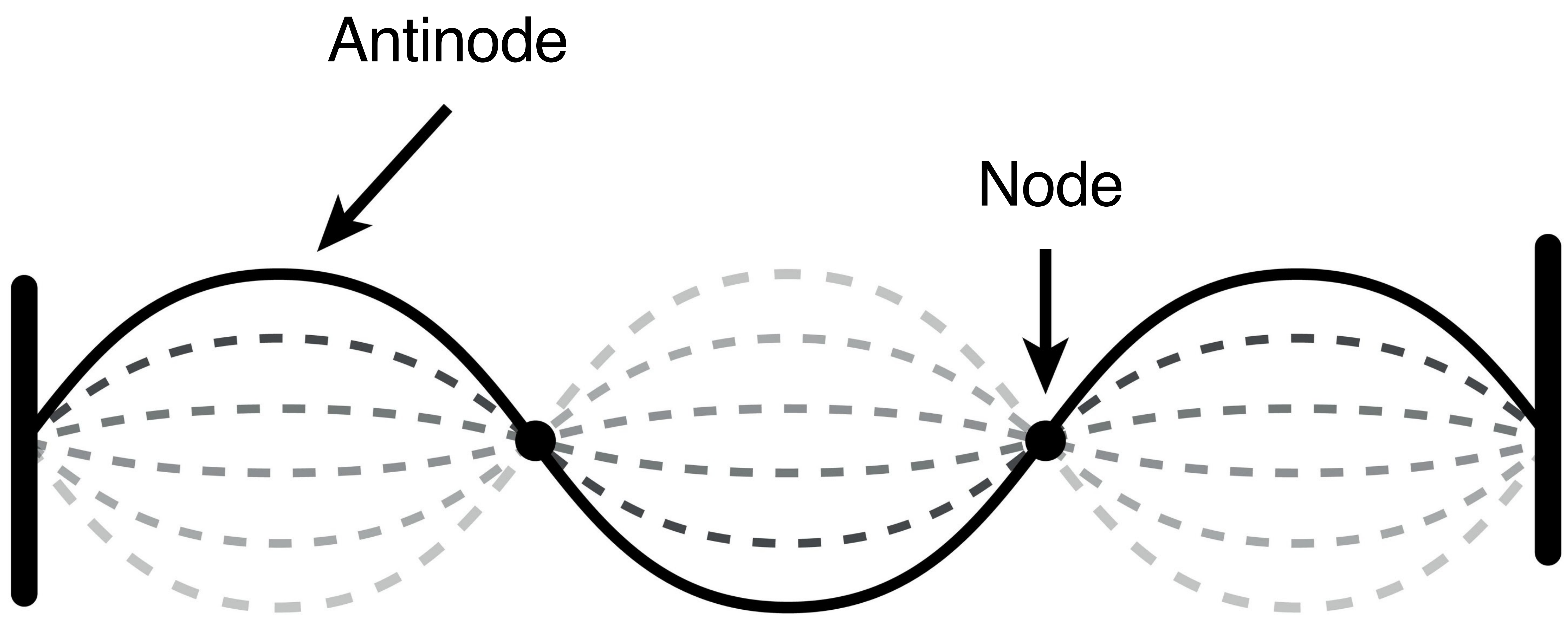
B. close end → phase is not reversed





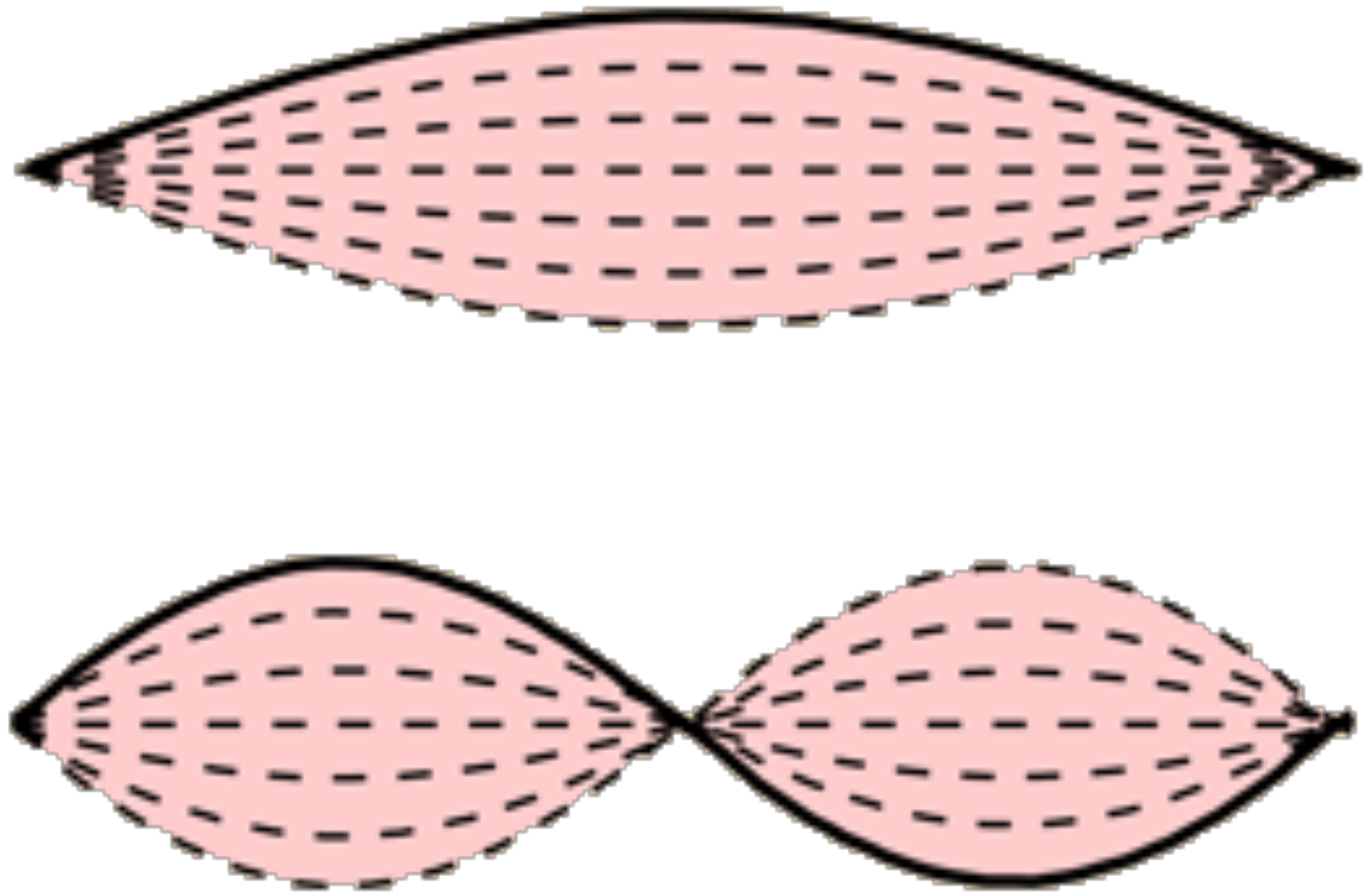
Nodes and Antinodes

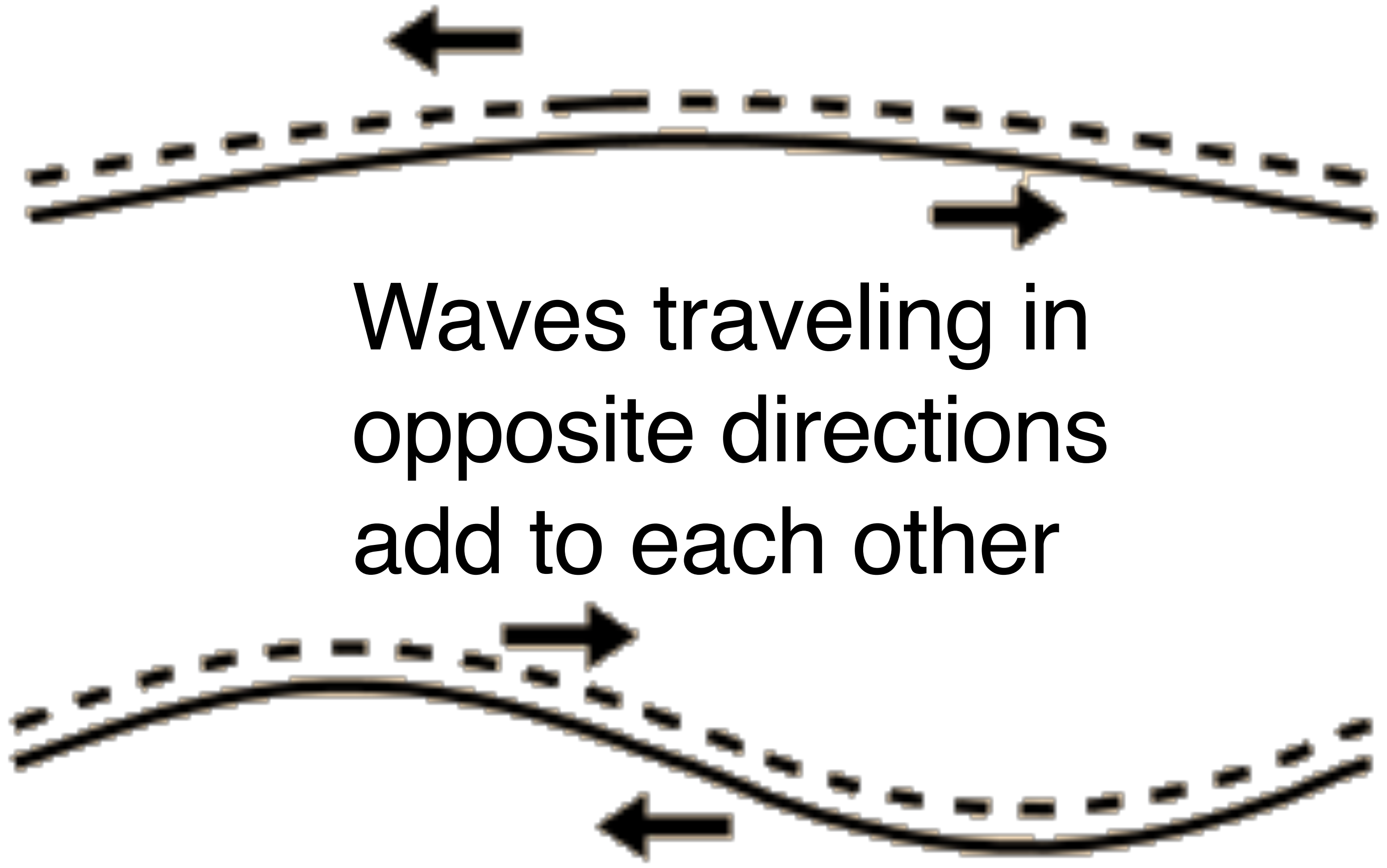
- Nodes and antinodes are specific points along a standing wave
- They represent the two extremes of how the medium (like a guitar string or air in a pipe) behaves during vibration



Nodes

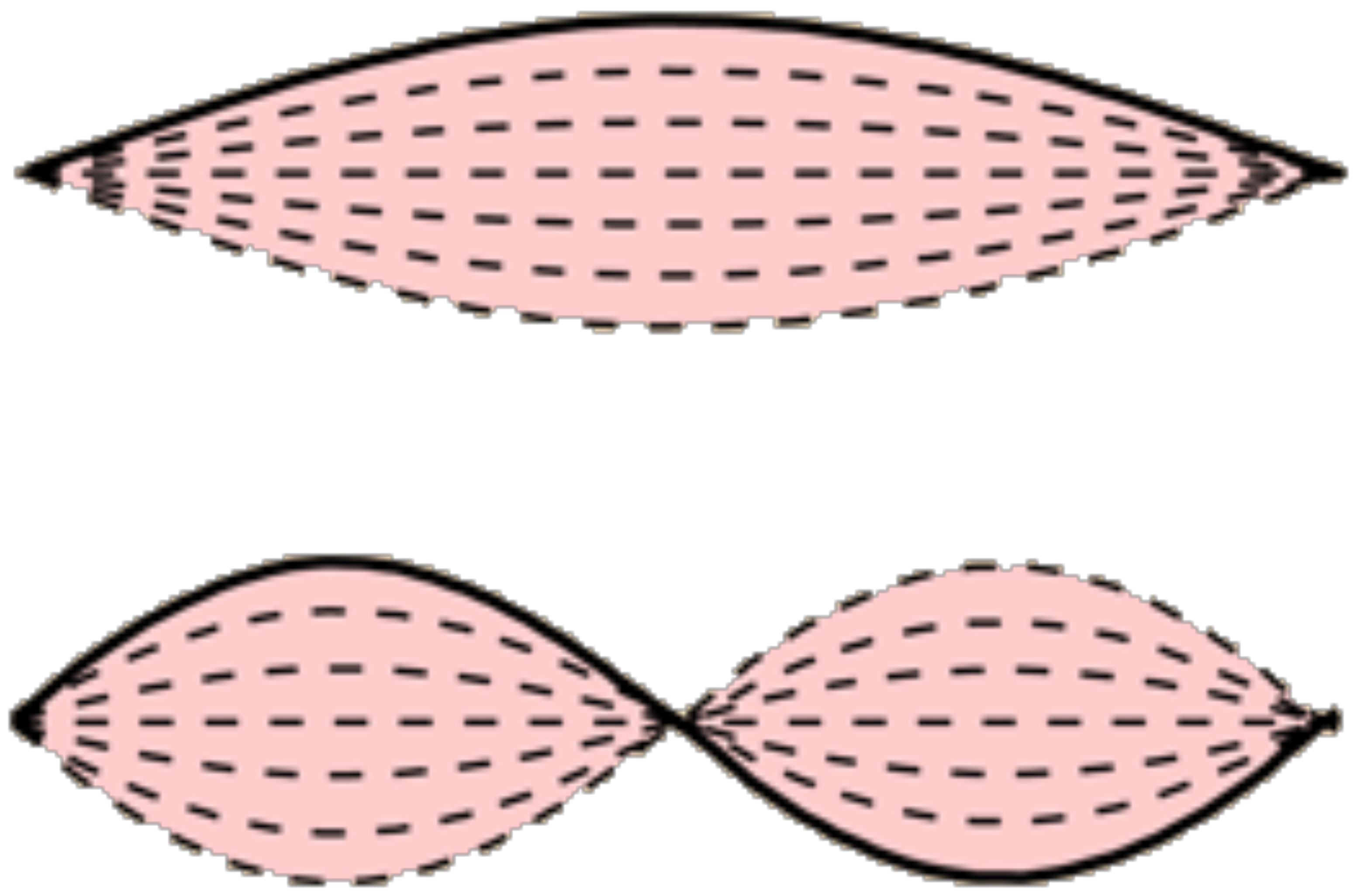
- A node is a point along a standing wave where the wave has zero amplitude
- This point point appears to be standing still; there is no displacement of the medium
- Nodes are the result of complete destructive interference, where two waves of the same frequency traveling in opposite directions perfectly cancel each other out

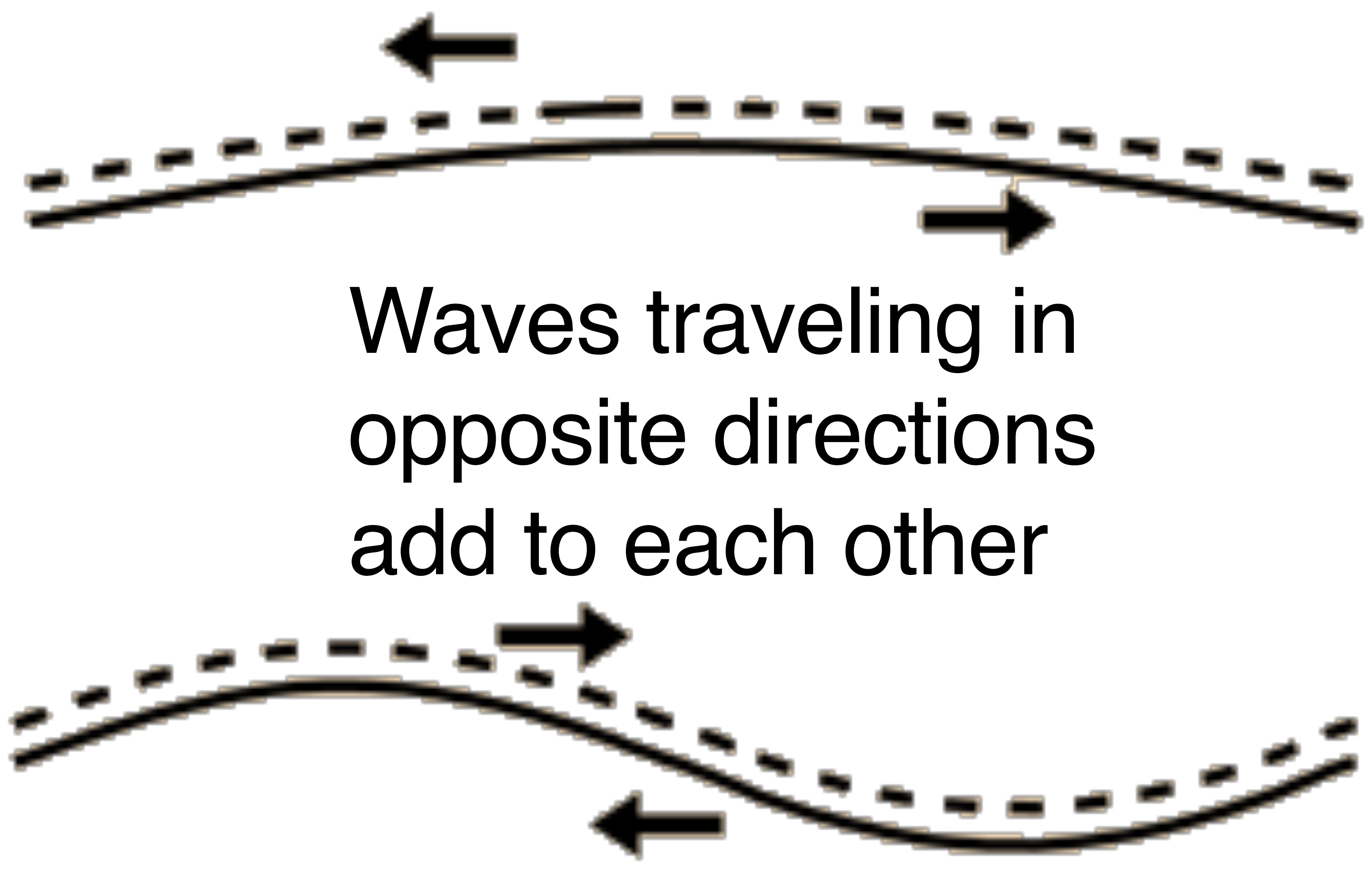




Antinodes

- An antinode is a point along a standing wave where the wave has maximum amplitude
- This point undergos the most intense vibration, moving back and forth between large positive and negative displacements
- Antinodes occur due to constructive interference, where the overlapping waves reinforce each other

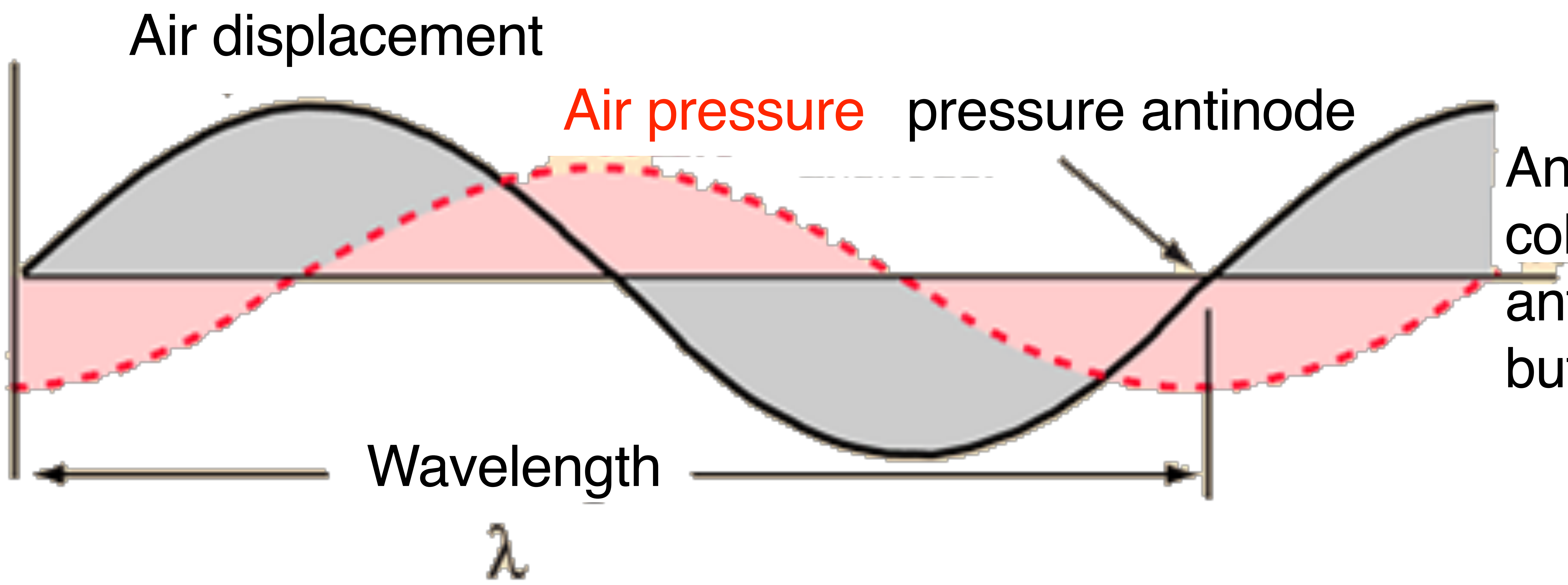




Pressure and Displacement

- We have seen that resonant air columns can be analyzed by pressure and air displacement
- A key rule is that a displacement node is always a pressure antinode (maximum pressure change), and vice versa
- At a displacement node, air is squeezed and expanded, creating maximum pressure variation

A node of motion, like a close end, will be an antinode of pressure



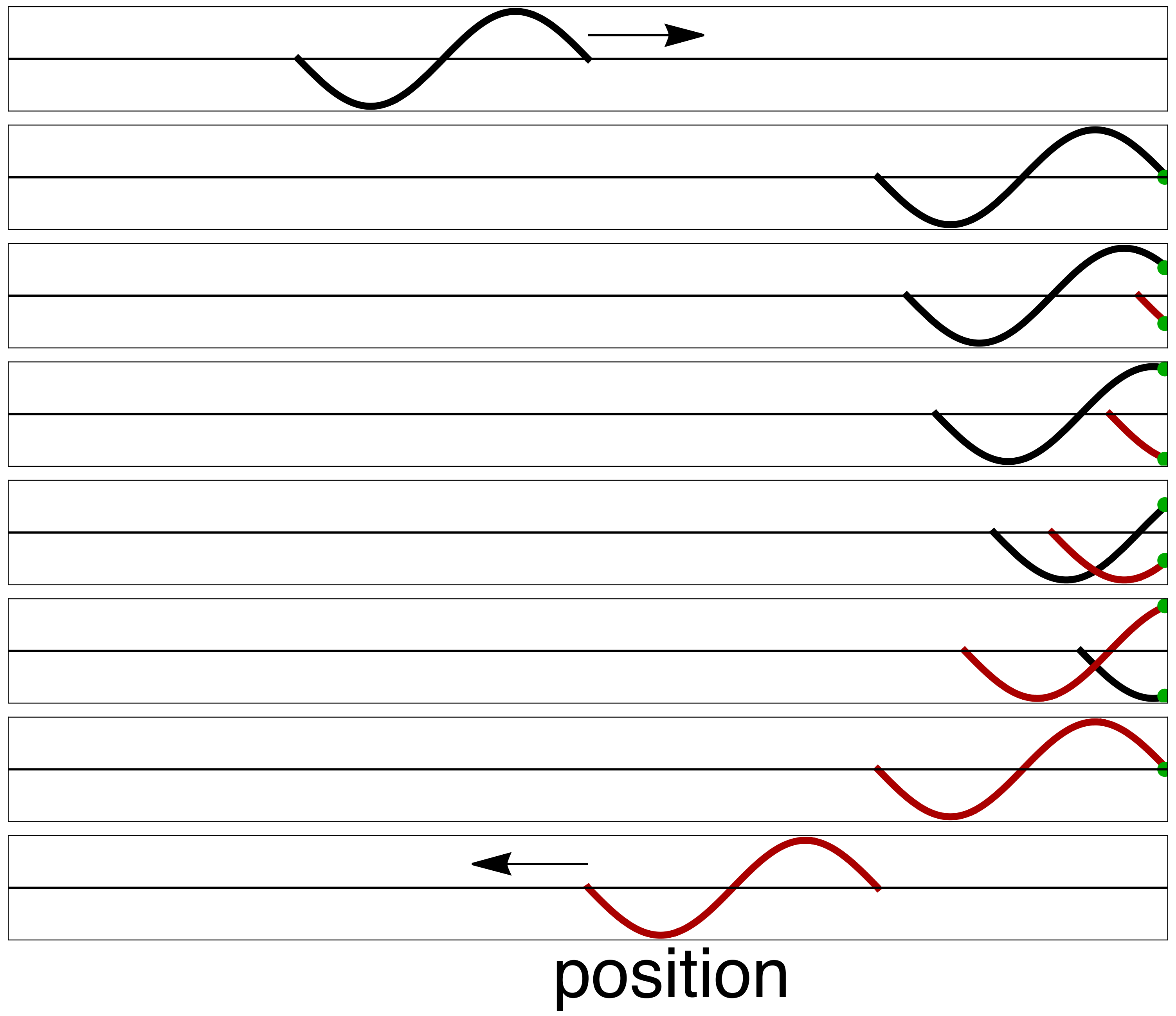
An open end of an air column would be an antinode of motion, but a node of pressure

Open and Closed Boundaries

- Within a cavity, oppositely traveling waves are interdependent, originating from the reflection of an initial wave at both boundaries
- These reflections create distinct boundary conditions at the ends, which are essential for understanding how standing waves are formed
- To analyze this, we must separately examine systems with open (fixed) versus closed (free) boundaries

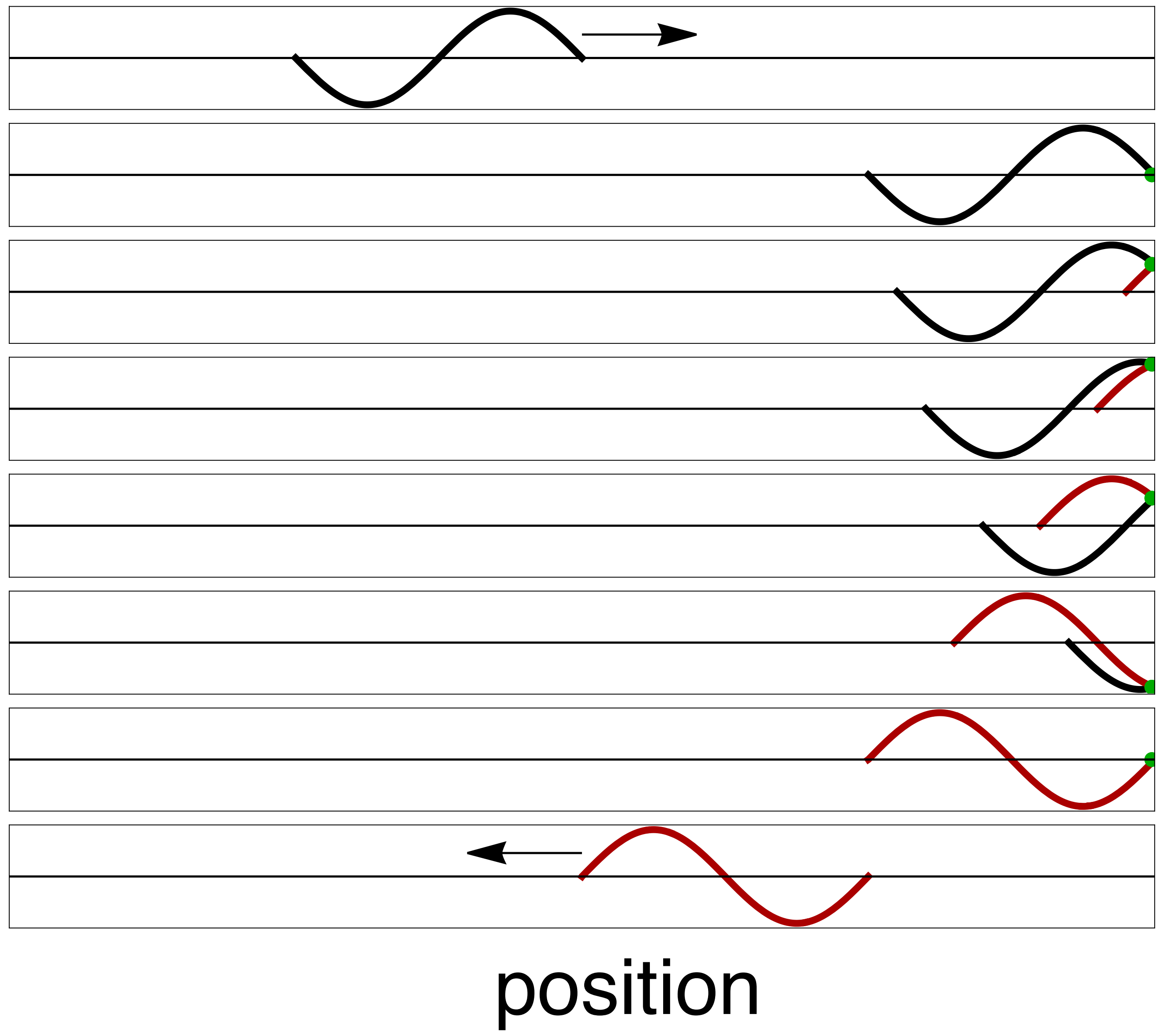
Open Boundary

- When a cavity end is open, the reflected wave undergoes a phase reversal, meaning a compression returns as a rarefaction (and vice-versa)
- Because the incident and reflected waves are equal in magnitude but opposite in sign, they cancel each other out
- The green dots in the figure illustrate that their sum is always zero, creating a node at the open end

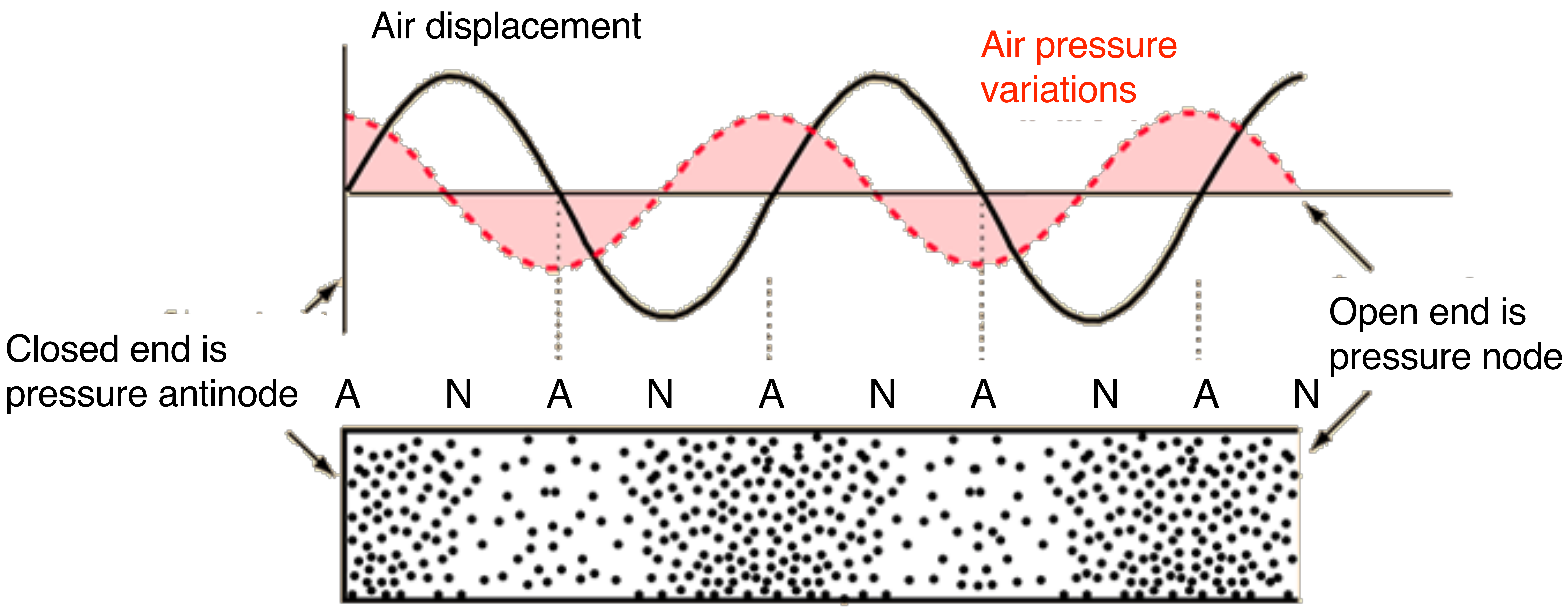


Closed Boundary

- When a cavity end is closed, reflected waves do not undergo a phase reversal
- This means an incoming compression reflects as a compression, and a rarefaction reflects as a rarefaction
- Unlike an open end, where waves cancel out, the incoming band reflected waves at a closed end reinforce each other
- Because they have the same value, they create a maximum oscillation between compression and rarefaction, producing a pressure antinode



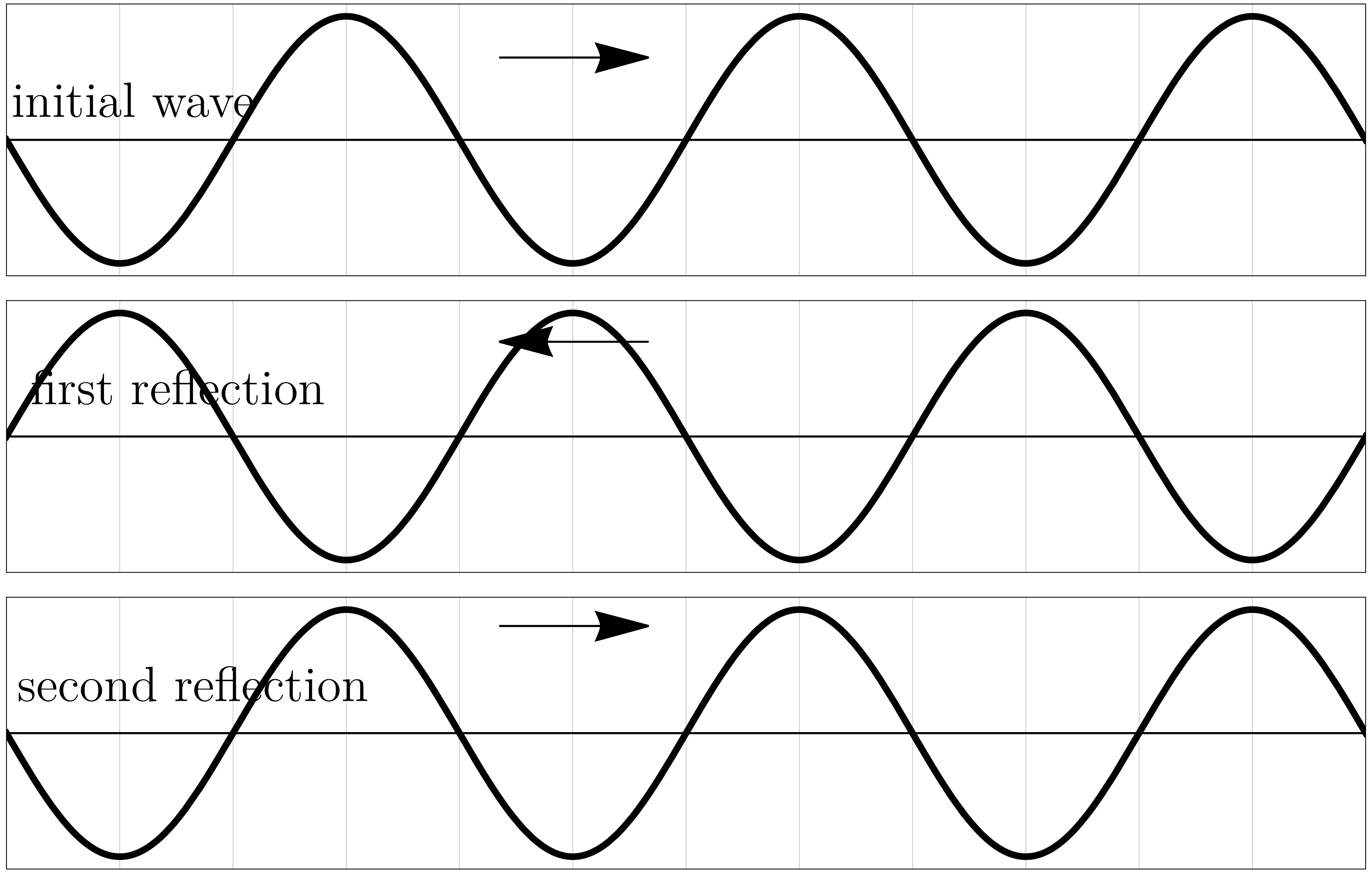
Effect of Open and Closed Ends on Pressure and Displacement



Constructive Interference

- For waves bouncing inside a cavity to produce constructive interference, they must have a wavelength that ensures consecutive reflections align properly

Example 1 ☛

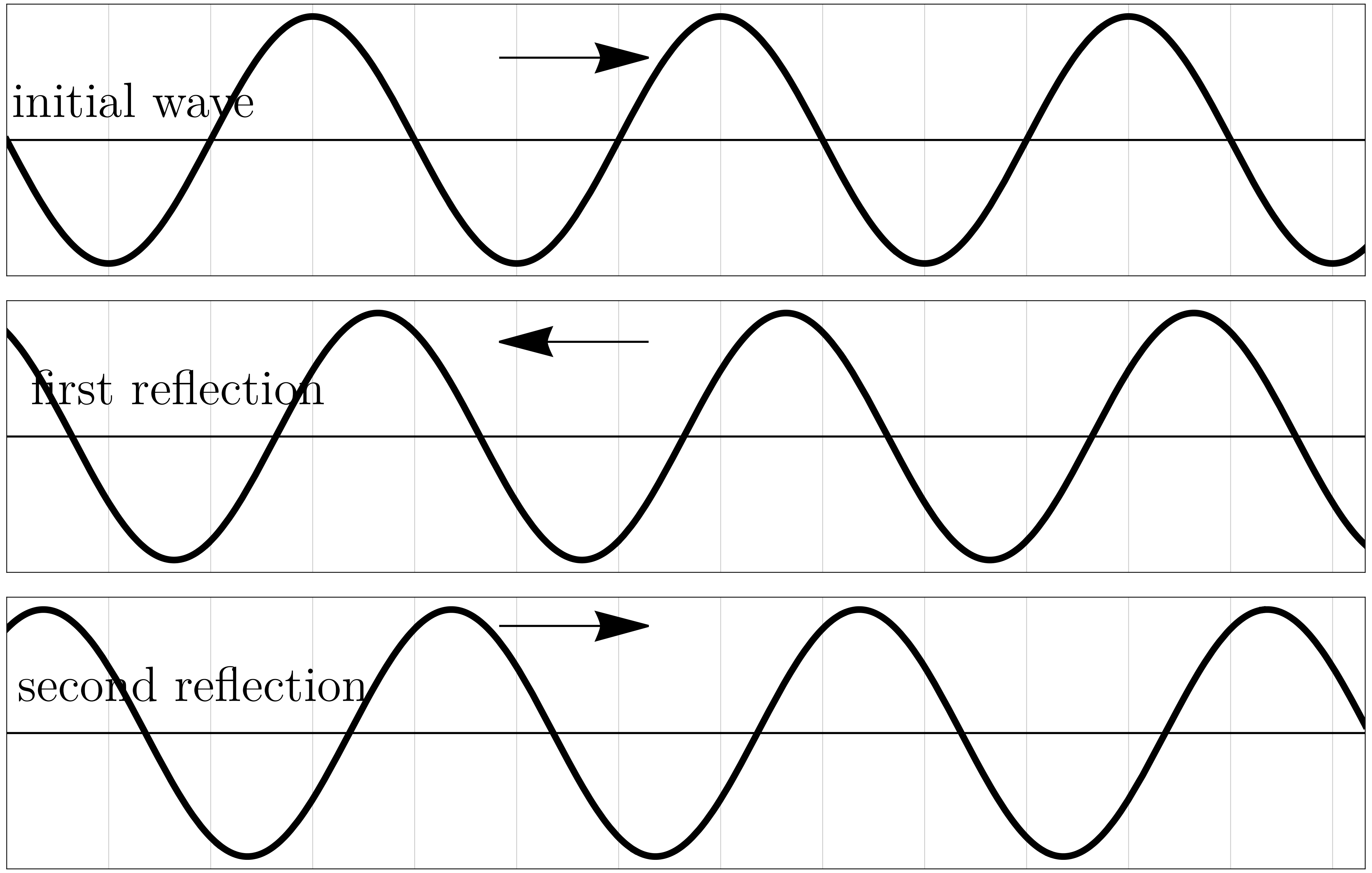


- Initial wave and its second reflection, both moving right, perfectly aligned
- This leads to fully constructive interference, where their amplitudes combine

Destructive Interference

➢ For waves bouncing inside a cavity to produce constructive interference, they must have a wavelength that ensures consecutive reflections align properly

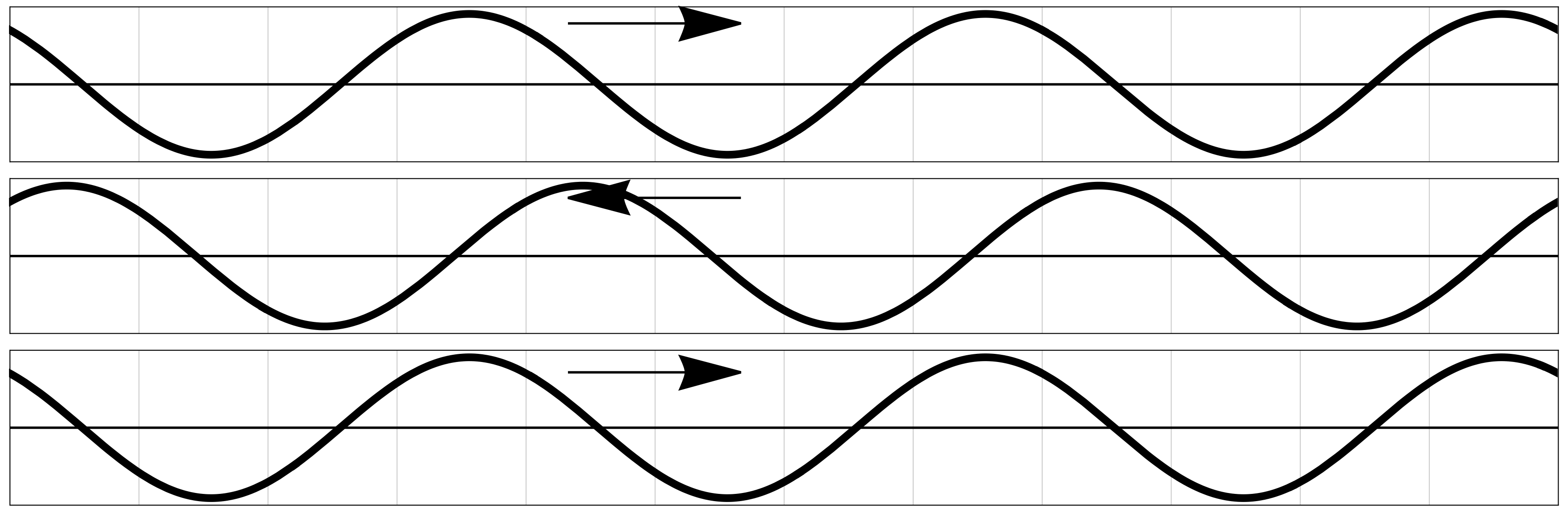
Example 2 ☛



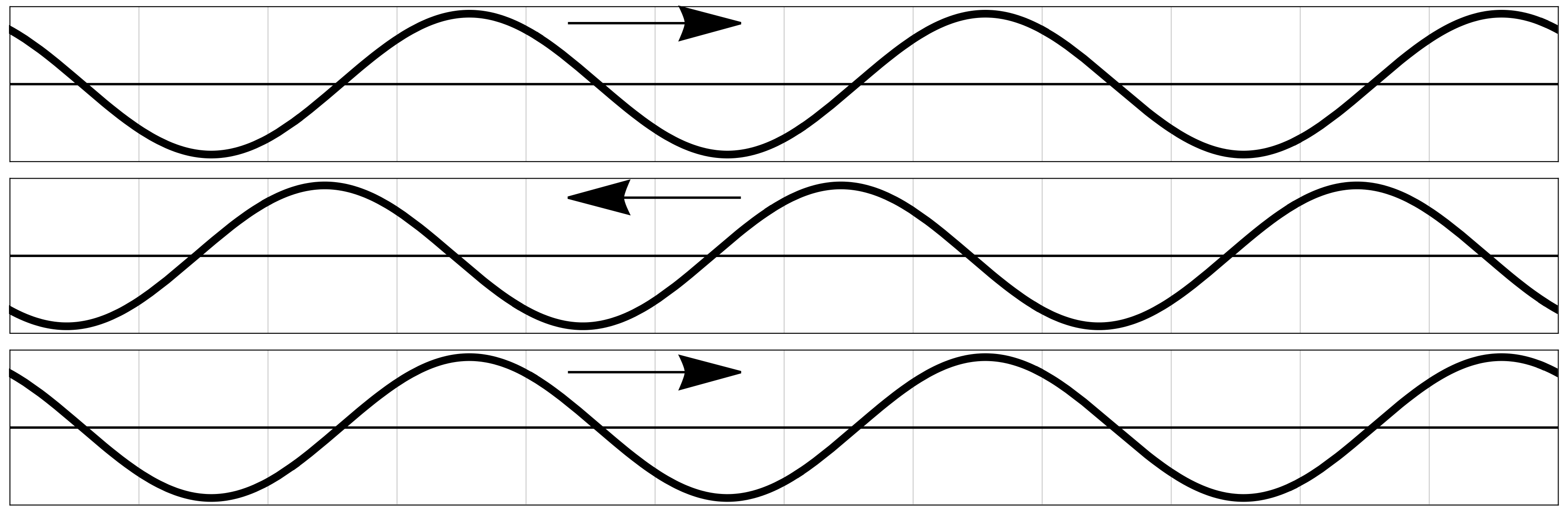
➢ The initial wave and its second reflection are misaligned, causing them to cancel each other out over multiple reflections

Equal Ends

- For a cavity with closed boundaries at both ends, waves reflect without inverting phase (compressions remain compressions)



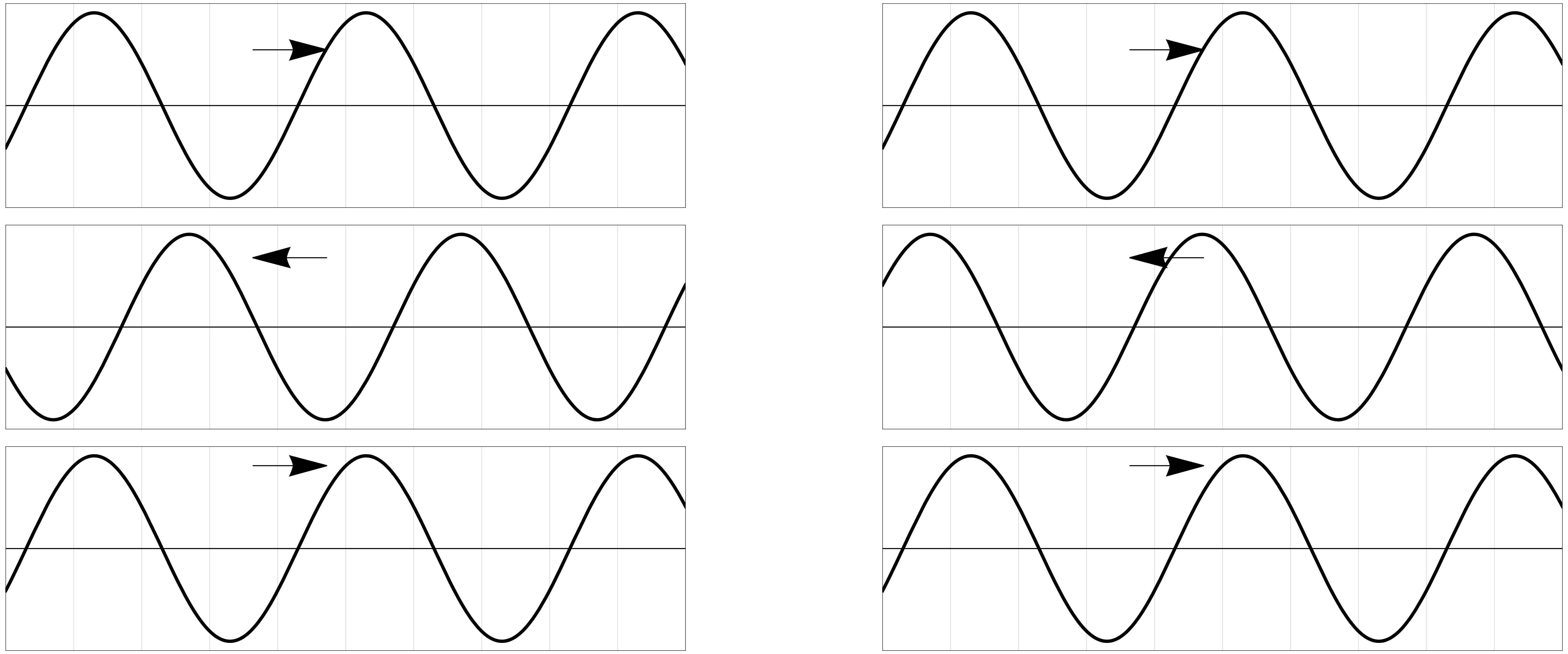
- For open ends, reflection causes phase reversal (compressions become rarefactions)



- As shown in the figures, although the first reflection differs, the second reflection is aligned with the incident wave in both cases, leading to constructive interference

Resonance

- This resonance occurs only when the cavity length is an exact multiple of half-wavelengths
- These examples illustrate a general rule relating wave wavelength to the cavity length required for constructive interference, which will be explored further below



closed-closed and open-open for 2.5 wavelength

Resonance

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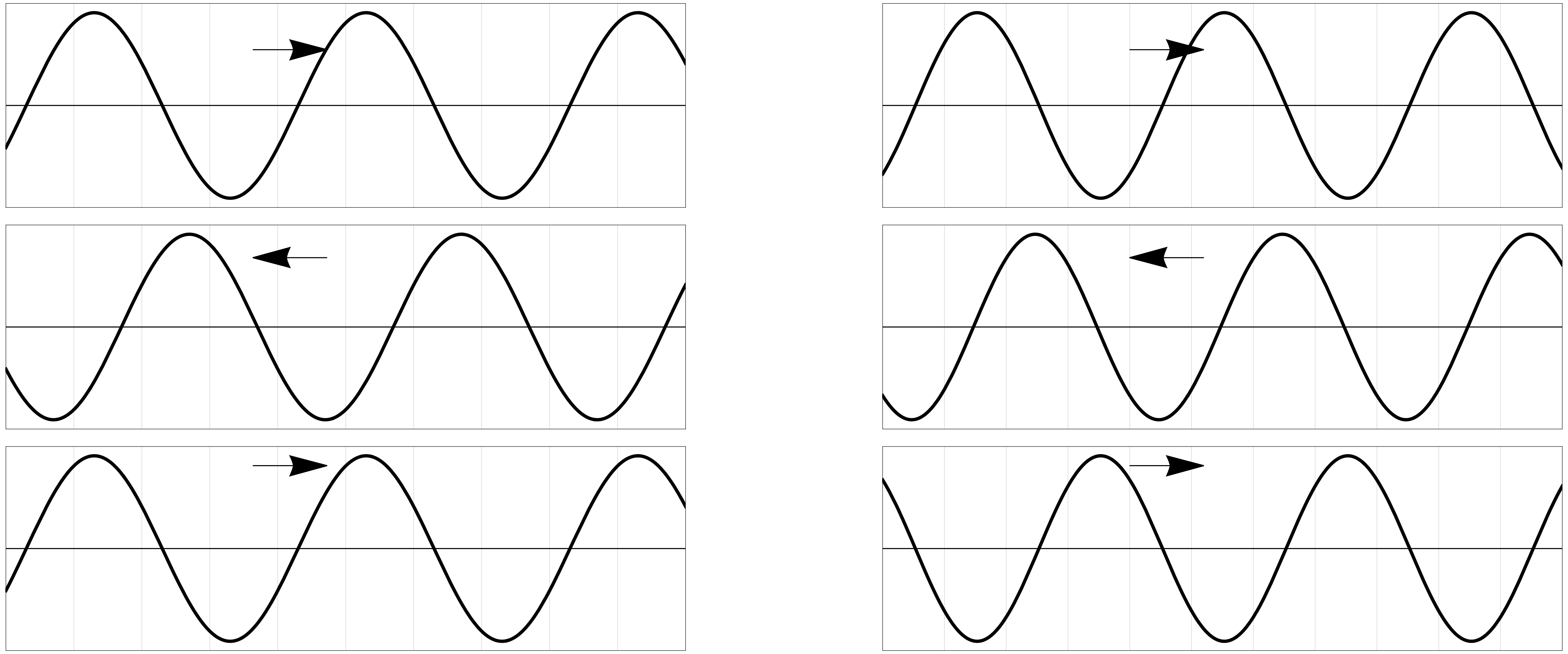

closed-closed and open-open for 2.75 wavelength

Resonance

- This resonance occurs only when the cavity length is an exact multiple of half-wavelengths
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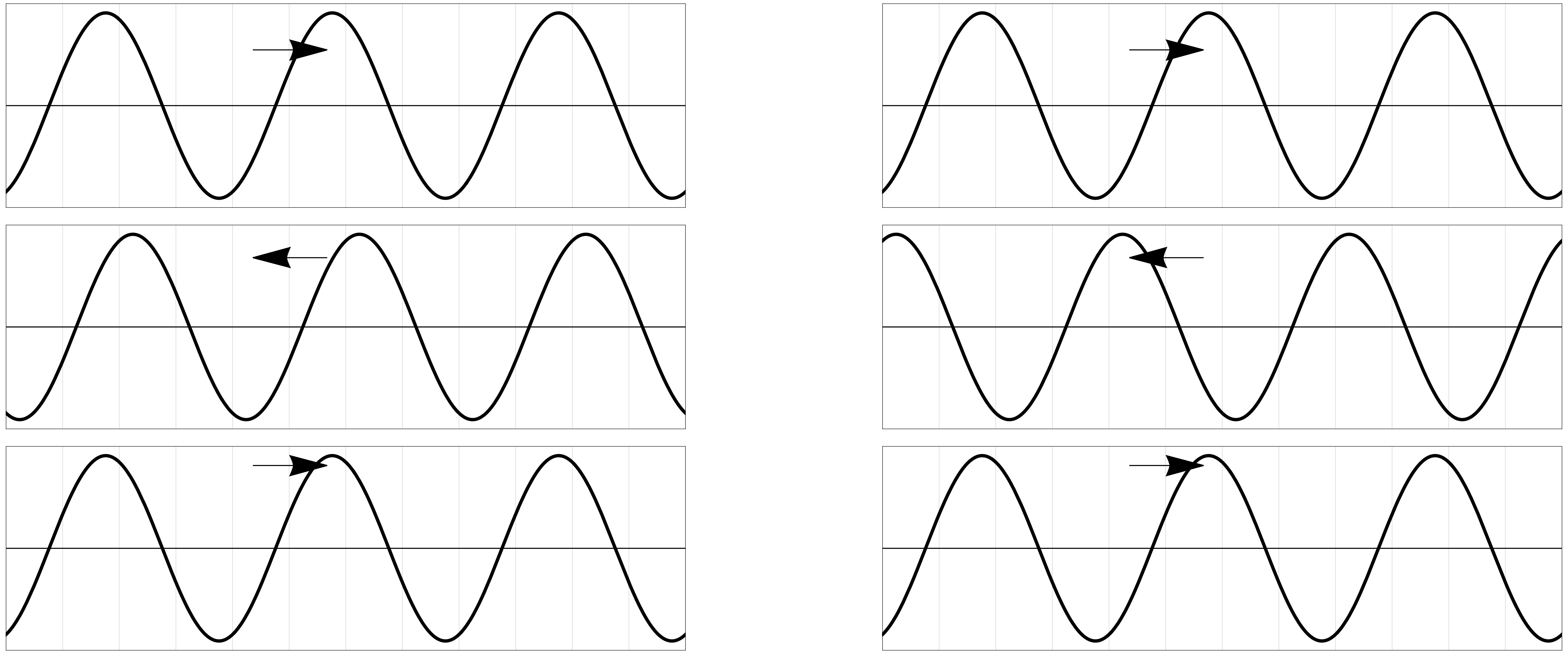
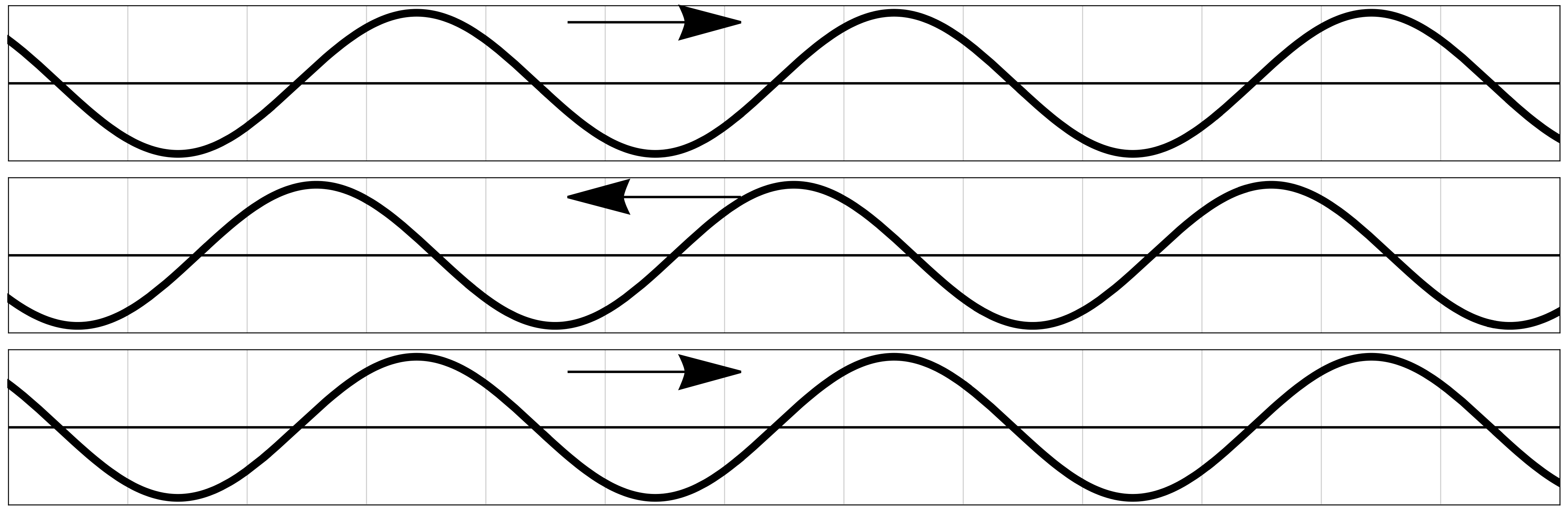
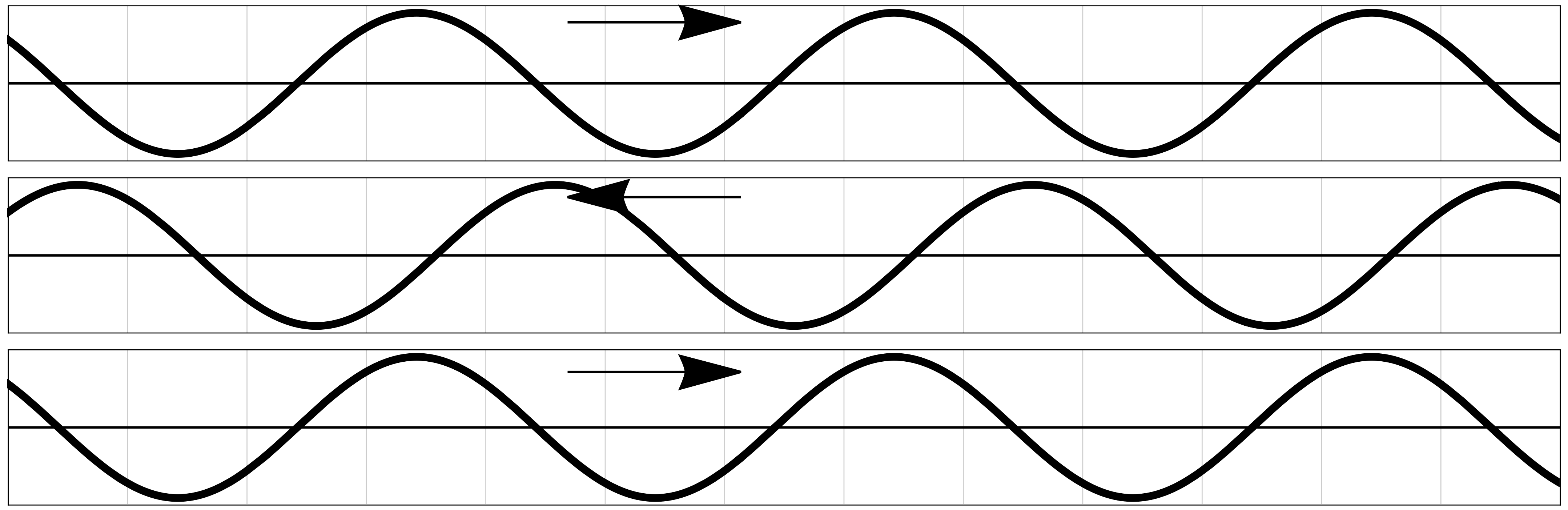


closed-closed and open-open for 3.0 wavelength

Different Ends

➢ When a cavity has one open and one closed end, only one of the two reflections undergoes a phase reversal

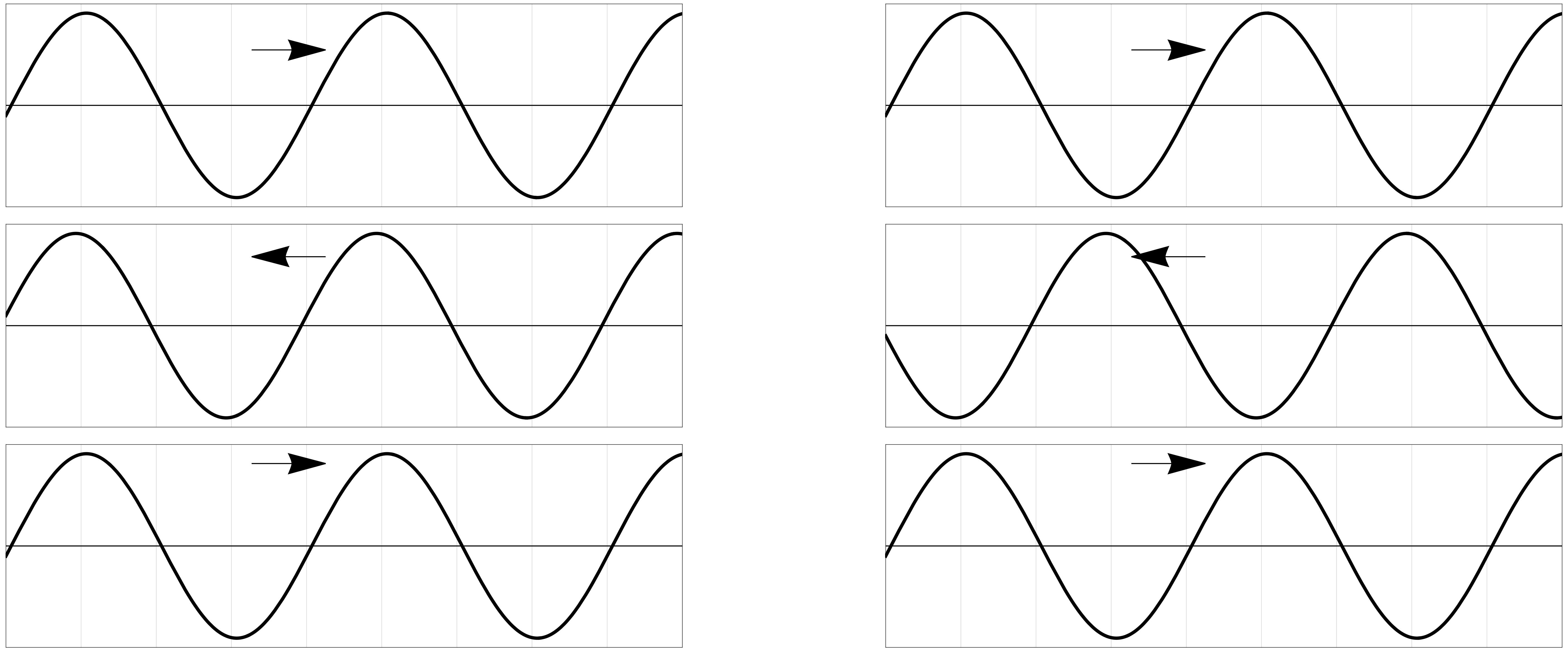




open-closed and closed-open

When do we have alignment?

- For the initial wave and its second reflection to align and create constructive interference, the cavity length must equal an integer number of full wavelengths plus an additional quarter or three-quarters of a wavelength
- This alignment occurs at 2.25 and 2.75 wavelengths, but fails at 2.5
- This specific behavior follows a broader rule that we will explore later



open-closed and closed-open 2.25 wavelengths

When do we have alignment?

- For the initial wave and its second reflection to align and create constructive interference, the cavity length must equal an integer number of full wavelengths plus an additional quarter or three-quarters of a wavelength
- This alignment occurs at 2.25 and 2.75 wavelengths, but fails at 2.5
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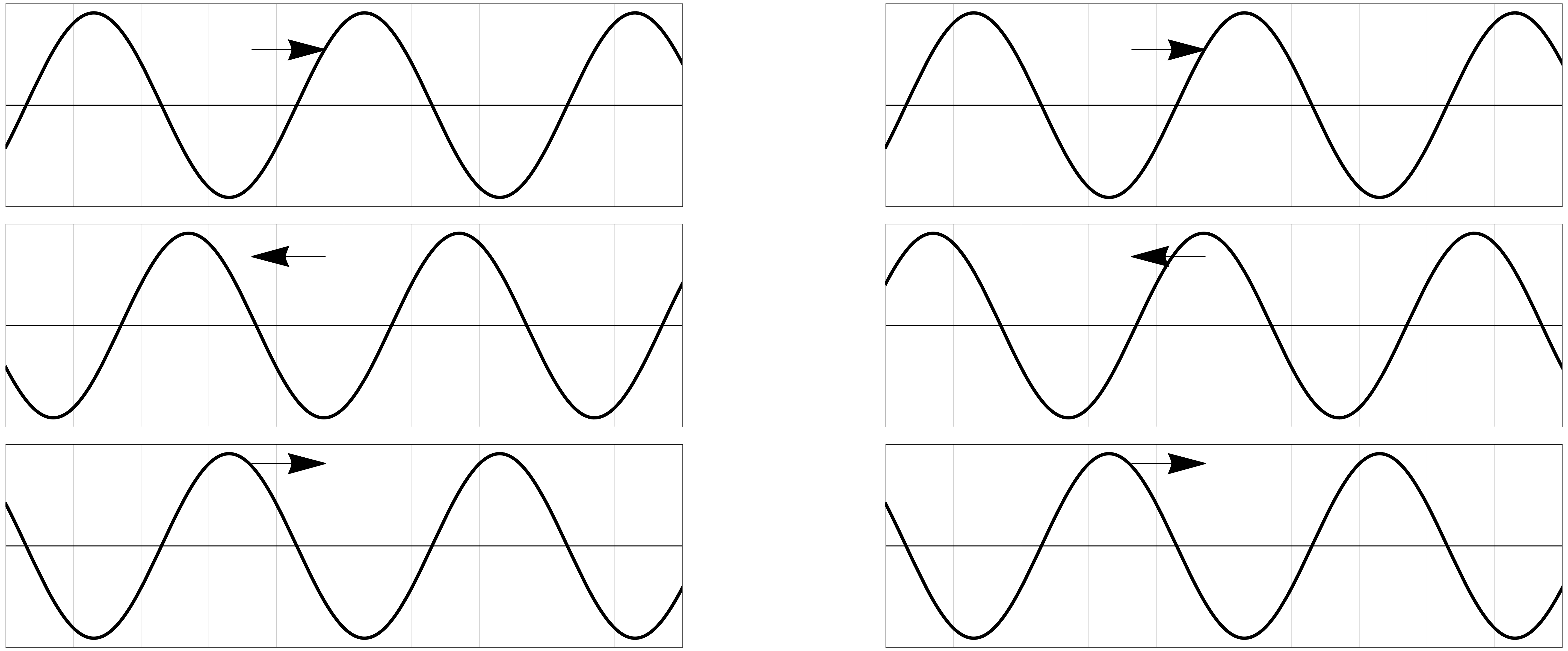

open-closed and closed-open 2.5 wavelengths

When do we have alignment?

- For the initial wave and its second reflection to align and create constructive interference, the cavity length must equal an integer number of full wavelengths plus an additional quarter or three-quarters of a wavelength
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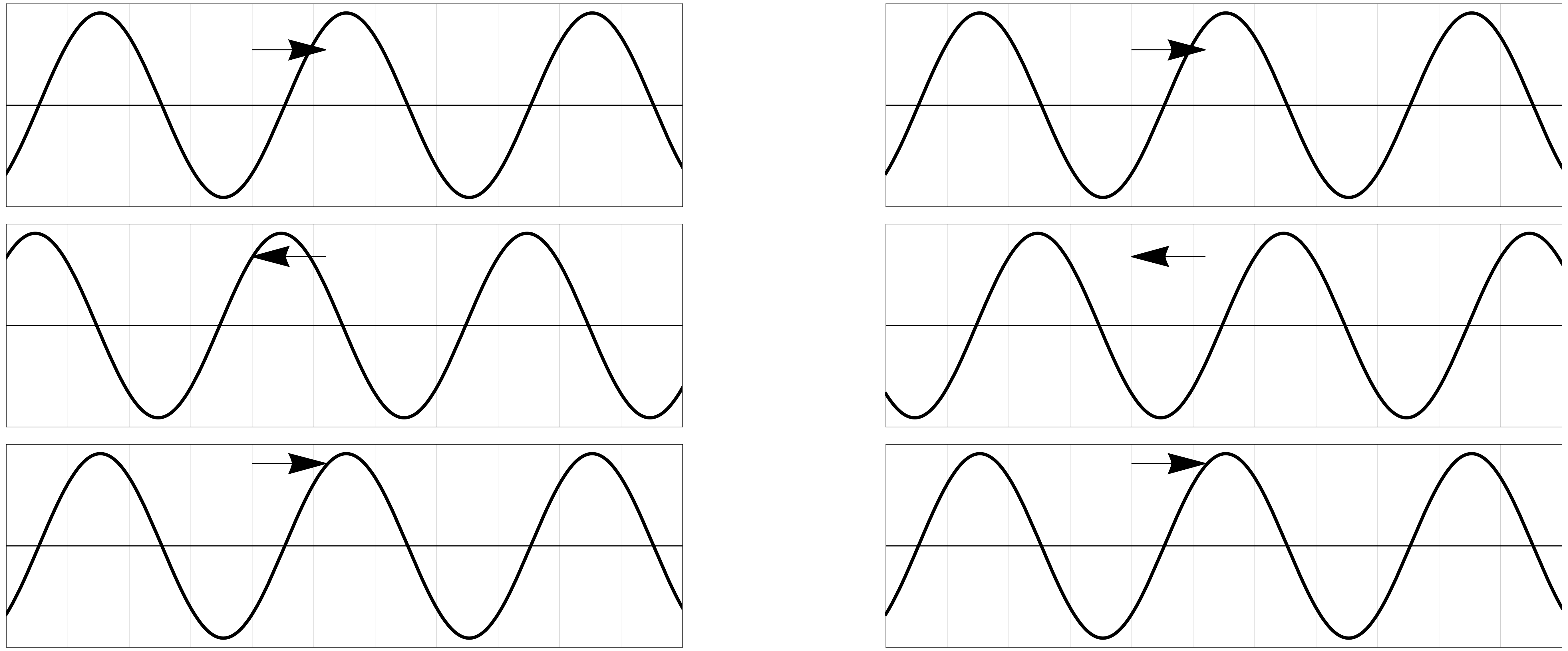


open-closed and closed-open 2.75 wavelengths

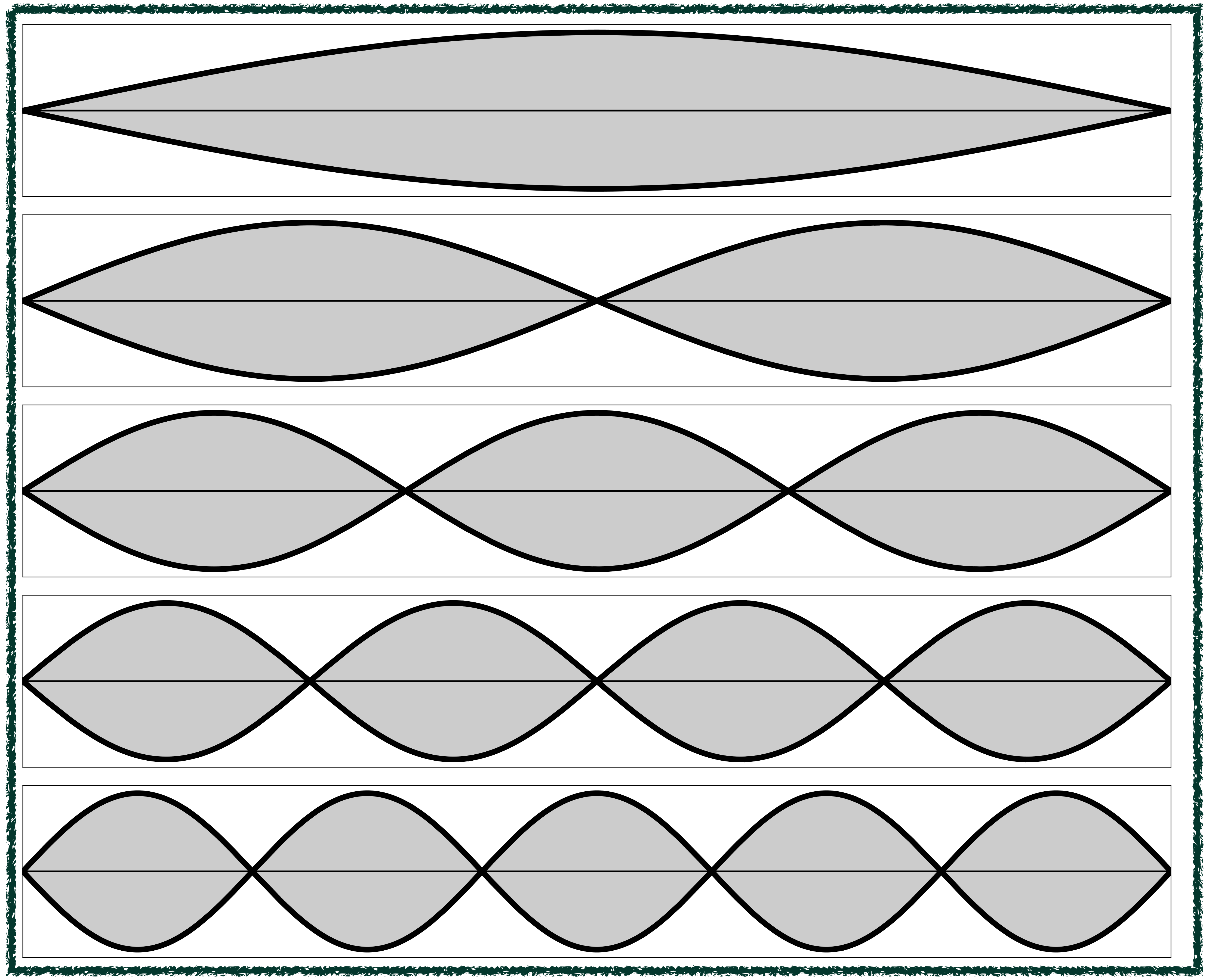
Visualizing Standing Waves: Graphical Analysis

- Let us now determine the conditions required for the formation of standing waves
- By applying a graphical method, we can sketch waves that satisfy two key boundary conditions

  ☛ pressure nodes form at open ends, while antinodes form at closed ends

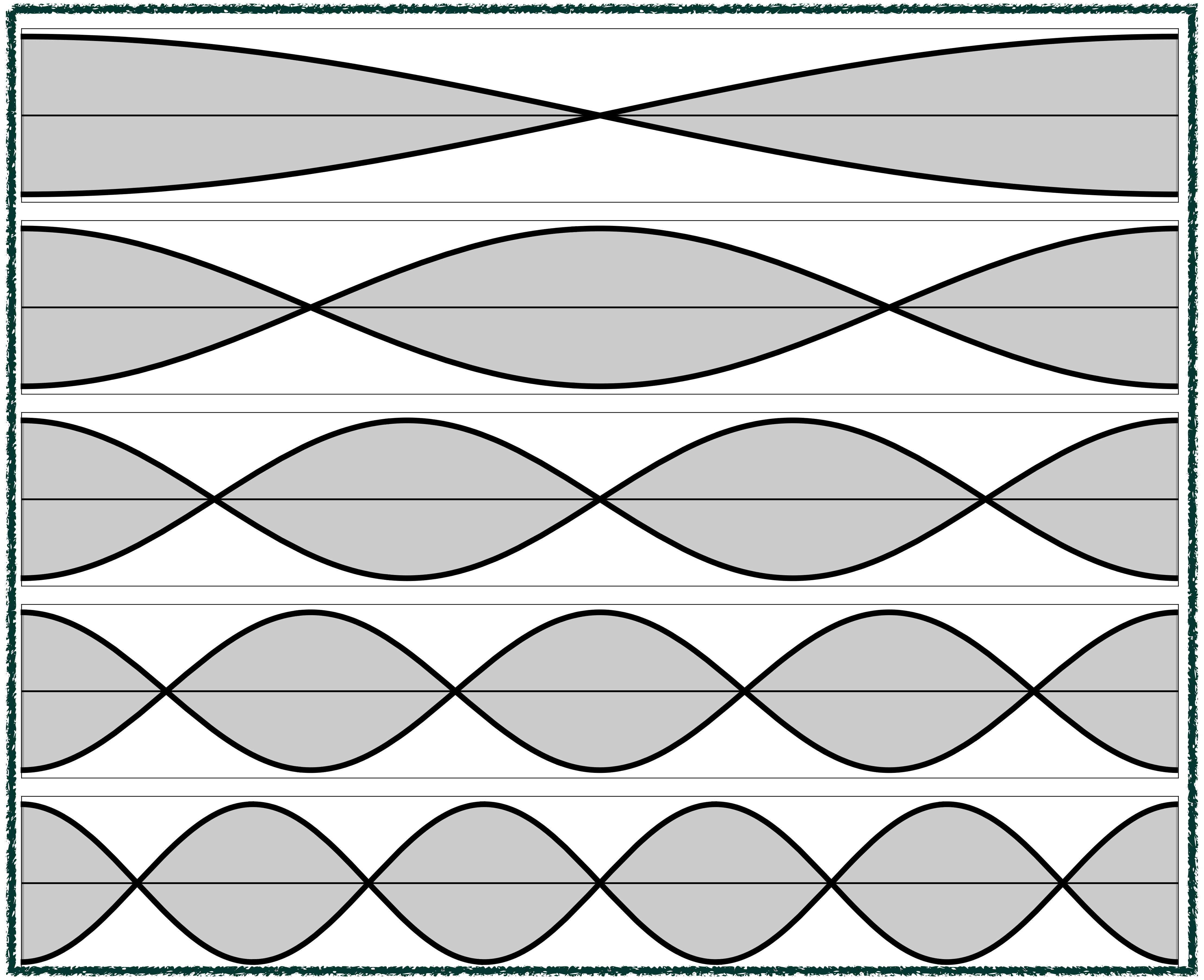
- We will examine four cavity configurations ☛ open-open, closed-closed, open-closed, and closed-open (noting that the latter two are functionally identical, as one can be rotated into the other)

Acoustic Pressure Mode in Open-Open Cavity



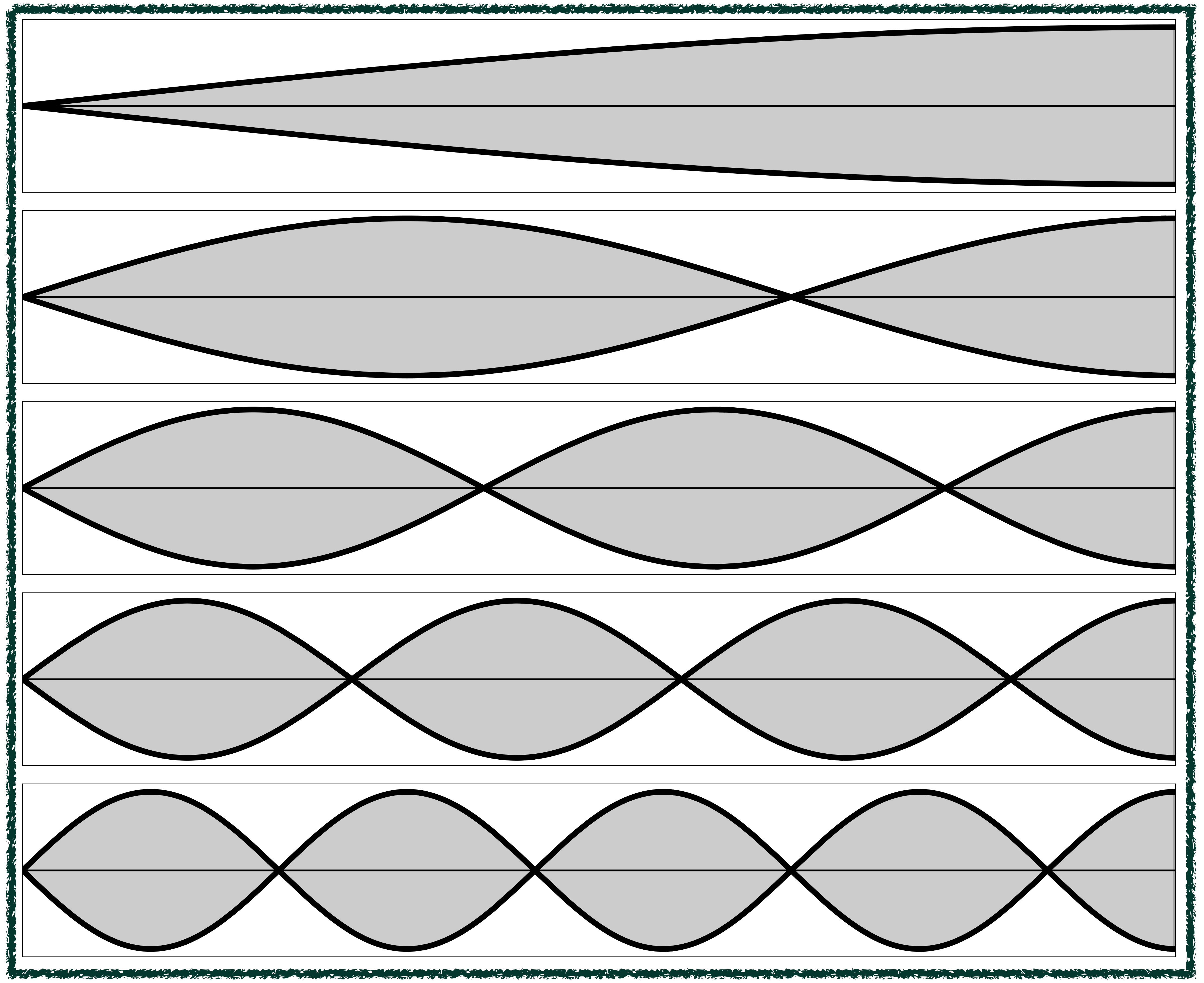
$$n = 1,2,3,4,5$$

Acoustic Pressure Mode in Closed-Closed Cavity



$$n = 1,2,3,4,5$$

Acoustic Pressure Mode in Open-Closed Cavity



$$n = 1,3,5,7,9$$

Visualizing Standing Waves: Governing Equations

- Based on our previous discussion a pattern emerges that allows for the classification of standing waves within different types of cavities

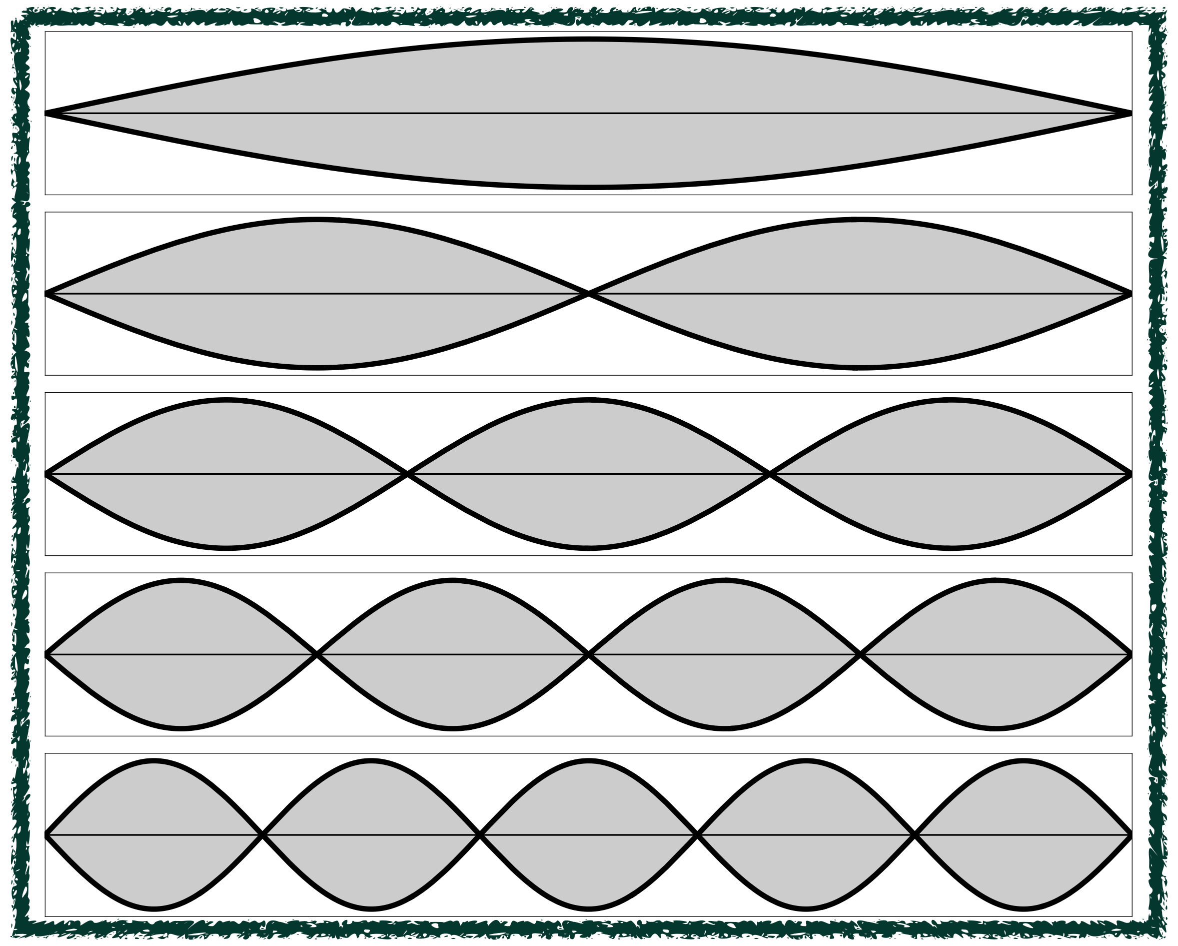
Standing Waves with Equal Ends

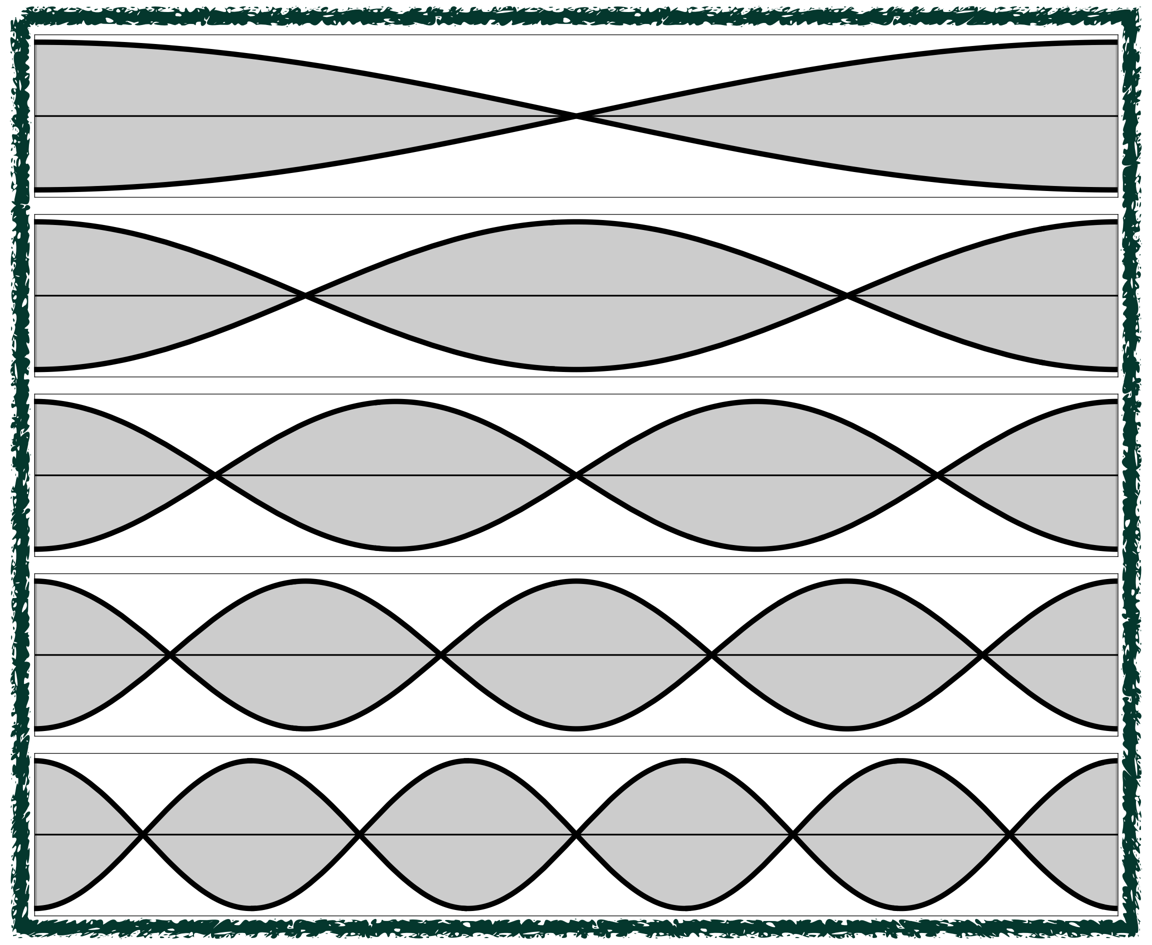
- Open-open and closed-closed cavities share similar wave patterns, specifically in the number of waves they can support

- In both scenarios, the cavity holds an integer or half-integer number of wavelengths ☛ one-half, one, one-and-a-half, and so on ($0.5, 1, 1.5, 2, 2.5, \cdots$ full waves)

- Because these cavities have matching boundary conditions at both ends, the relationship between the cavity length (L) and the wavelength (λ) is consistent for both types ☛

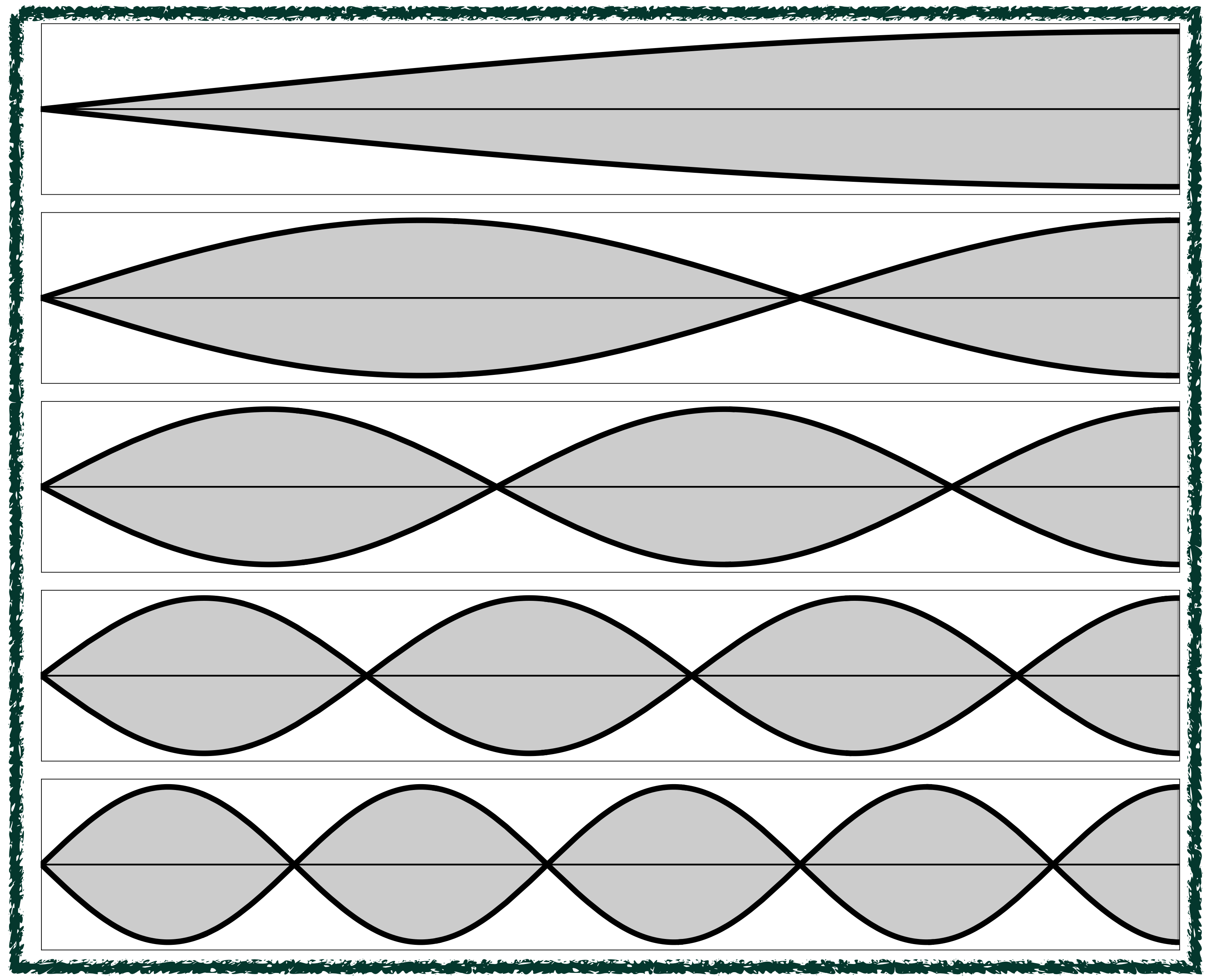
$\dfrac{L}{\lambda} = 0.5, 1, 1.5, 2, 2.5, \cdots$ which can be expressed in general as $L = \dfrac{n}{2}\lambda$ whith $n = 1, 2, 3, \cdots$





Standing Waves with Unequal Ends

- For mismatched end conditions, the cavity supports resonances at odd multiples of a quarter-wavelength $(0.25, 0.75, 1.25, 1.75, 2.25 \cdots)$ full wave periods
- The resonant wavelength (λ) is therefore related to the cavity length (L) by ☛ $\frac{L}{\lambda} = 0.25, 0.75, 1.25, 1.75, 2.25 \cdots$ which can be expressed in general as $L = \frac{n}{4}\lambda$ with $n = 1, 3, 5, \cdots$



Harmonics

- A cavity can support various standing waves, known as harmonics, depending on the integer n used in the resonance formulas

- The simplest of these $(n = 1)$ is the fundamental harmonic

- For a cavity with identical ends, this fundamental mode determines the longest possible resonant wavelength $\dfrac{L}{\lambda_1} = \dfrac{1}{2} \Rightarrow \lambda_1 = 2L$ which is twice the length of the cavity

- In the case of a cavity with unequal ends, the fundamental harmonic has a wavelength given by

$$\frac{L}{\lambda_1} = \frac{1}{4} \Rightarrow \lambda_1 = 4L$$

Frequency Domain

- For pressure waves, it is more practical to express the cavity length in terms of frequency rather than wavelength

- Substitting the wave speed equation $u = \lambda f$ into $L = \dfrac{n}{2}\lambda$ and $L = \dfrac{n}{4}\lambda$ we obtain

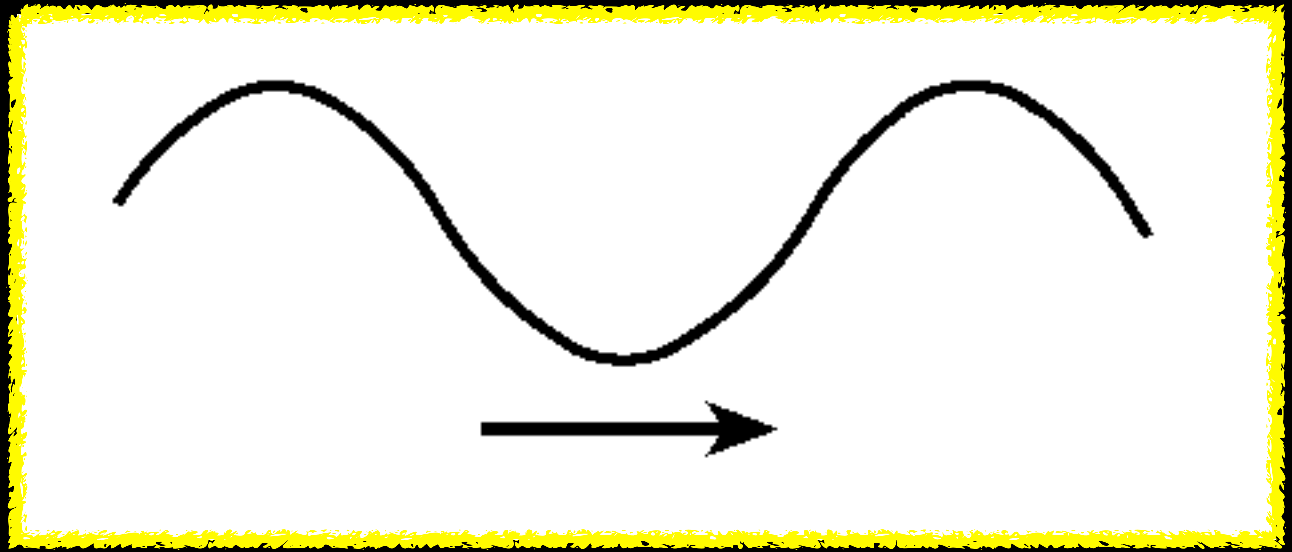
$$f_n = \frac{nu}{2L}, \quad n = 1,2,3,\cdots \qquad \text{and} \qquad f_n = \frac{nu}{4L}, \quad n = 1,3,5,\cdots$$

- It is straightforward to see that for equal ends the fundamental frequency is $f_1 = \dfrac{u}{2L}$ whereas for unequal ends we have $f_1 = \dfrac{u}{4L}$

Frequency Domain (cont'd)

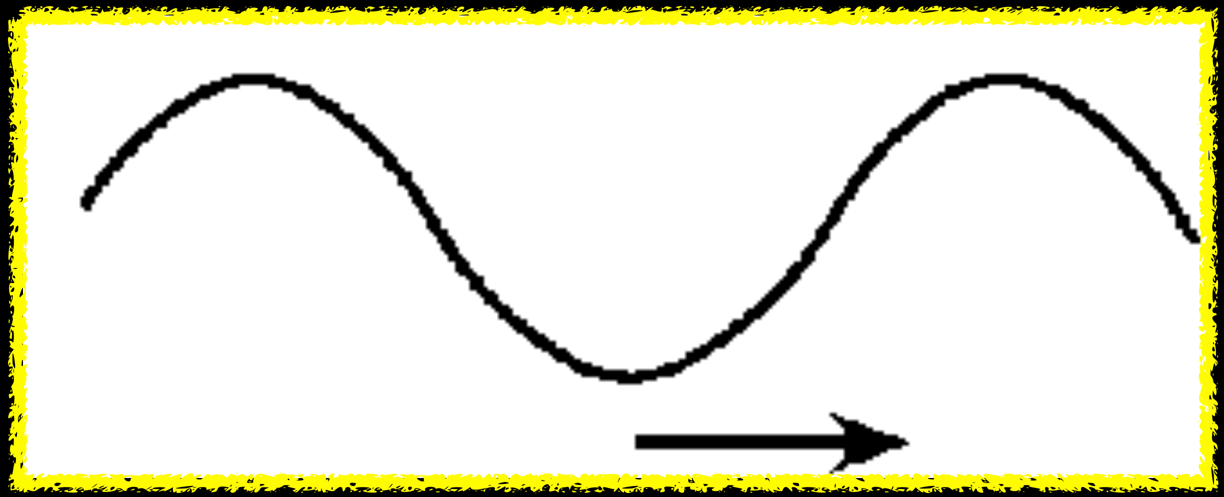
- Ultimately, we can see that all frequencies (f_n) are simply integer multiples of the fundamental frequency f_1 using $f_n = nf_1$
- The relation holds for both boundary conditions, where equal ends allow for all harmonics $(n = 1,2,3,\cdots)$ and unequal ends allow only for odd harmonics $(n = 1,3,5,\cdots)$
- A cavity with identical ends produces harmonics at every integer multiple of the fundamental frequency (e.g. $50,\ 100,\ 150$ Hz)
- Conversely, a cavity with dissimilar ends generates only odd-numbered multiples (e.g. $50,\ 150,\ 250$ Hz)

The diagram below represents a wave moving toward the right side of this page



Which wave shown below could produce a standing wave with the original wave?

A.



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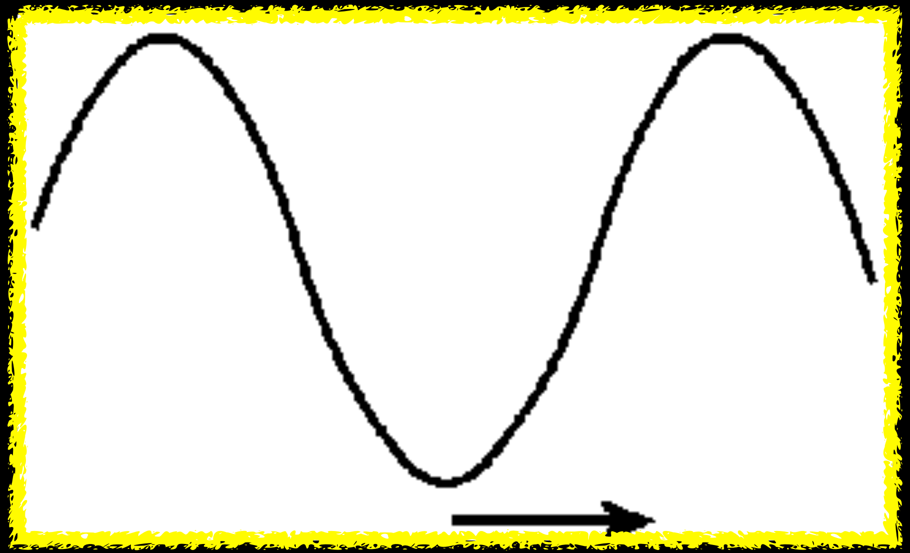

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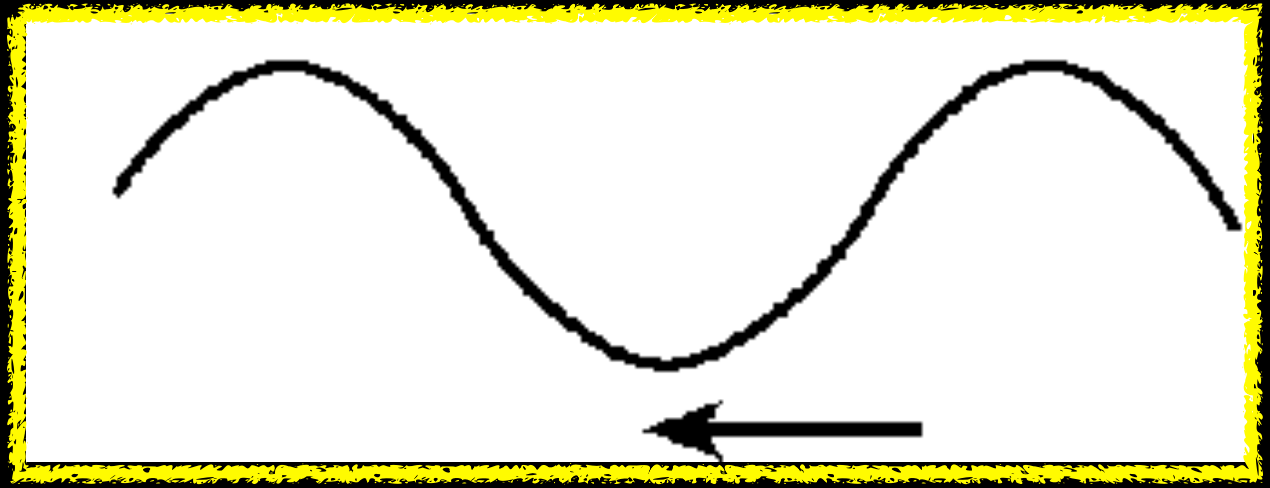
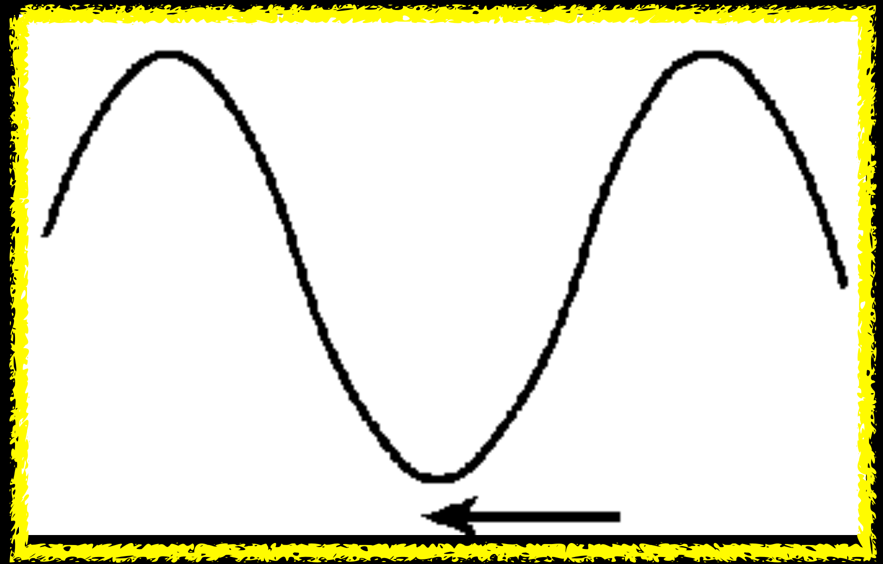


D.



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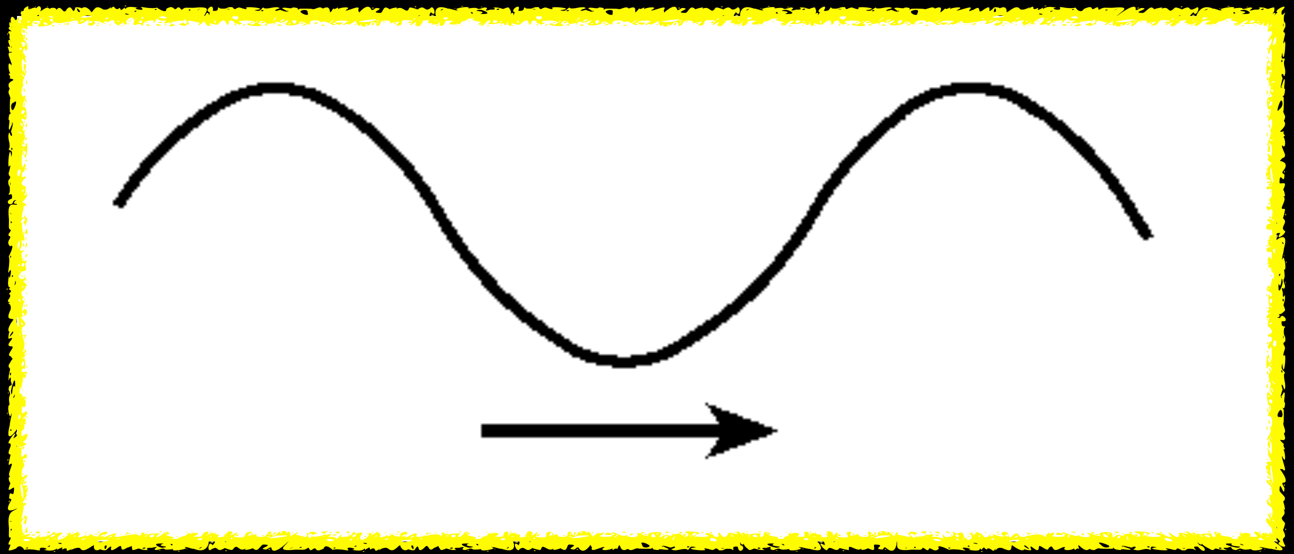

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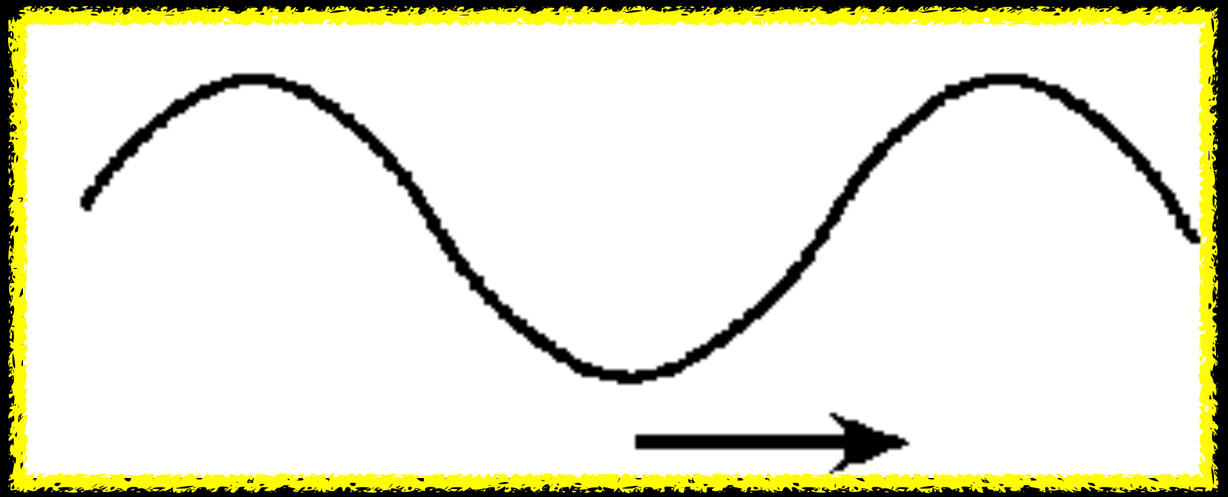
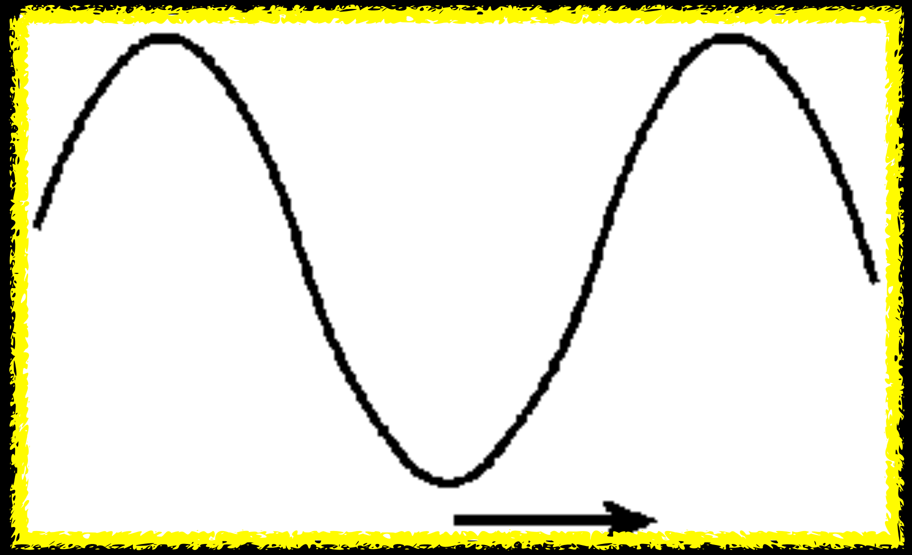
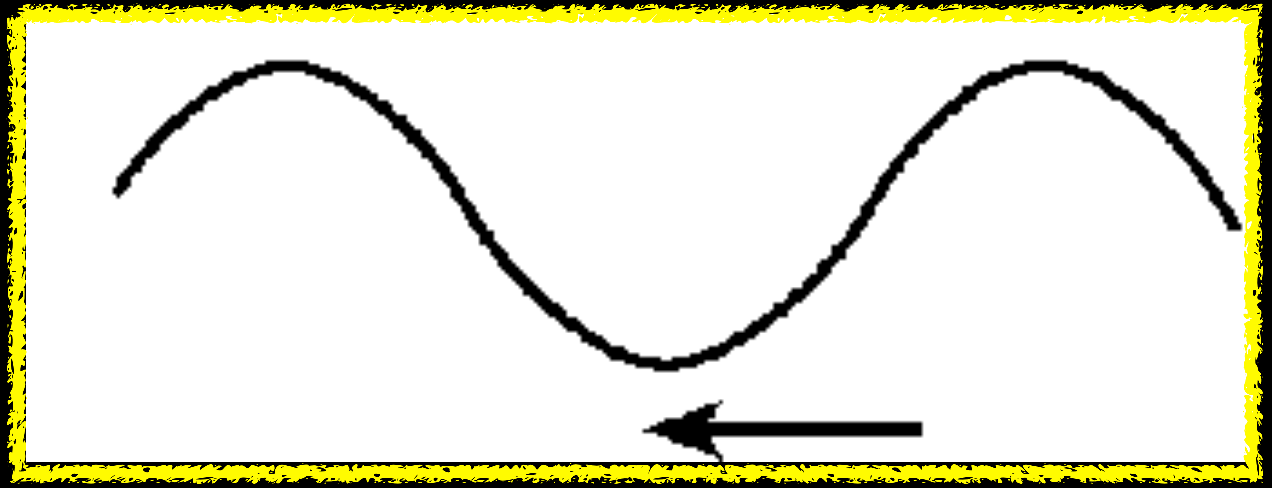


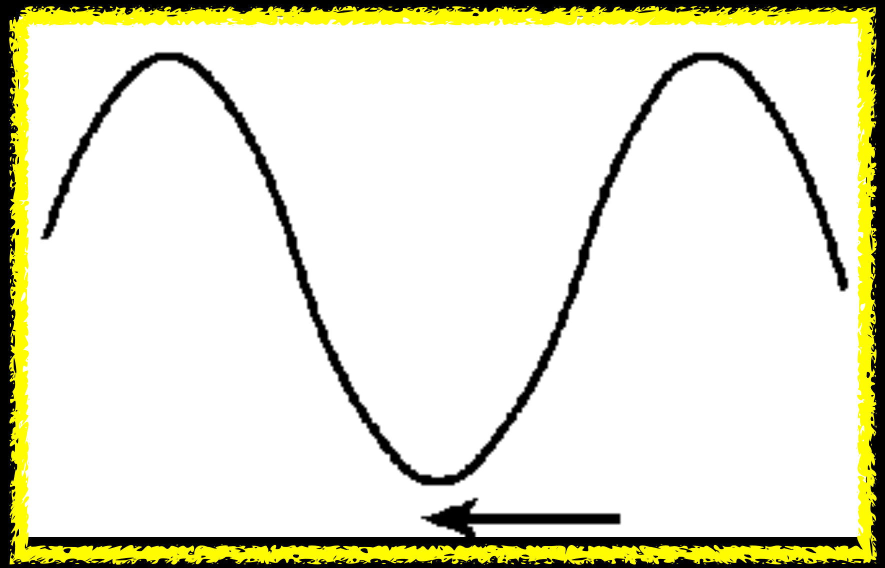
B.



C.



D.



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B. Have different amplitudes

C. Have different wavelengths

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What is the distance between two consecutive nodes on this standing wave?

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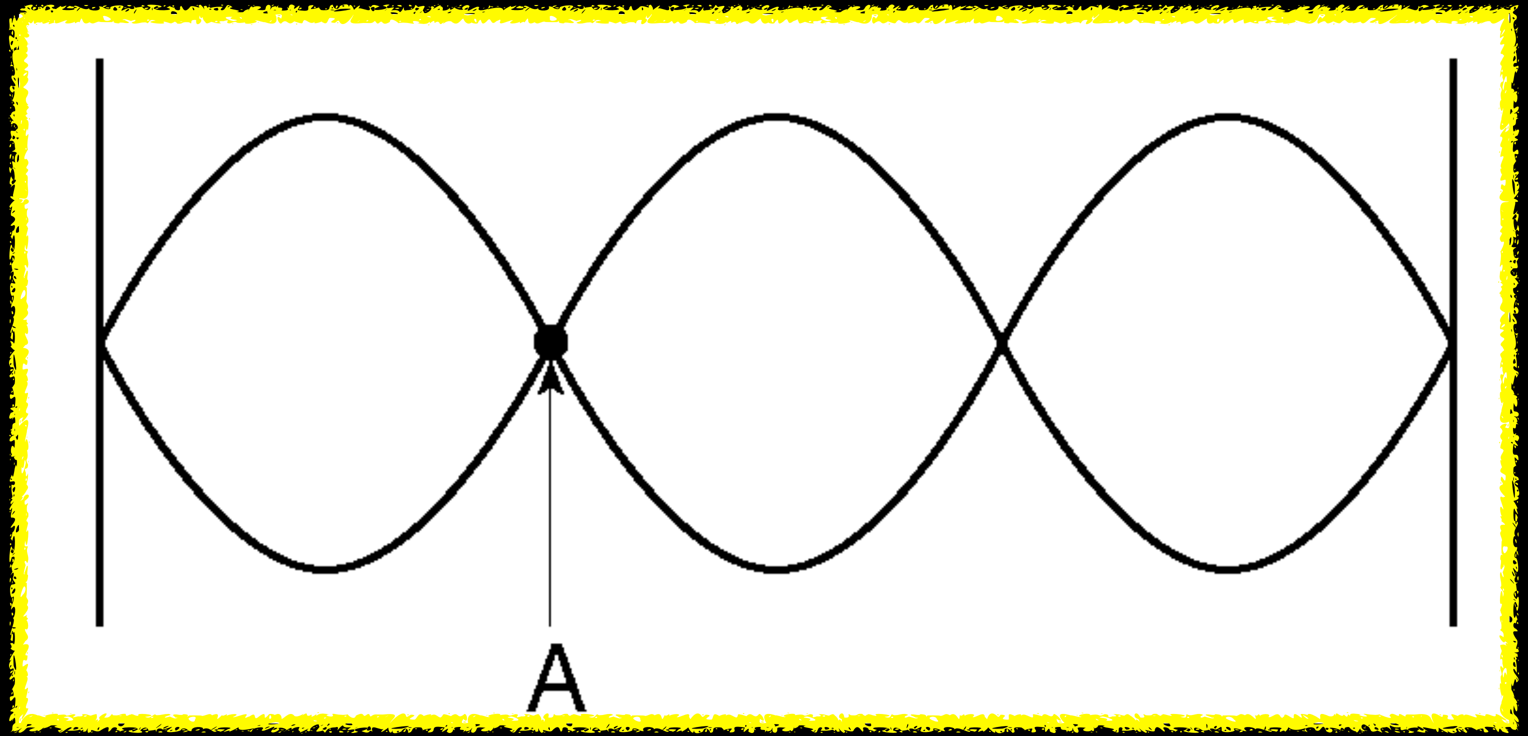
The diagram below shows a standing wave



Point A on the standing wave is

A. a node resulting from constructive interference

B. a node resulting from destructive interference

C. an antinode resulting from constructive interference

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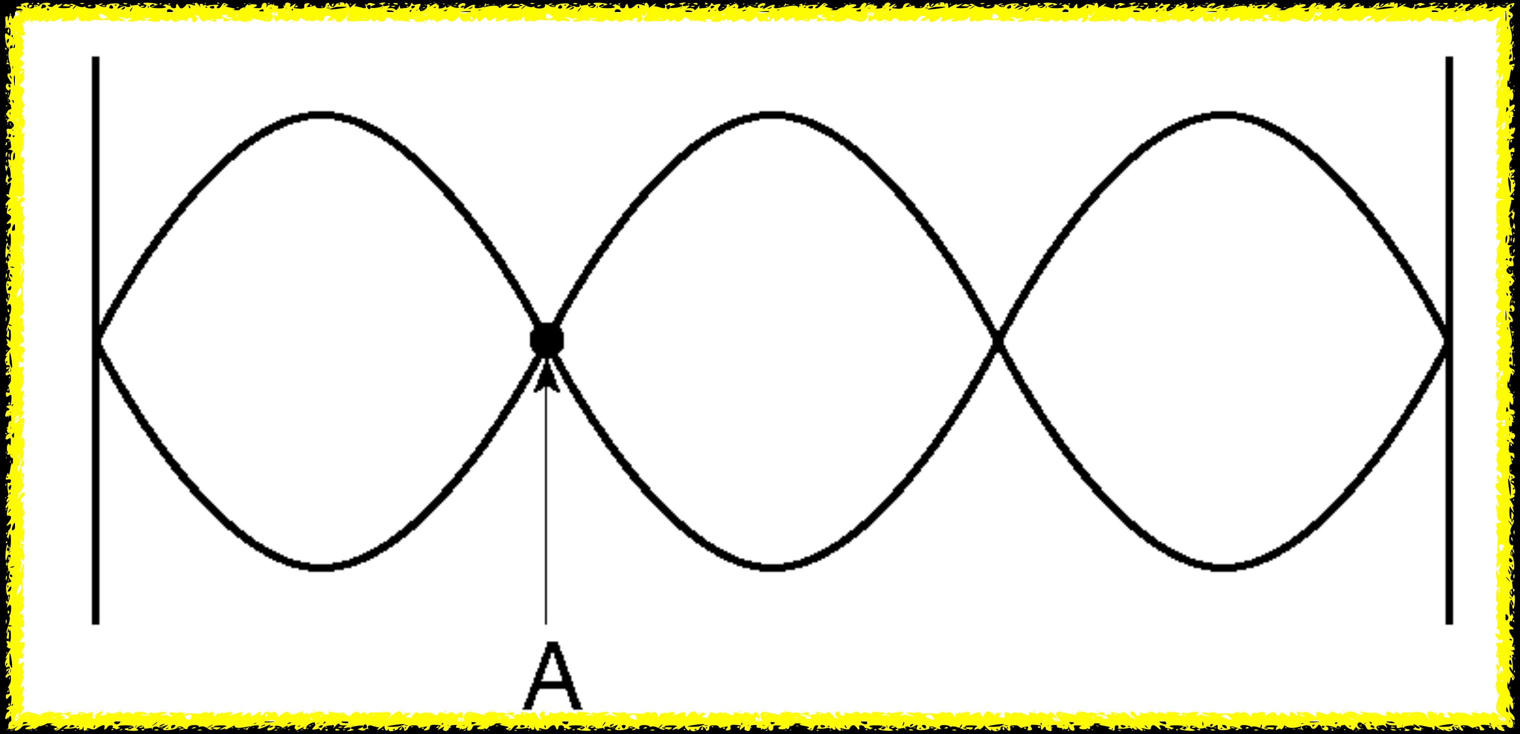

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