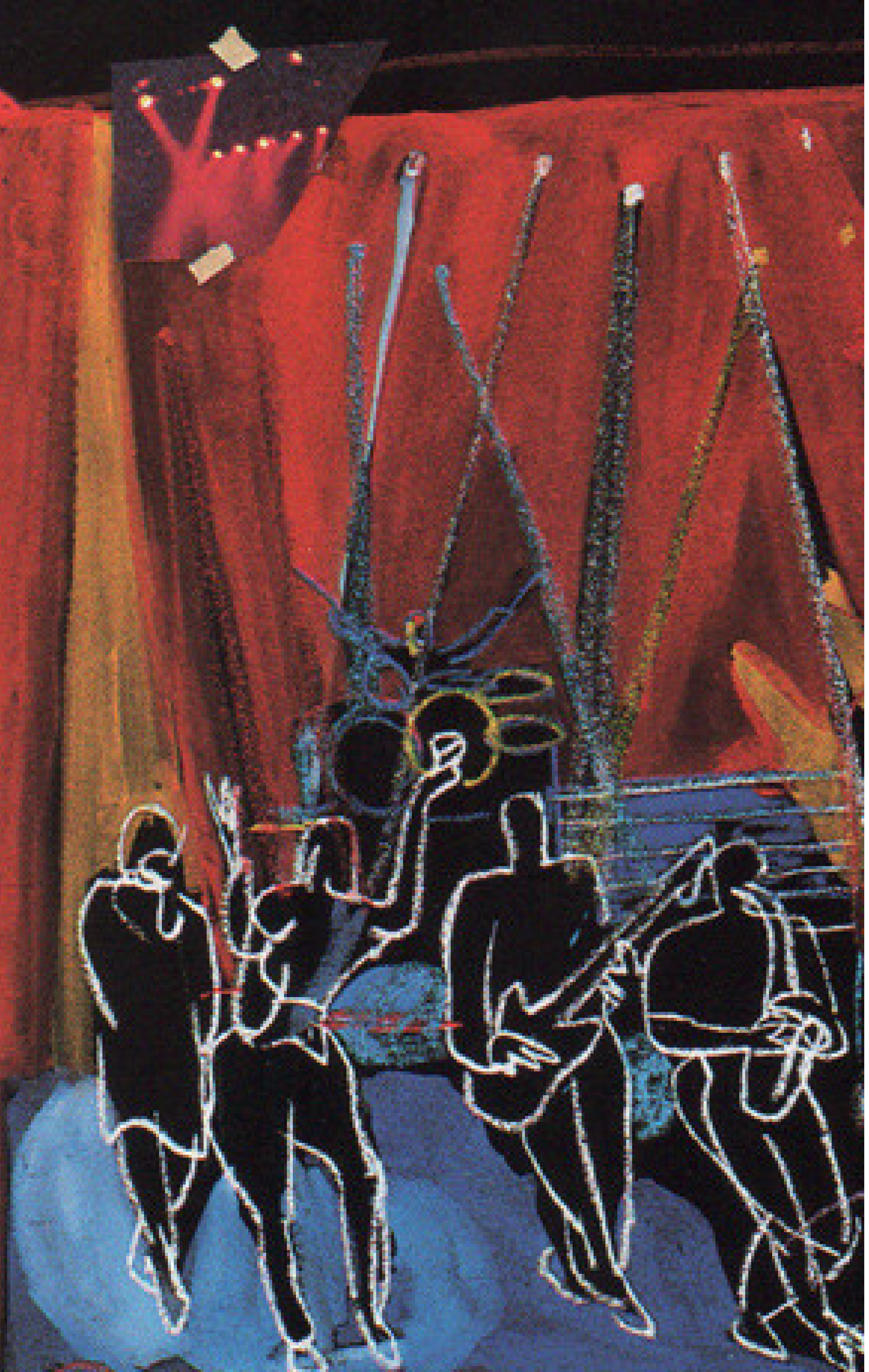


PHY-141

Wave Superposition

Luis Anchordoqui

Outline

- ➢ Wave superposition dictates that the combined disturbance of multiple overlapping waves is the algebraic sum of their individual displacements

- ➢ When waves occupy the same space, they interfere to produce a net, resultant wave without altering their original forms

- ➢ In today's class we examine this fundamental physical phenomenon

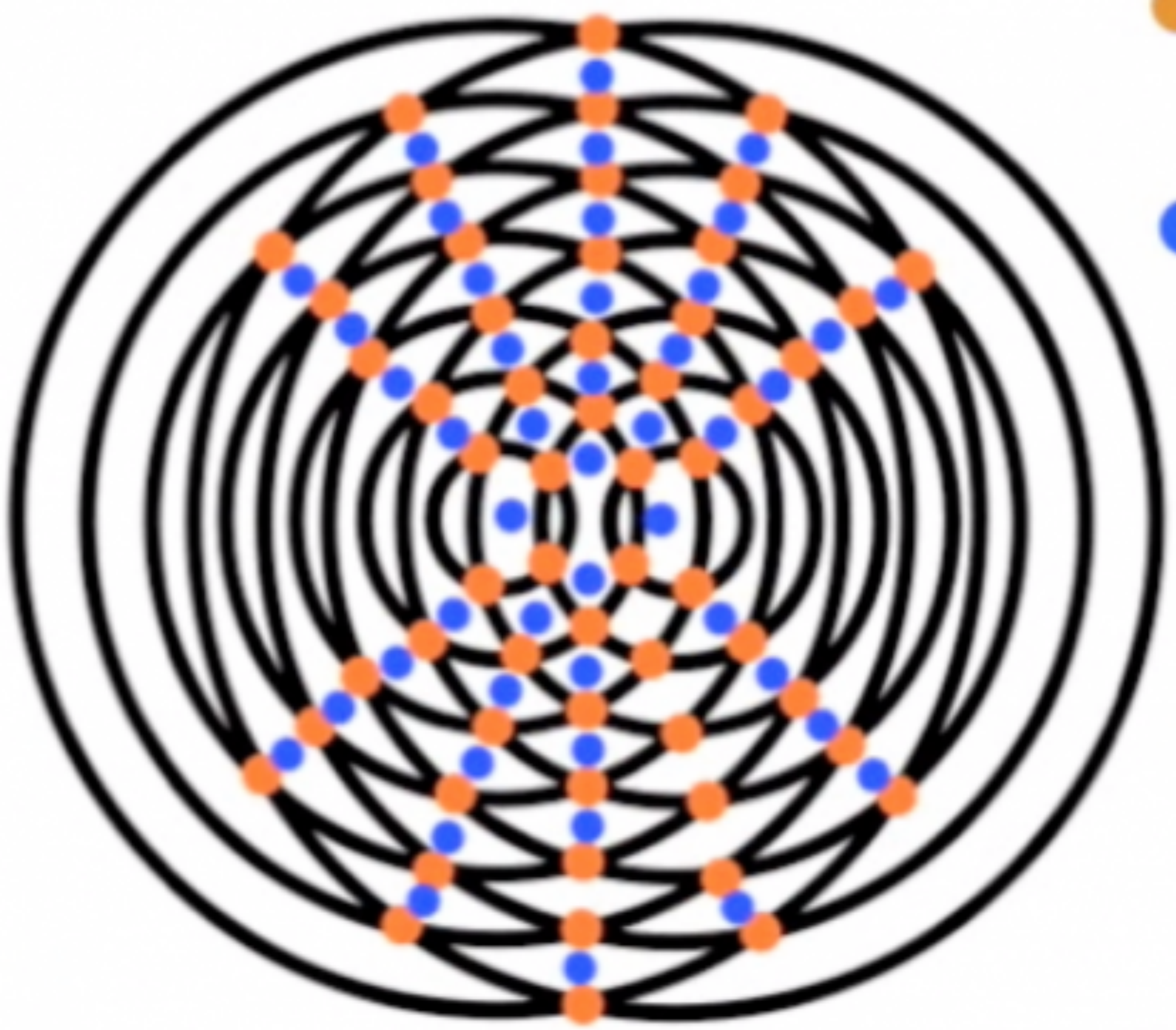
Interference

- To understand the basic principles underlying interference
- One of the most important properties of waves is the principle of superposition
- The principle of superposition for waves states that when two waves occupy the same point, their effect on the medium adds algebraically
- If two waves would individually have the effect $+1$ on a specific point in the medium, then when they are both at that point the effect on the medium is $+2$
- If a third wave with effect -2 happens also to be at that point, then the total effect on the medium is zero

- This idea of waves adding their effects, or canceling each other's effects, is the source of interference

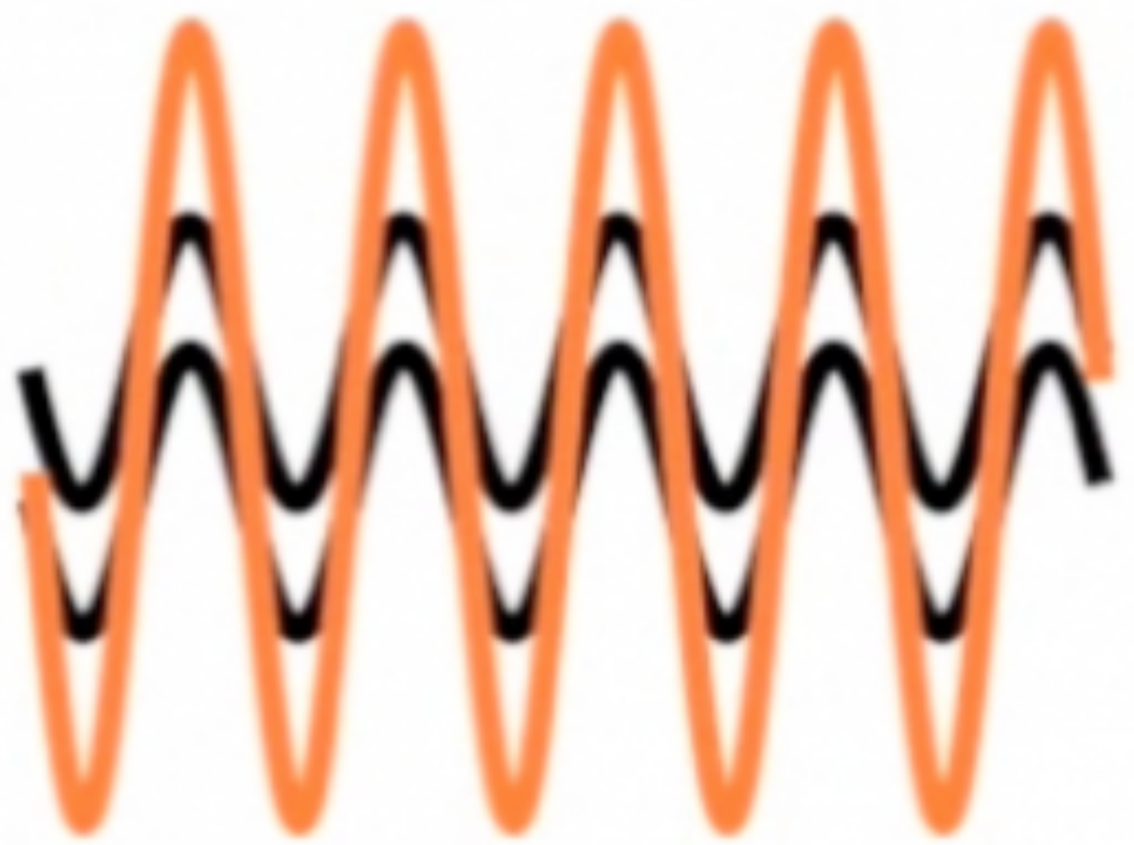
Constructive Interference

- *Constructive interference* occurs when overlapping waves combine to form a resultant wave with a larger amplitude than the individual waves



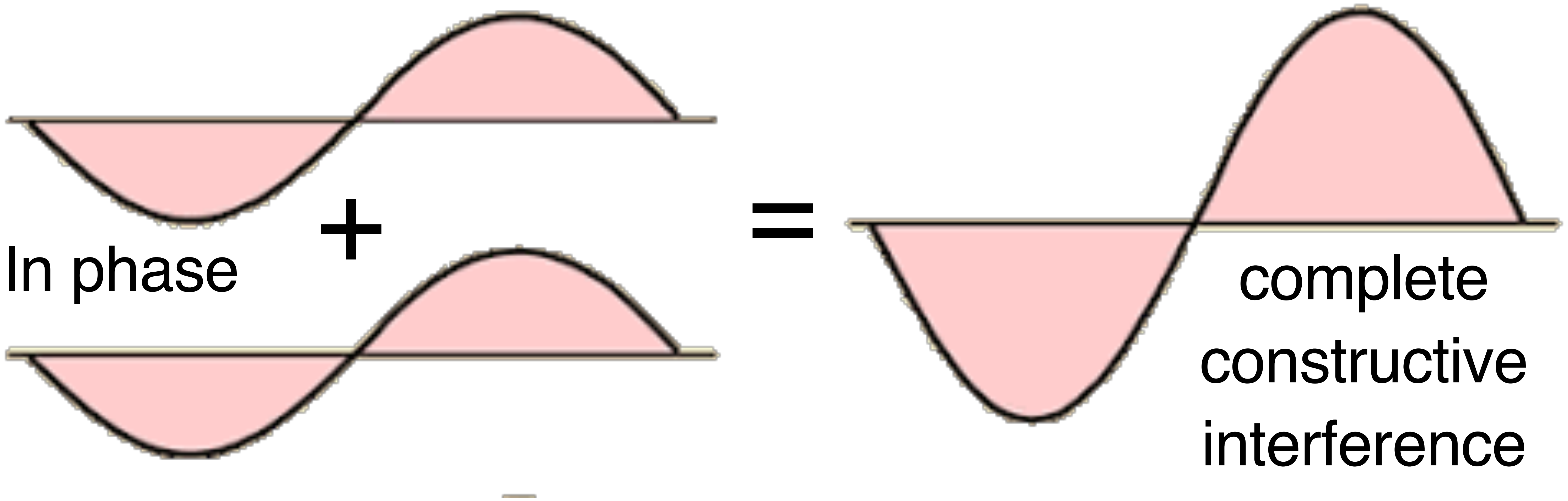
Marks the spot where two crests meet

Marks the spot where two troughs meet



Complete Constructive Interference

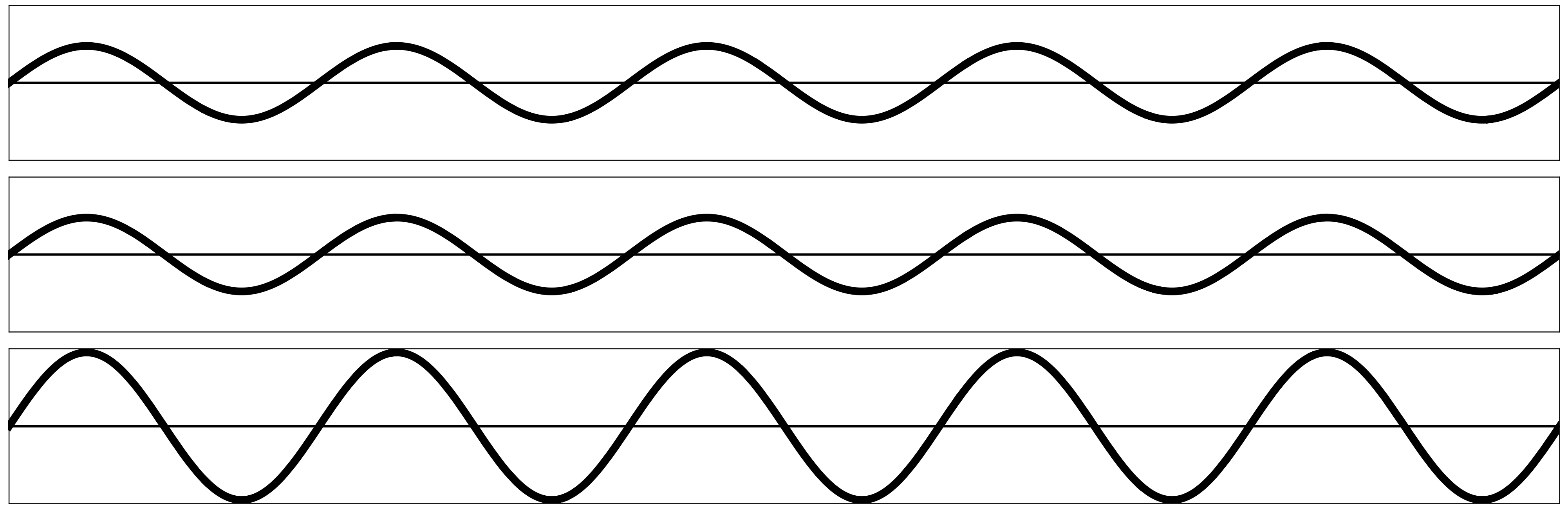
- When two waves with the same frequency and phase (in-phase) meet, they undergo complete constructive interference
- Because their crests and troughs perfectly align, their displacements add together at every point, resulting in a single wave with an amplitude equal to the sum of the individual amplitudes ($A_{\text{total}} = A_1 + A_2$)



In algebraic terms…

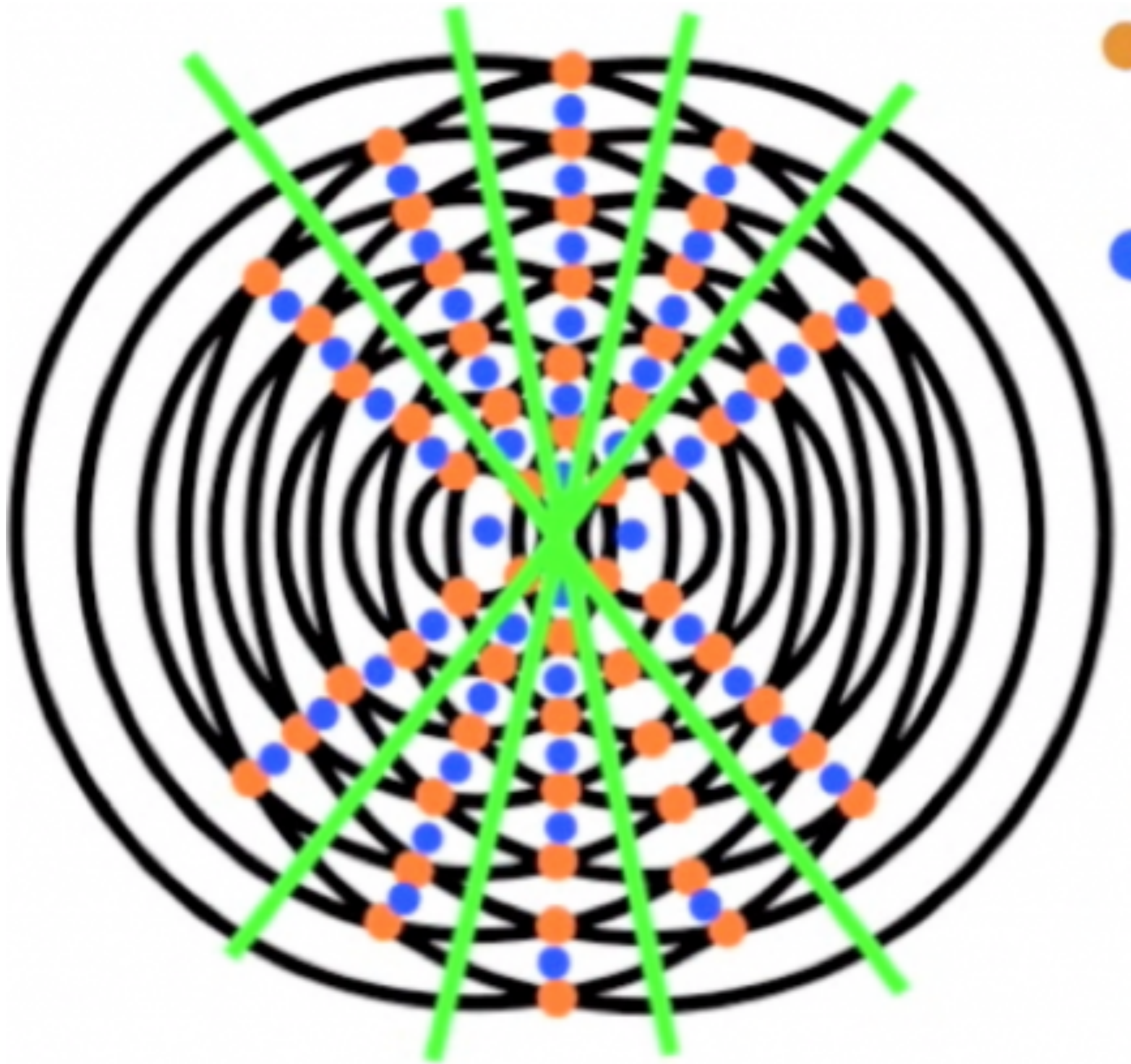
- Imagine two waves with displacements $x_1 = A_1 \sin(\omega_1 t + \delta_1)$ and $x_2 = A_2 \sin(\omega_2 t + \delta_2)$ such that $\omega_1 = \omega_2 = \omega$ and $\delta_1 = \delta_2 = \delta$
- Using the principle of superposition, the total displacement $x_{\rm total}$ is the sum of the individual displacements ☛ $x_{\rm total} = x_1 + x_2 = A_1 \sin(\omega t + \delta) + A_2 \sin(\omega t + \delta)$
- Because the $\sin(\omega t + \delta)$ term is common to both, we can factor it out to obtain

  ☛ $x_{\rm total} = (A_1 + A_2)\sin(\omega t + \delta)$



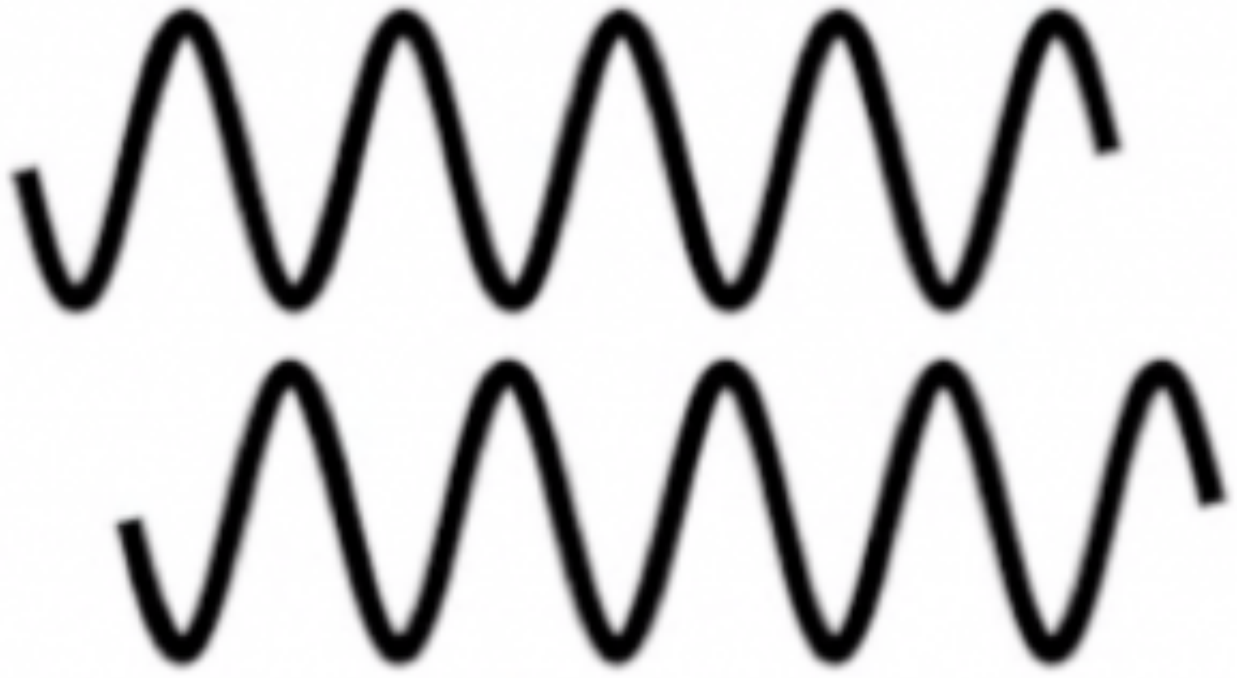
Destructive Interference

Destructive interference occurs when overlapping waves subtract, reducing the overall amplitude



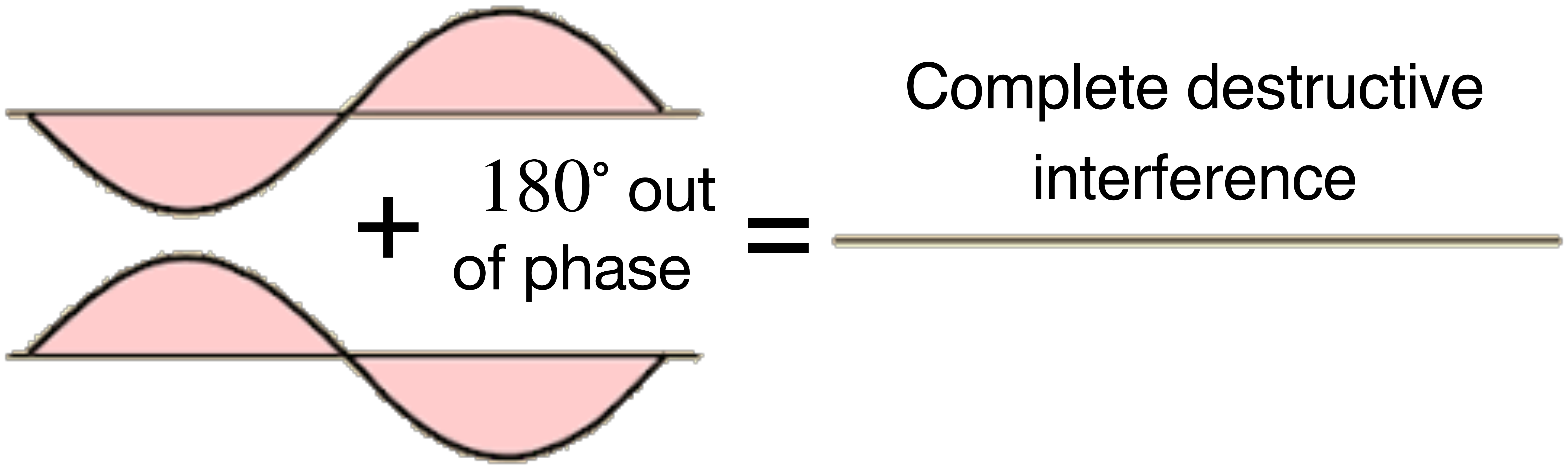
Marks the spot where two crests meet

Marks the spot where two troughs meet



Complete Destructive Interference

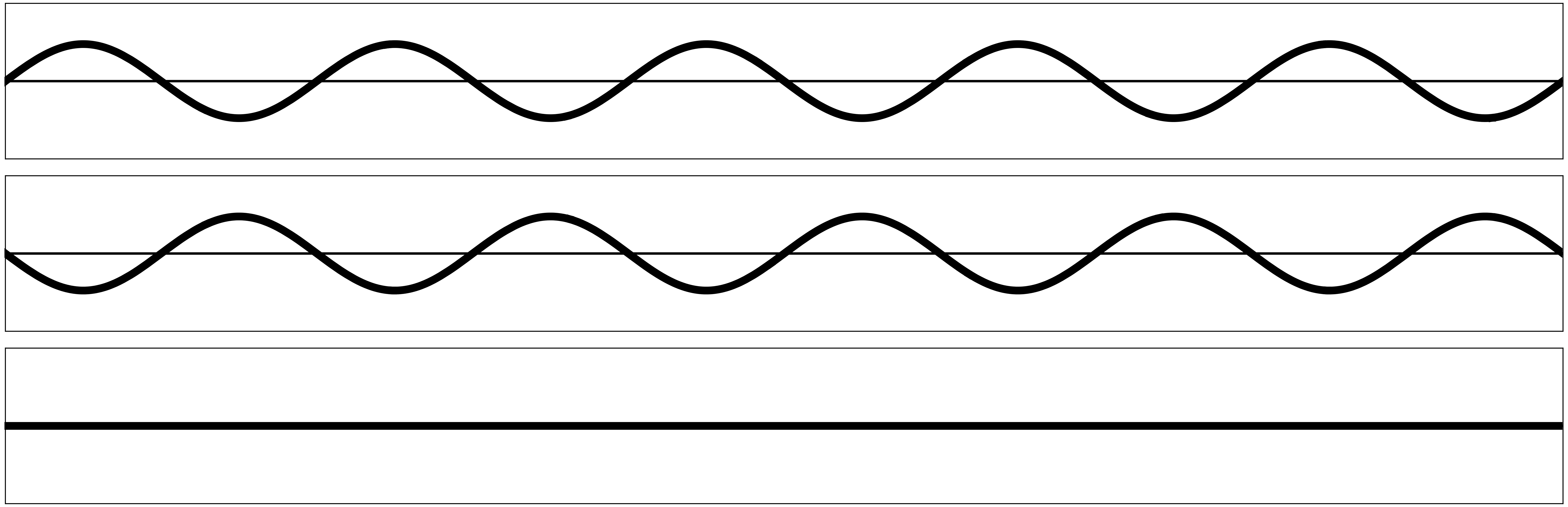
- Complete destructive interference is a phenomenon where two waves combine, causing their crests and troughs to cancel each other out
- In particular, complete destructive interference occurs when two waves of identical frequency and amplitude are 180° (π radians) out of phase, meaning the crest of one aligns with the trough of another
- This results in the complete cancellation of the waves, producing a net amplitude of zero



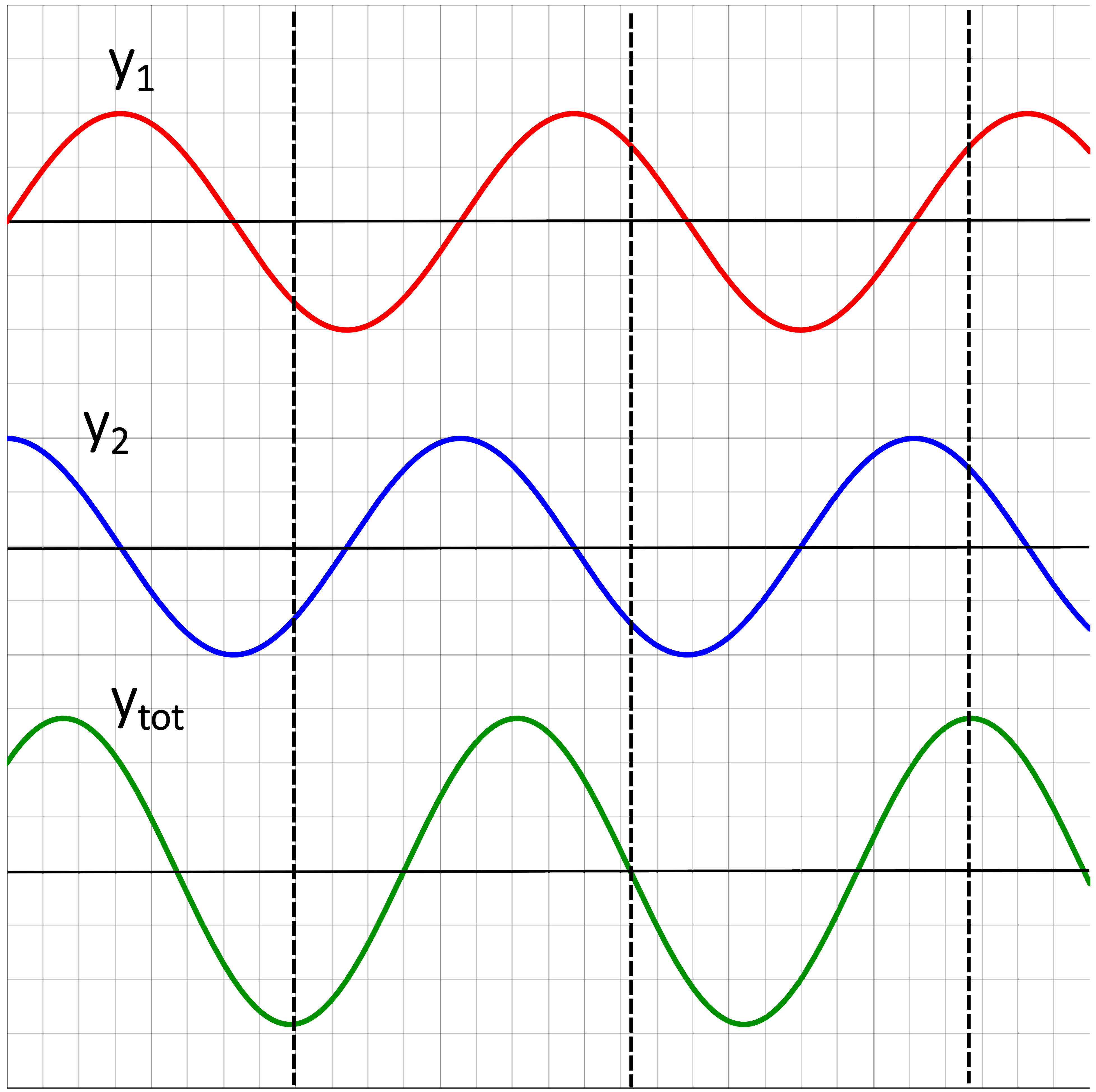
In algebraic terms…

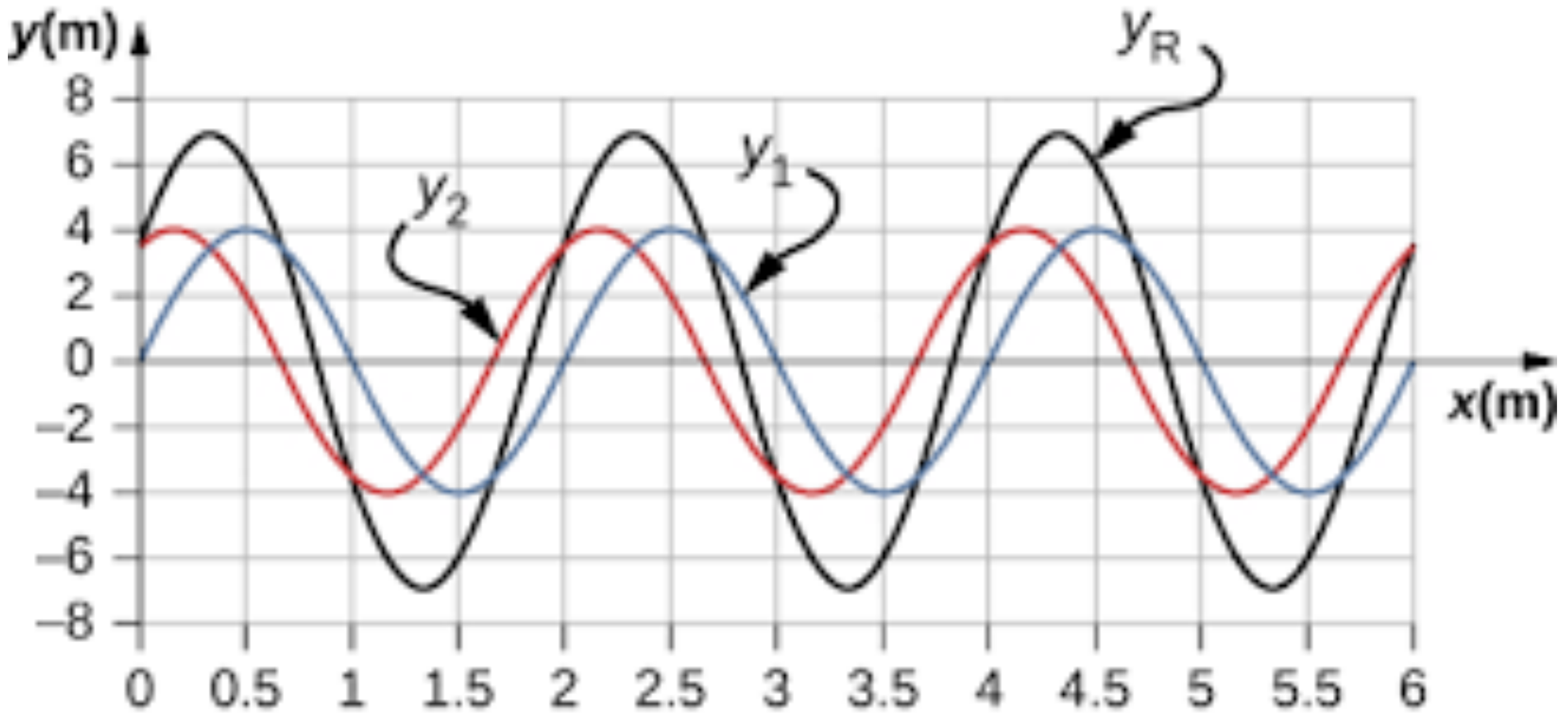
- $x_1 = A\sin(\omega t)$ and $x_2 = A\sin(\omega t + \pi) = -A\sin(\omega t)$

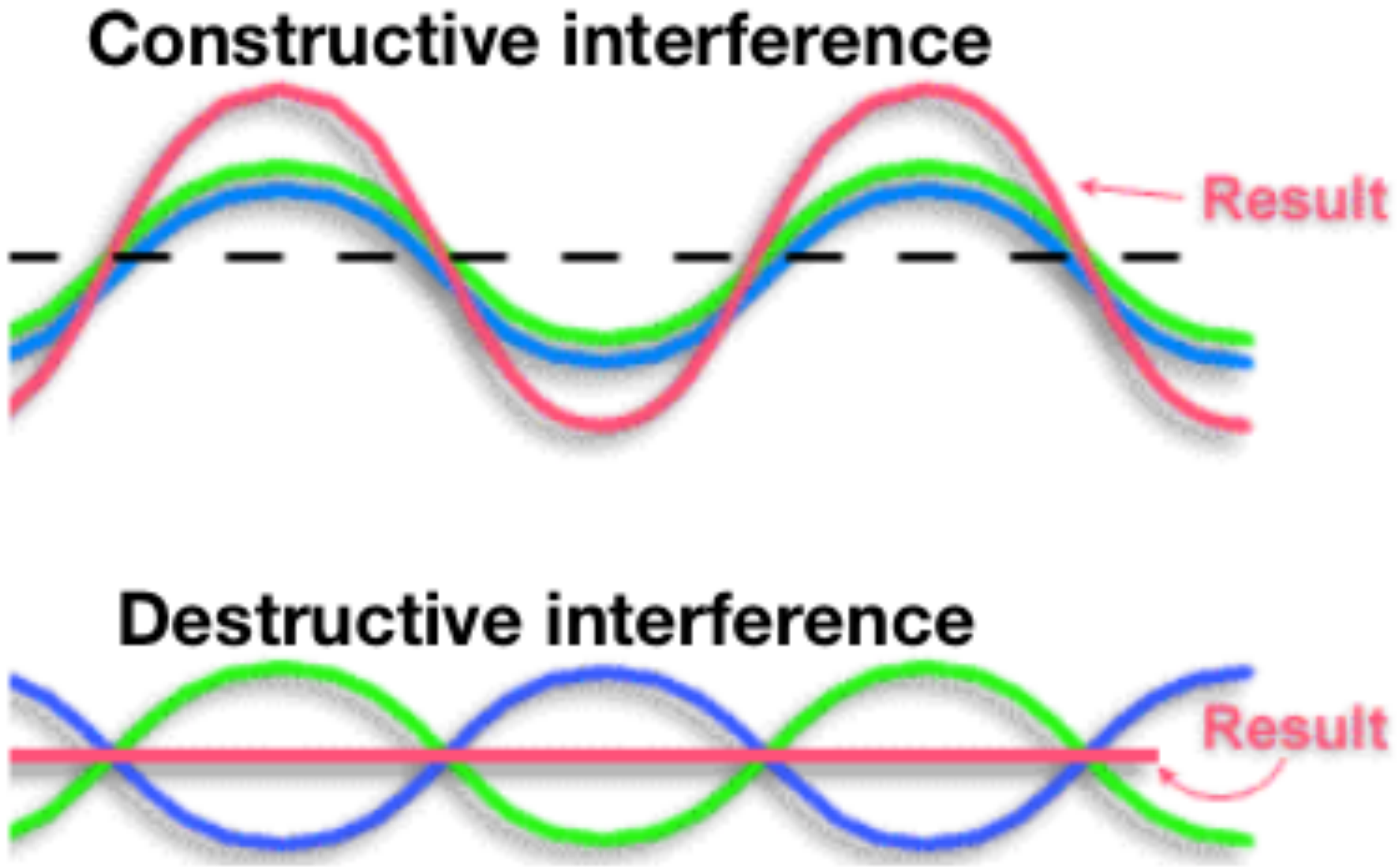
  where we used the trigonometric identity $\sin\theta = -\sin(\theta + \pi)$
- Note that $x_1 + x_2 = 0$



Examples of Interference and Complete Interference

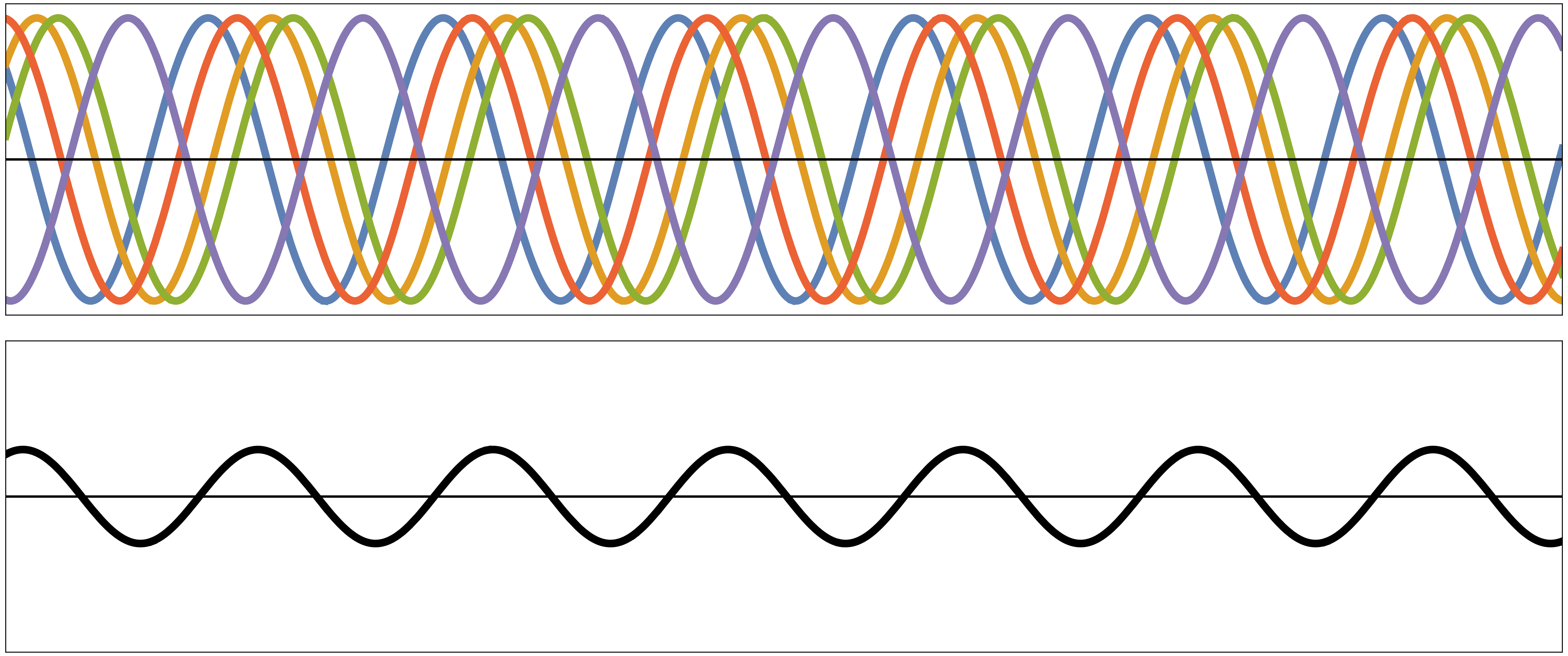






Interference of multiple waves

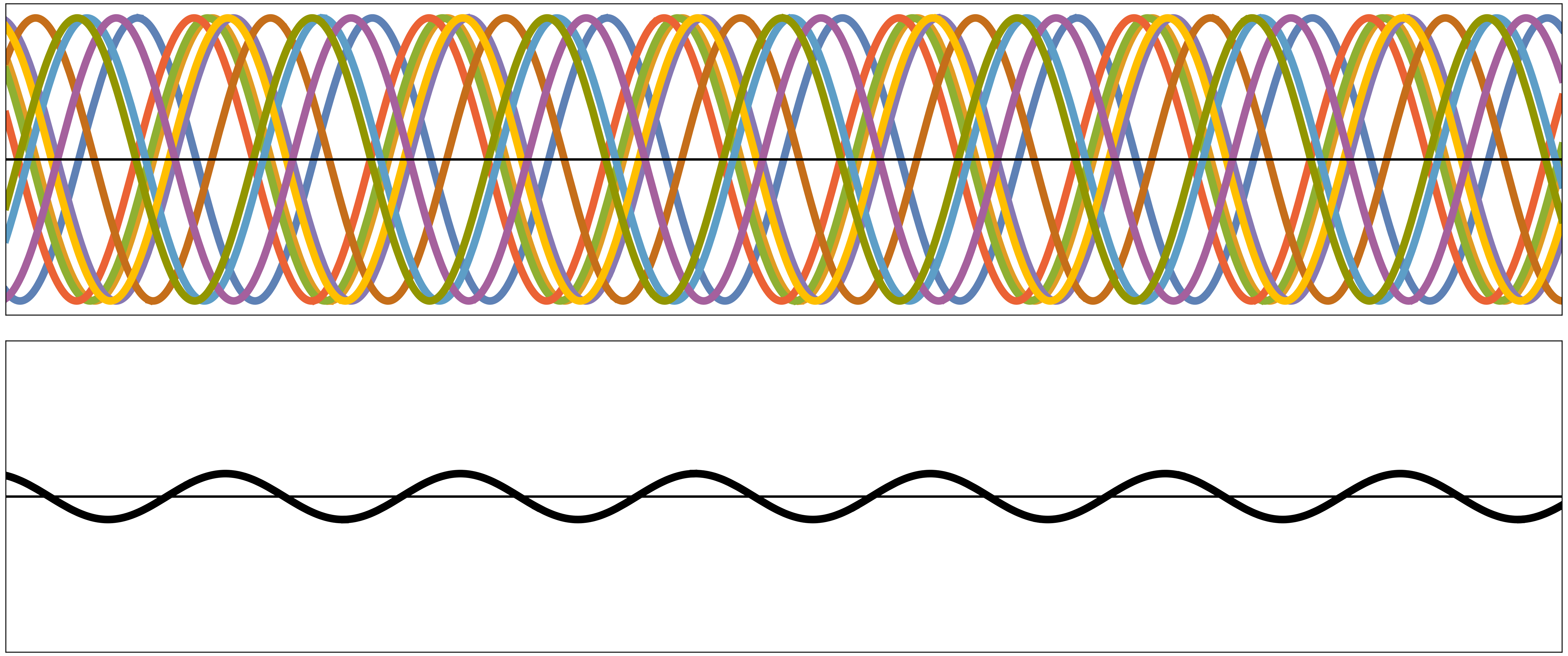
- Usually, waves traveling in the same direction are neither perfectly in phase nor perfectly in counter-phase
- That is, often, interference is neither constructive nor destructive, but somewhere in between



Sum of 5 randomly shifted waves

Interference of multiple waves

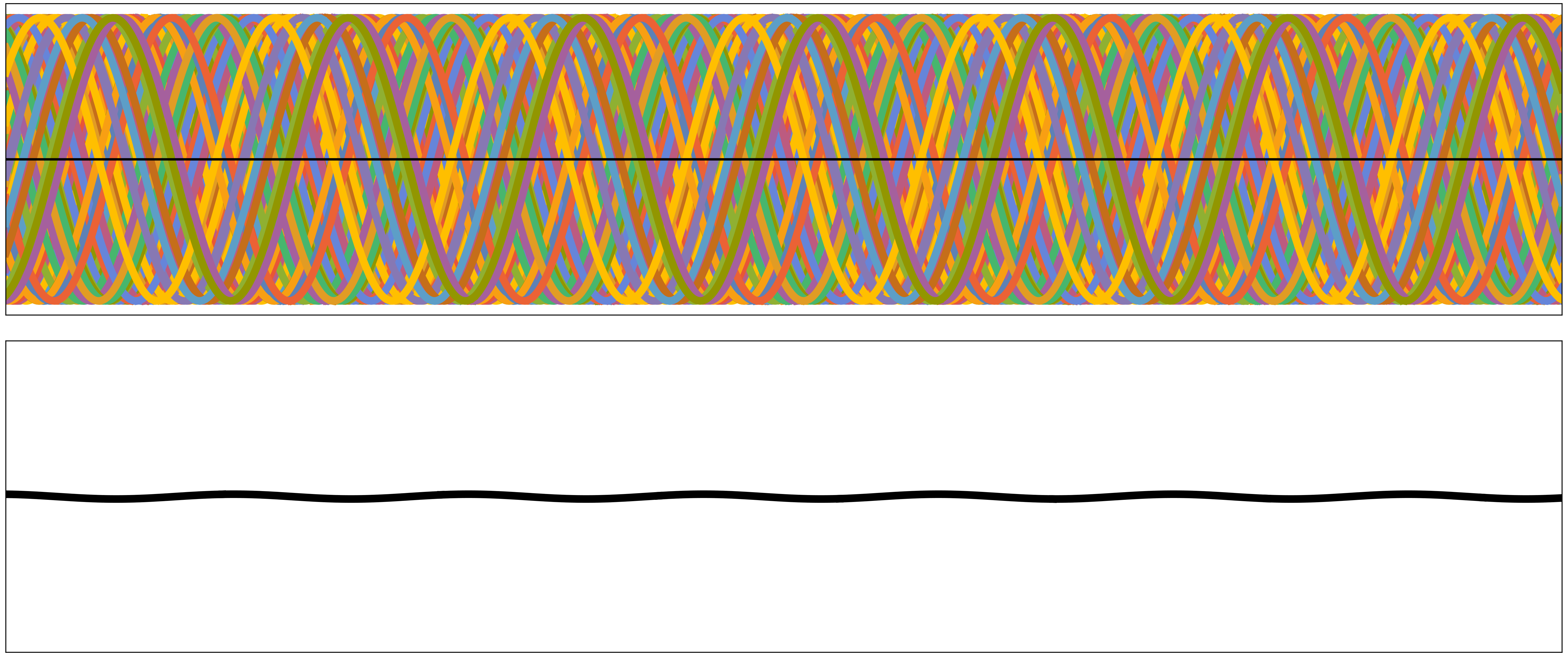
- Usually, waves traveling in the same direction are neither perfectly in phase nor perfectly in counter-phase
- That is, often, interference is neither constructive nor destructive, but somewhere in between



Sum of 10 randomly shifted waves

Interference of multiple waves

- Usually, waves traveling in the same direction are neither perfectly in phase nor perfectly in counter-phase
- That is, often, interference is neither constructive nor destructive, but somewhere in between



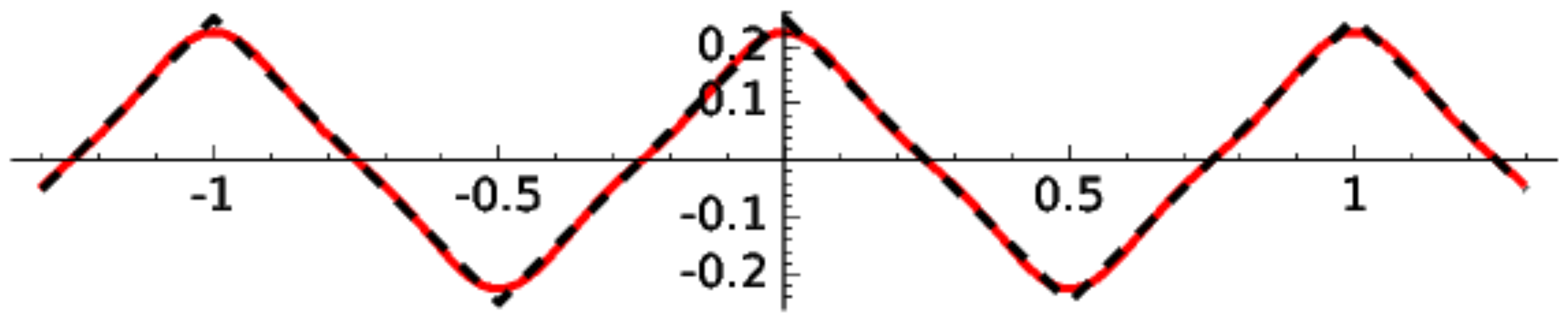
Sum of 100 randomly shifted waves

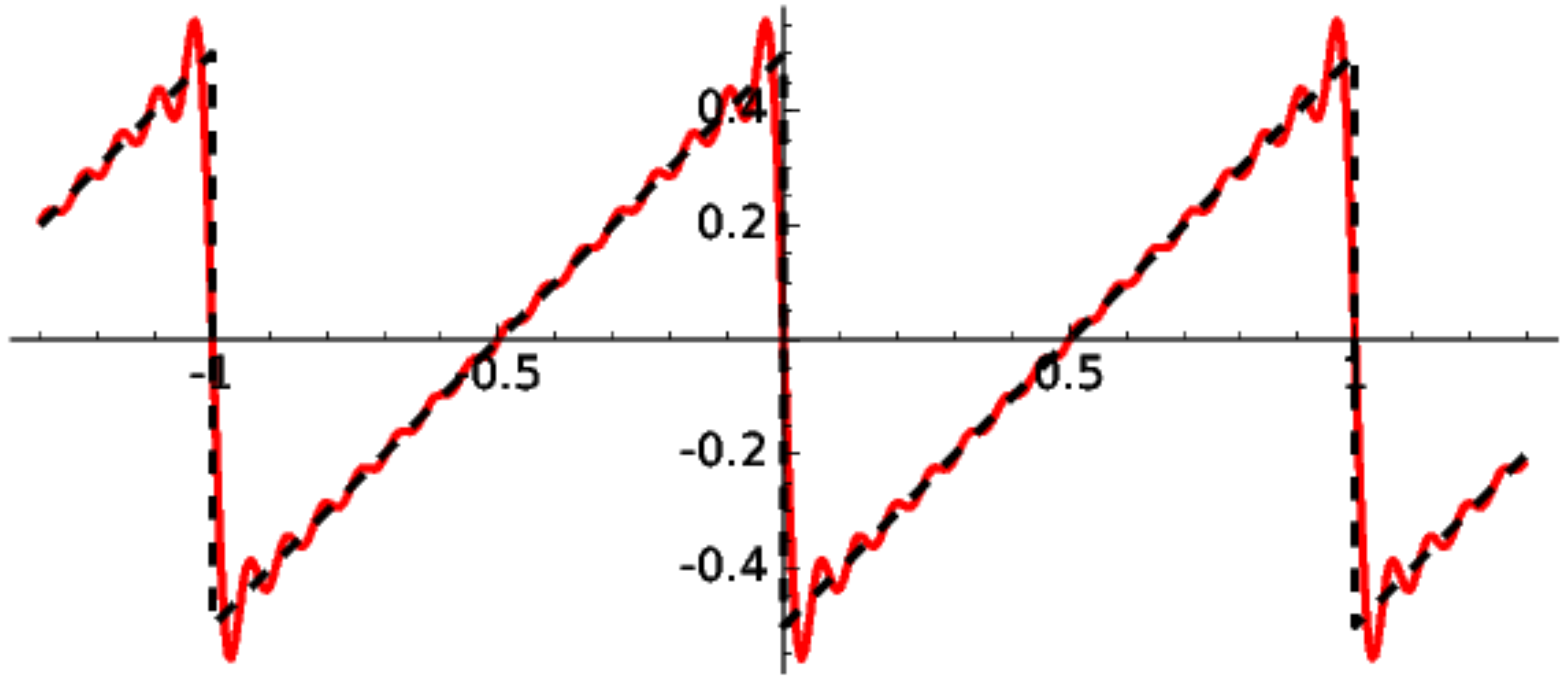
Resolving Wave Patterns into Component Sinusoids

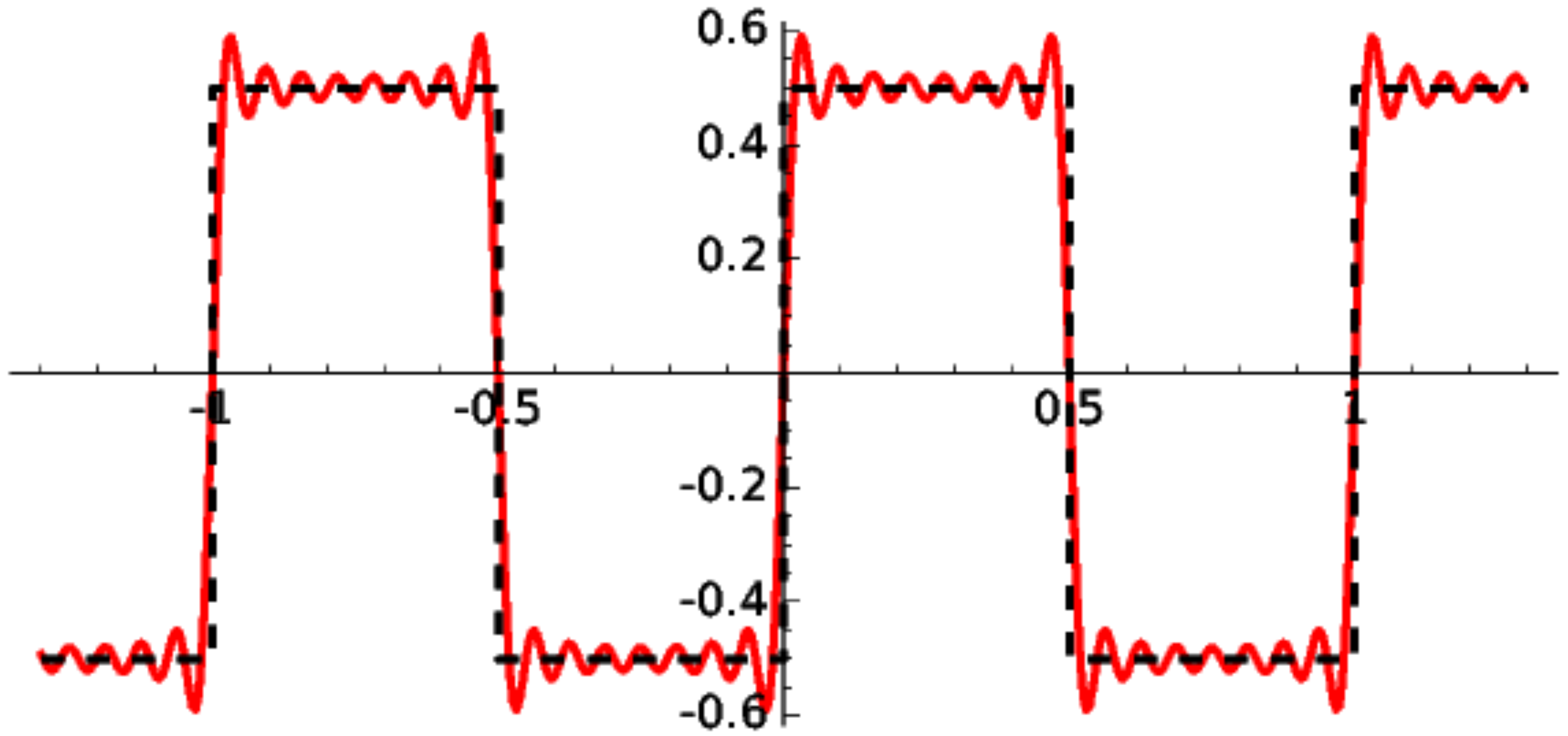
- Sinusoids occupy a foundational role in wave analysis,

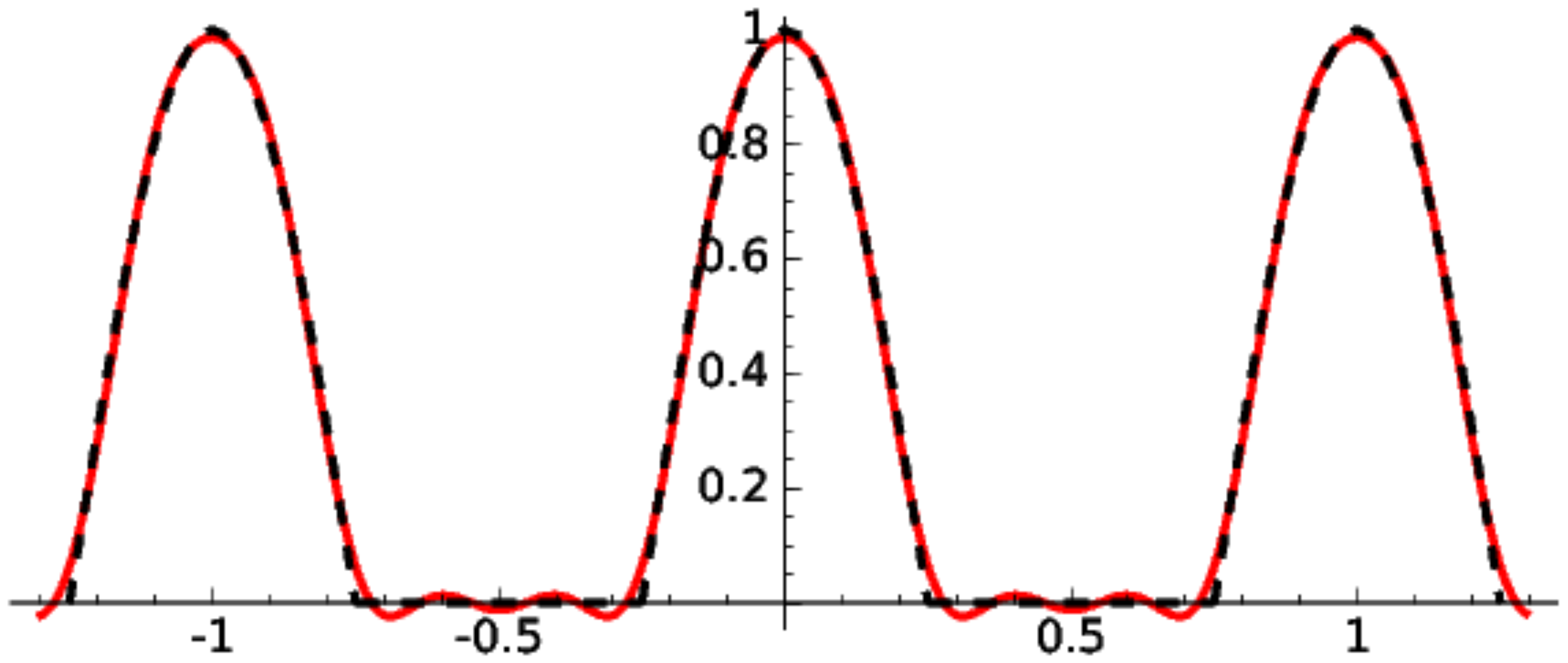
  as Fourier analysis permits any complex waveform to be decomposed into constituent sine and cosine functions

- Fourier's theorem asserts that any periodic, continuous wave can be broken down into a sum of simple sinusoidal waves (partial waves) with specific amplitudes and phases



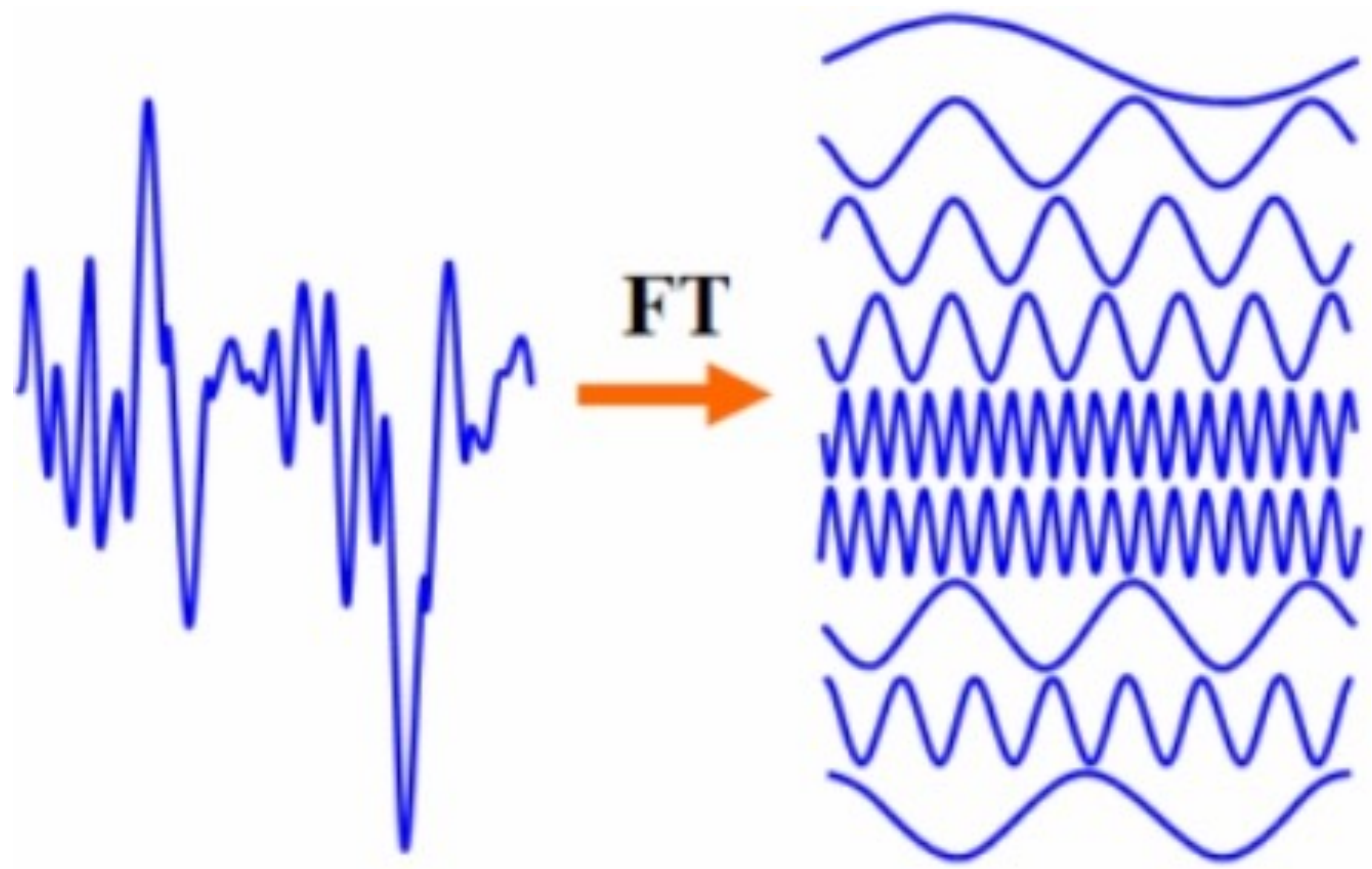






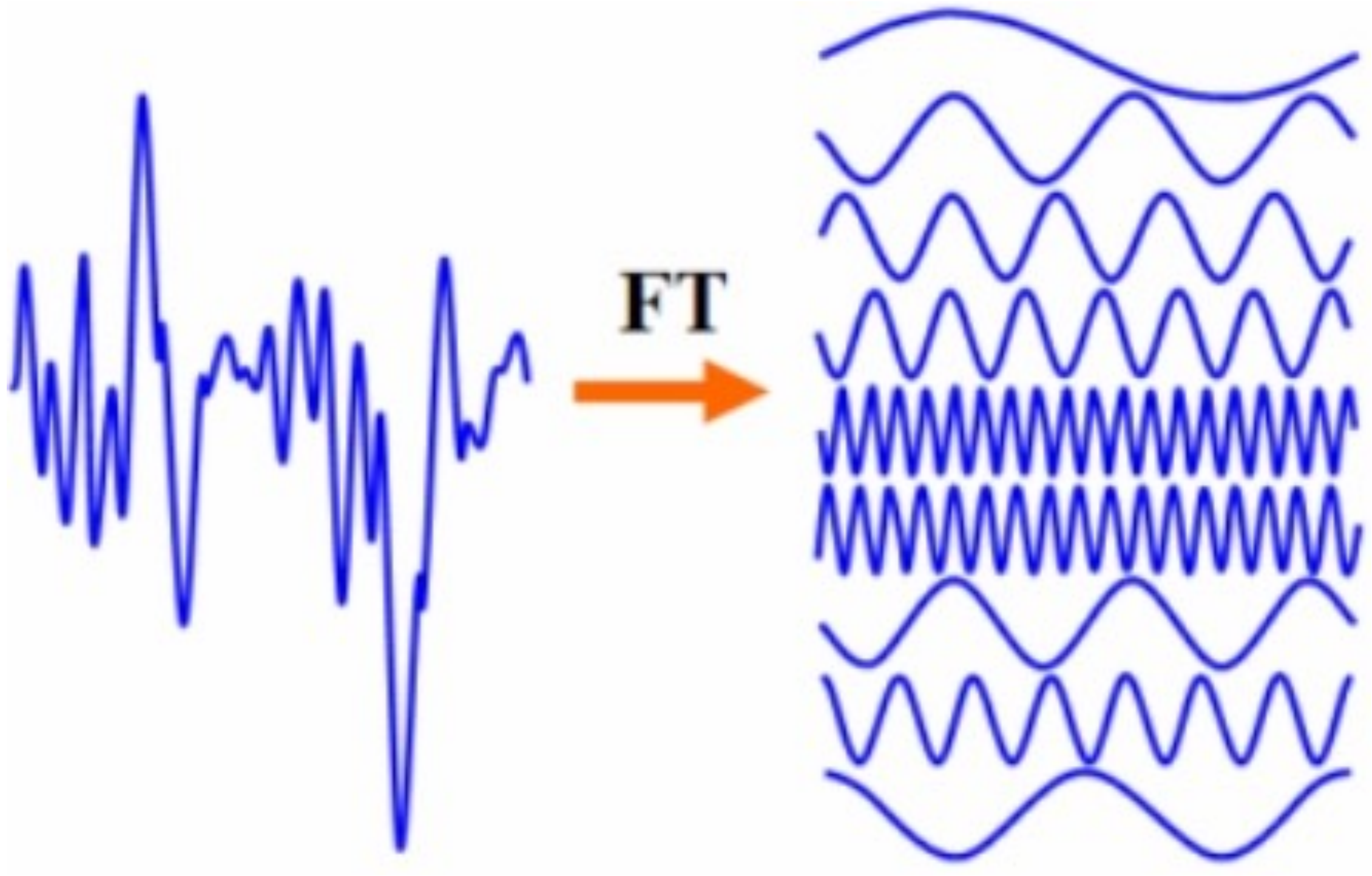
Fourier Theorem

- Imagine a highly complex, non-sinusoidal waven $W_{\text{non-sinusoidal}}$
- Fourier theorem states that $W_{\text{non-sinusoidal}} = \sum_{n=1}^{\infty} w(A_n, f_n)$ where each $w(A_n, f_n)$ is an individual sinusoidal paprtial wave with a specific frequency f_n and amplitude A_n
- Recall that the $\sum$ symbol indicates that we are adding together multiple waves,
  - $W_{\text{non-sinusoidal}} = w(A_1, f_1) + w(A_2, f_2) + w(A_3, f_3) + \cdots$



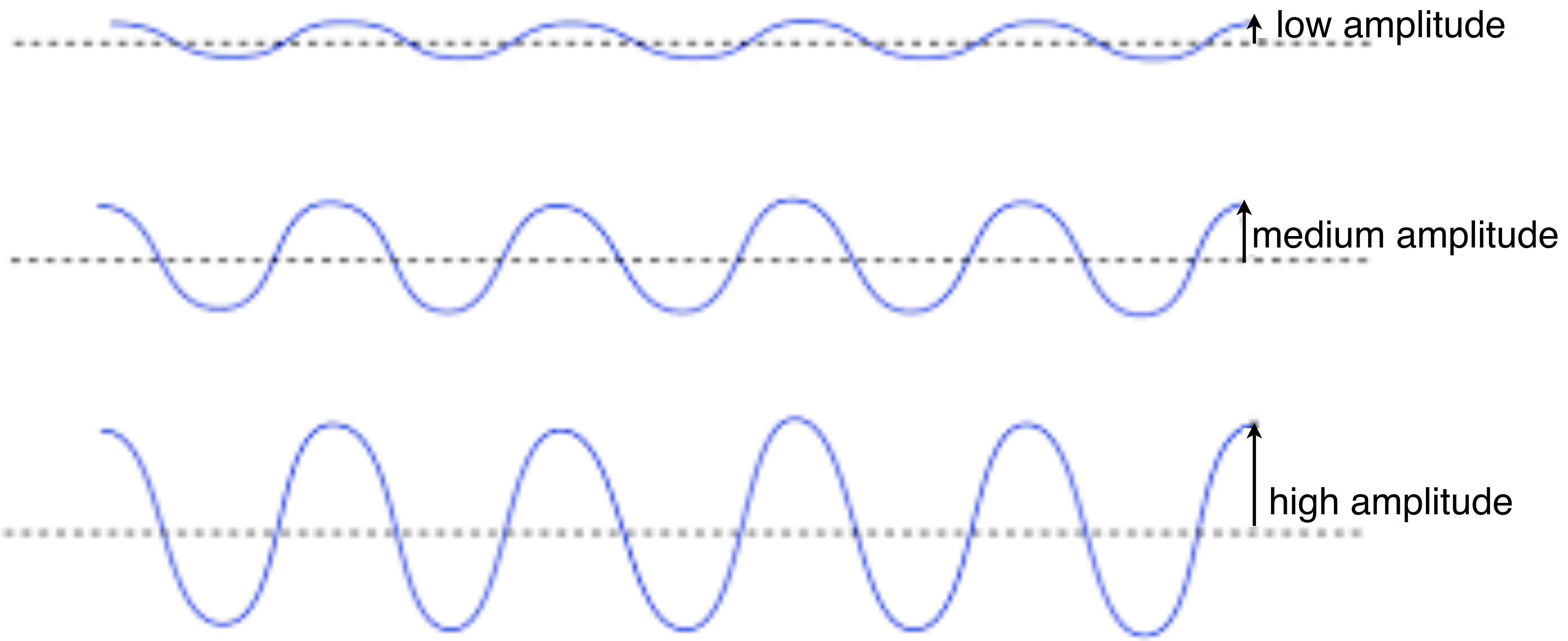
Remarkable Property of Fourier Sums

- Note that the sum consists of an infinite series of waves, which may appear computationally inefficient compared to simply using the single wave $W_{\text{non-sinusoidal}}$
- This is not a practical concern; because these waves are not equally significant, we do not need to use all of them



Example: Wave with Rapidly Decreasing Amplitude

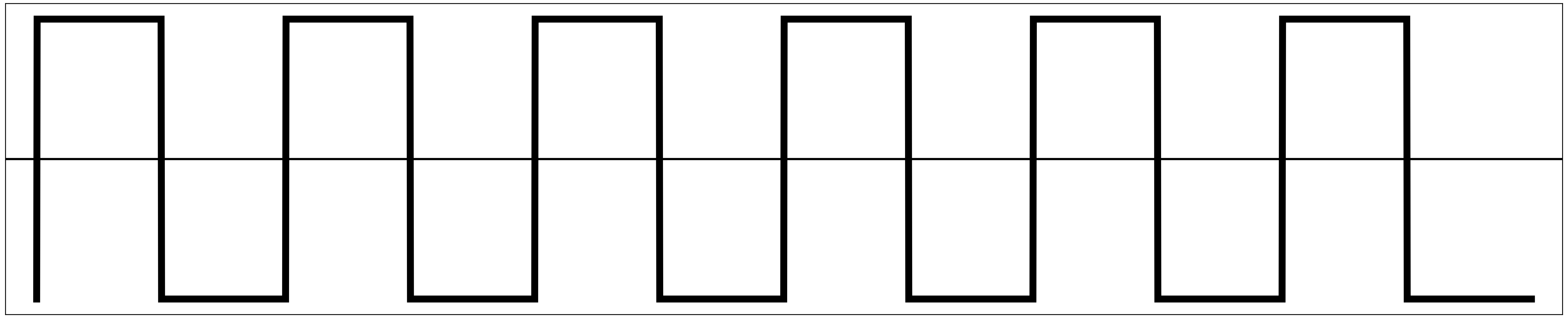
- Consider a wave decomposed into sinusoidal components with rapidly decreasing amplitudes (e.g., $A_1 = 100,\ A_2 = 10,\ A_3 = 1$)
- By the time you reach the tenth component $(A_{10} = 10^{-7})$, its amplitude is a billion times smaller than the first
- Because its contribution is statistically negligible, you can discard it *along with all subsequent, even smaller waves* without significantly altering the result
- You can achieve a highly accurate approximation using only the first nine waves rather than an infinite series
- While requirements vary, sometimes requiring 100, sometimes 1000, you do not need an infinite number of waves
- Always prioritize retaining the waves with the highest amplitudes and discard the smaller ones



Example: Square Wave

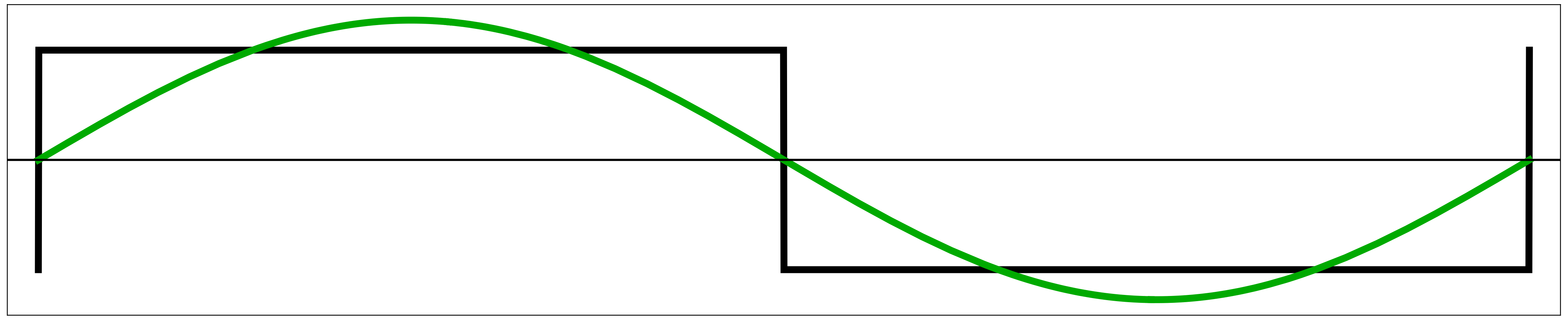
- Consider the square wave at frequency f_0
- By applying Fourier analysis (the derivation of which exceeds the scope of this course) the exact partial waves required to reconstruct this wave can be determined
- Omitting the complicated derivation ☛ these constituent waves are expressed as follow

$$A_n = \frac{4}{\pi}\frac{1}{n} \quad \text{and} \quad f_n = n f_0 \quad \text{with} \quad n = 1,3,5,\cdots$$

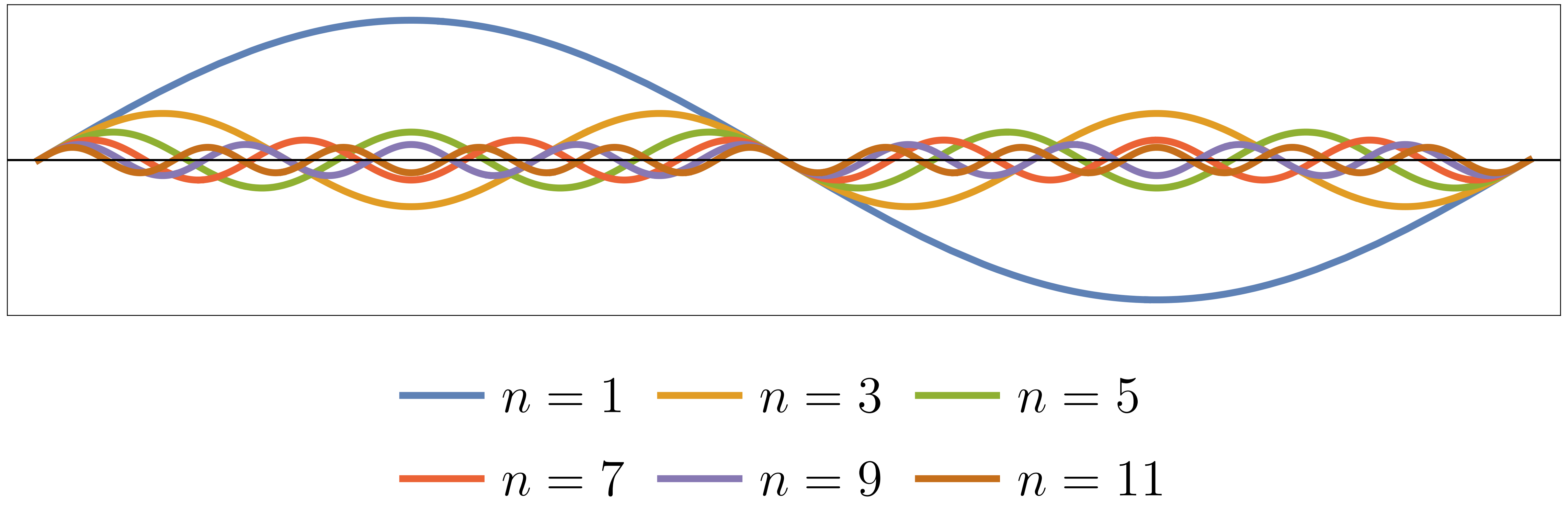


Example: Square Wave

- It is straightforward to see that the first wave ($n = 1$) possesses the highest amplitude
- Because the amplitude shrinks as n gets larger, the first partial wave is the most dominant, with all subsequent waves acting as minor refinements



Square wave with its fundamental harmonic

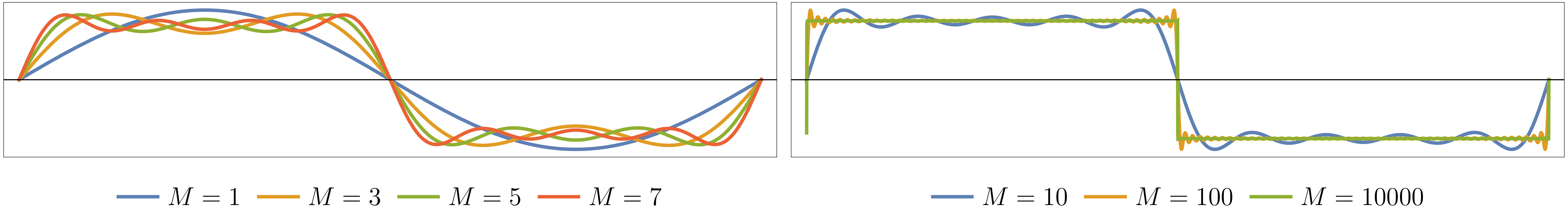


First six odd harmonics

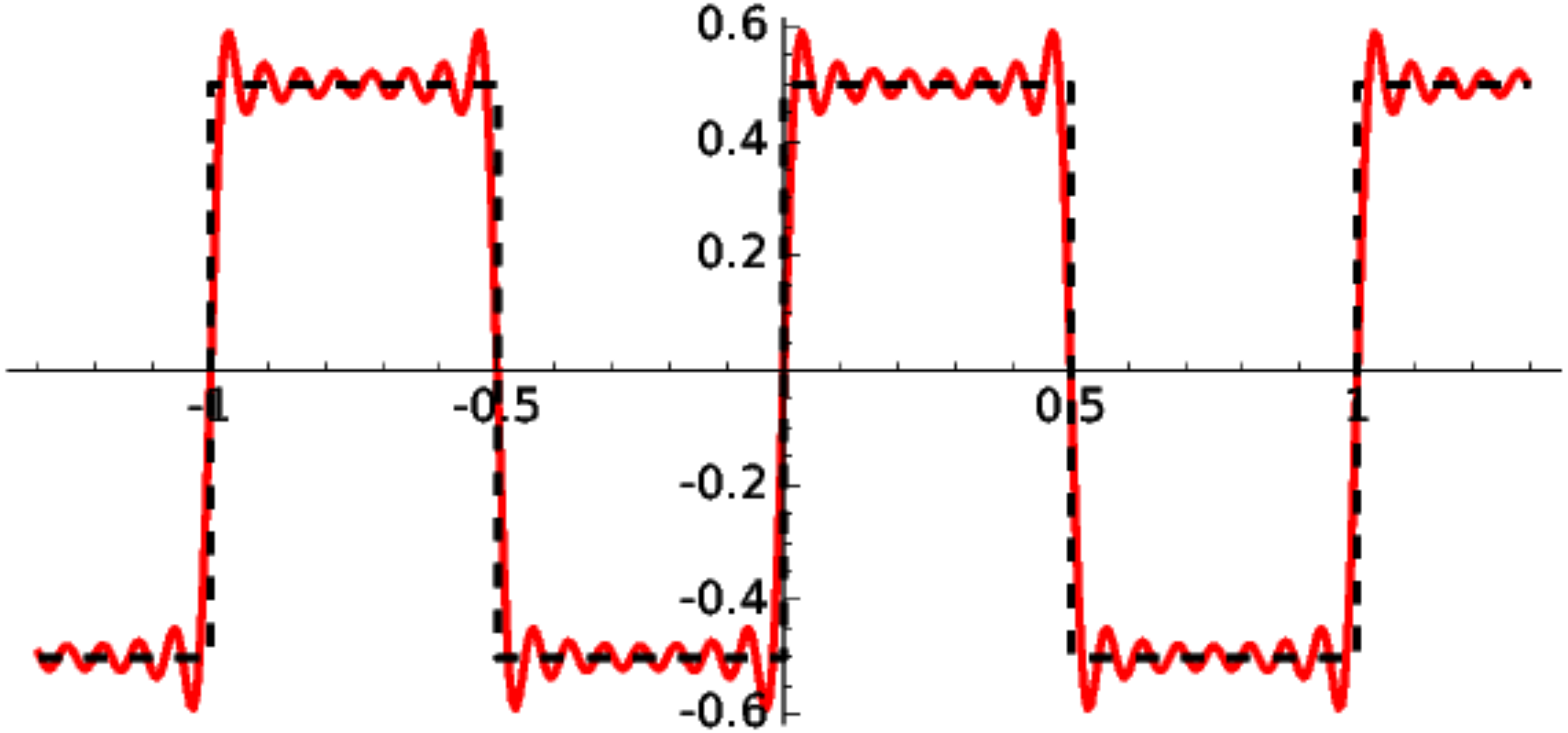
Example: Square Wave

➢ When these waves are combined, $W_{\text{square}} \approx \sum_{n=1}^{M} w(A_n, f_n)$

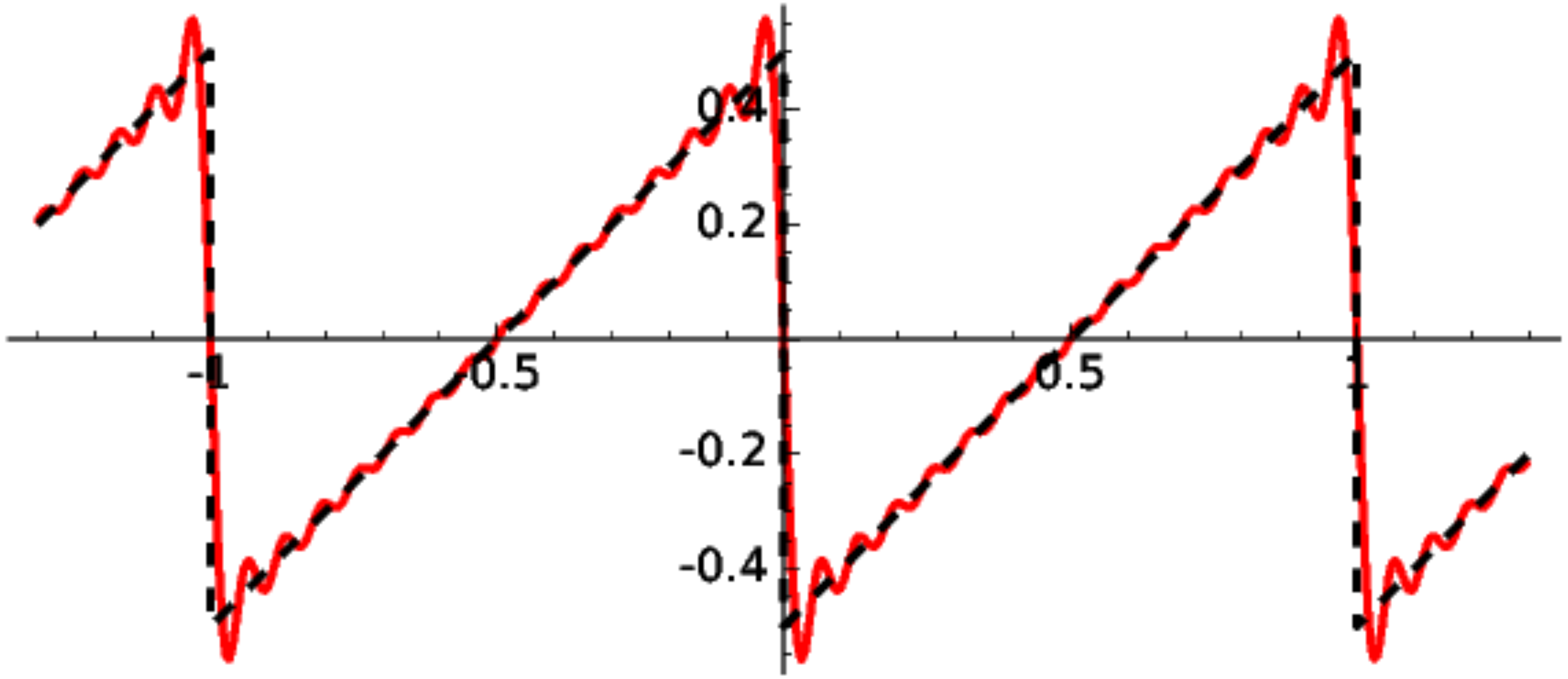
we obtain cumulative square wave approximations



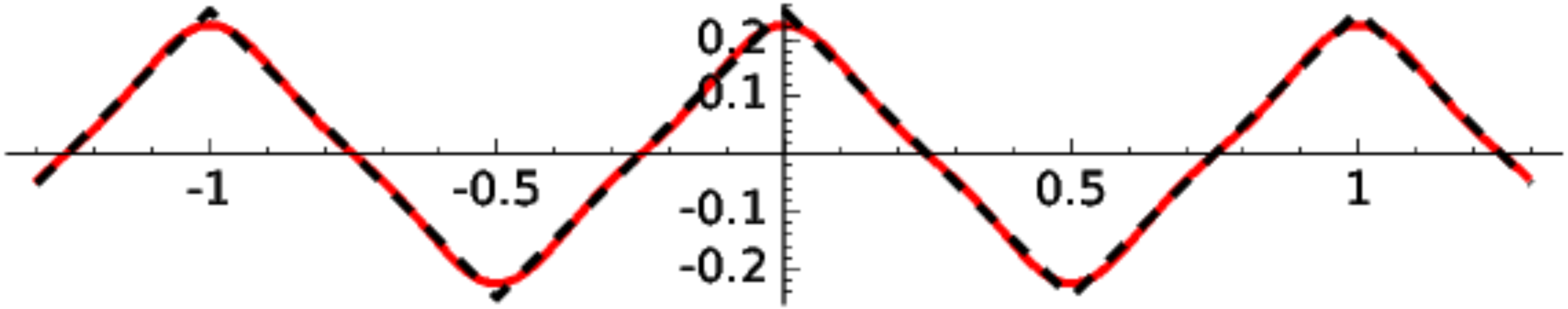
Example: Square Wave (16 frequencies)



Example: Sawtooth Wave (16 frequencies)

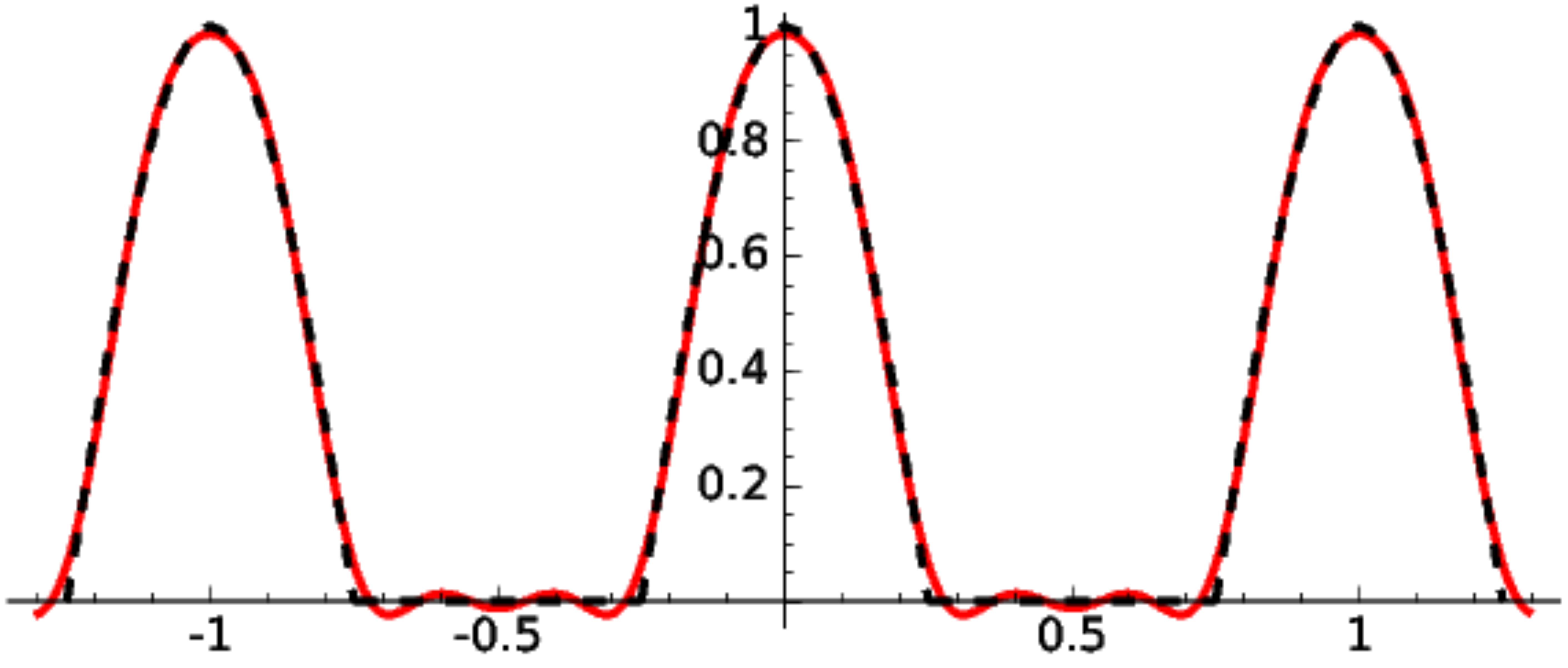


Example: Triangle Wave (3 frequencies)



Example: Half-Wave Rectified Cosine (4 frequencies)

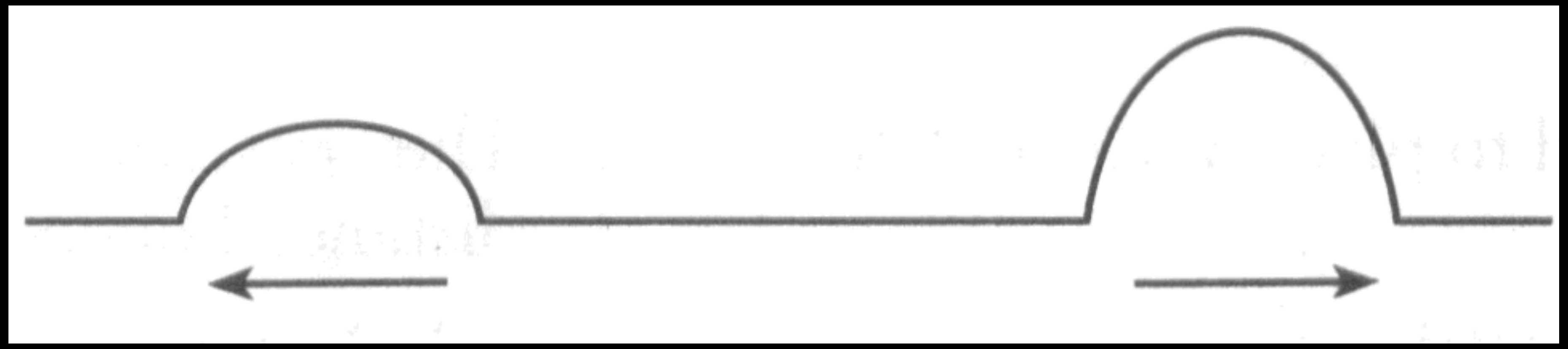
- The negative portions of the cosine wave are replaced by zeros, turning the signal off for half the period



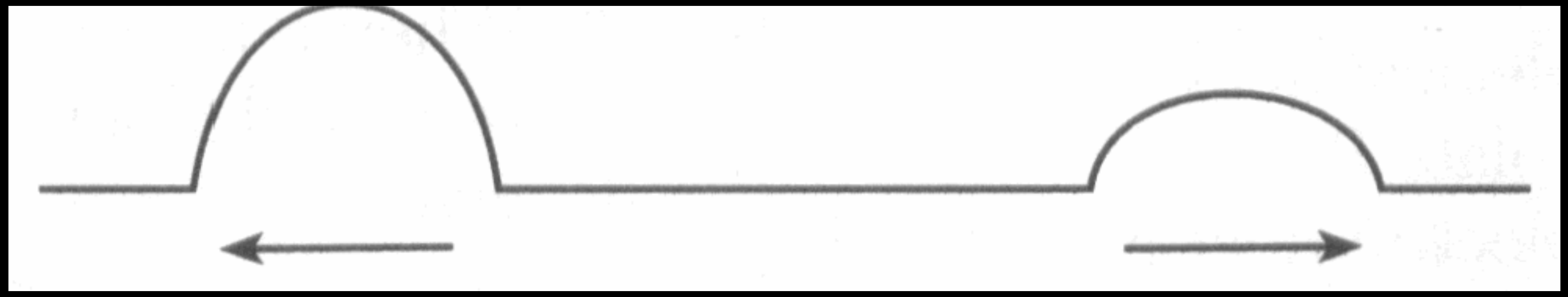
The diagram below represents two pulses approaching each other from opposite directions in the same medium

Which diagram best represents the medium after the pulses have passed through each other?

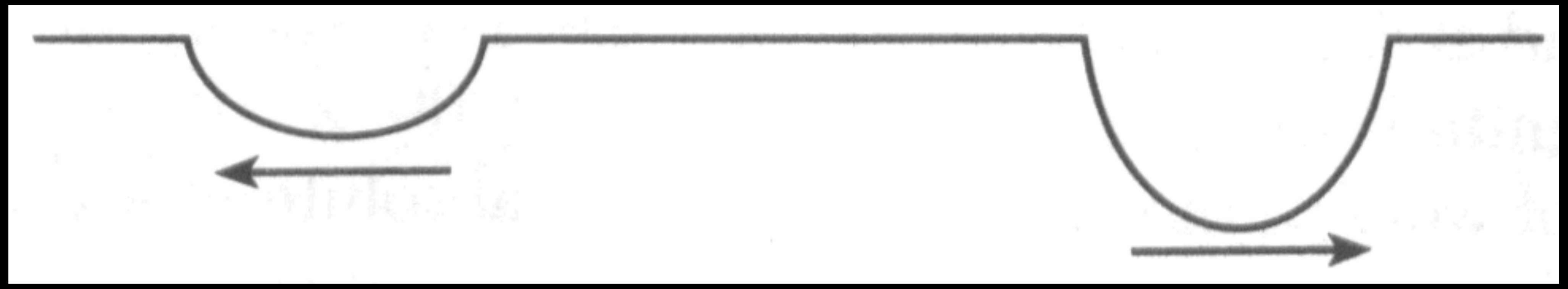
A.



B.



C.

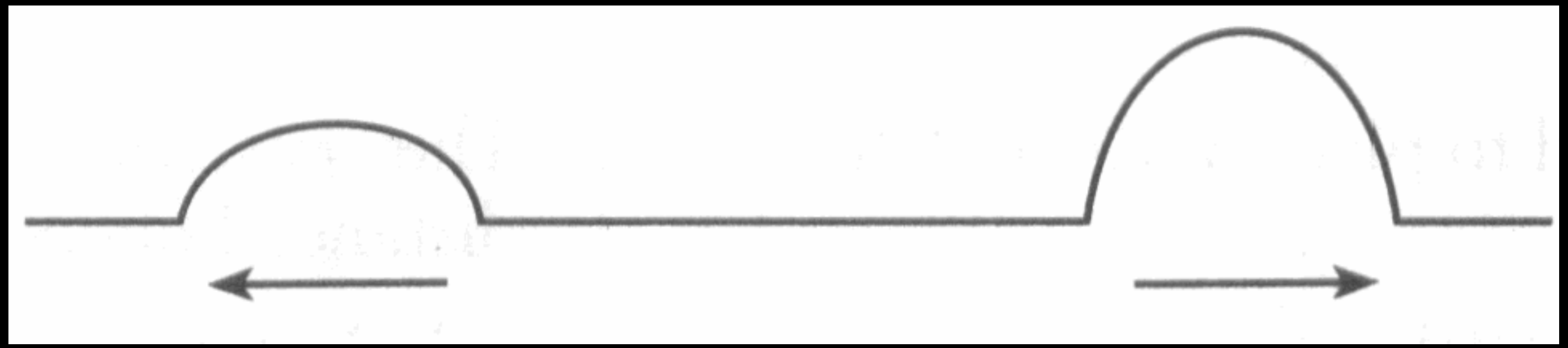


D.

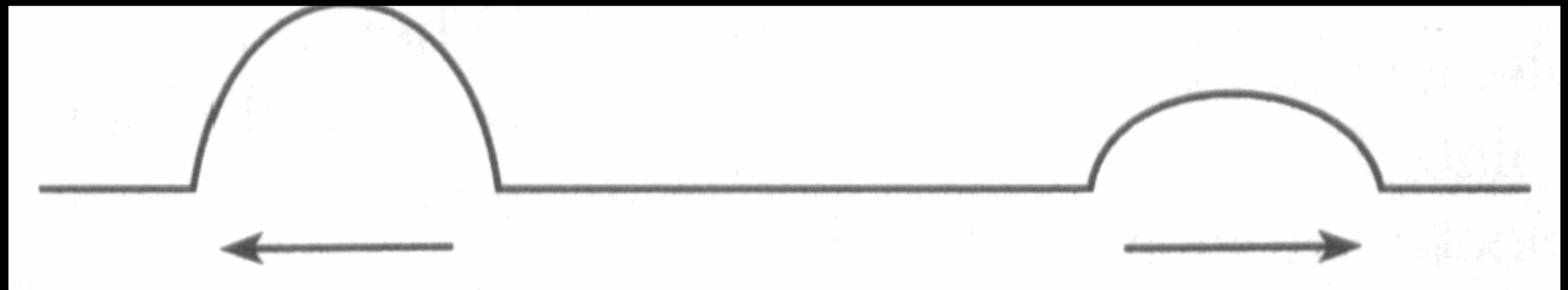
The diagram below represents two pulses approaching each other from opposite directions in the same medium

Which diagram best represents the medium after the pulses have passed through each other?

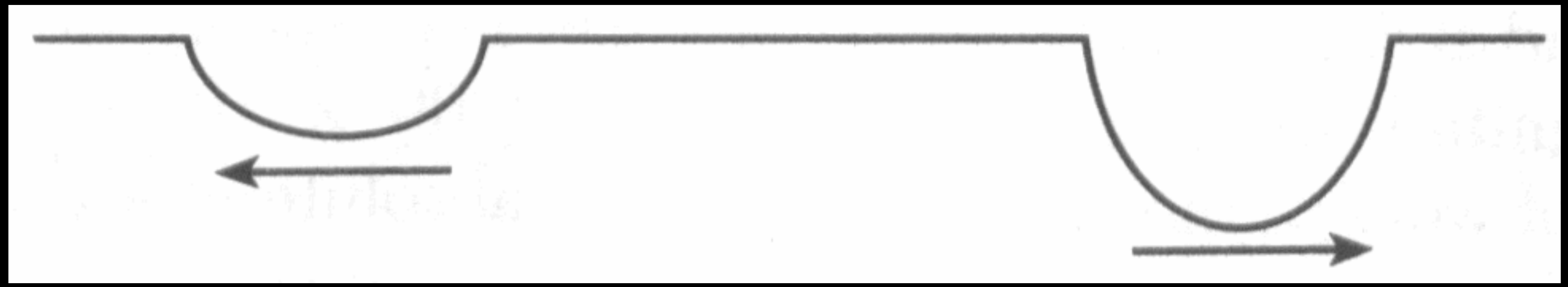
A.



B.



C.



D.

Two wave sources operating in phase in the same medium produce the circular wave patterns shown in the diagram below

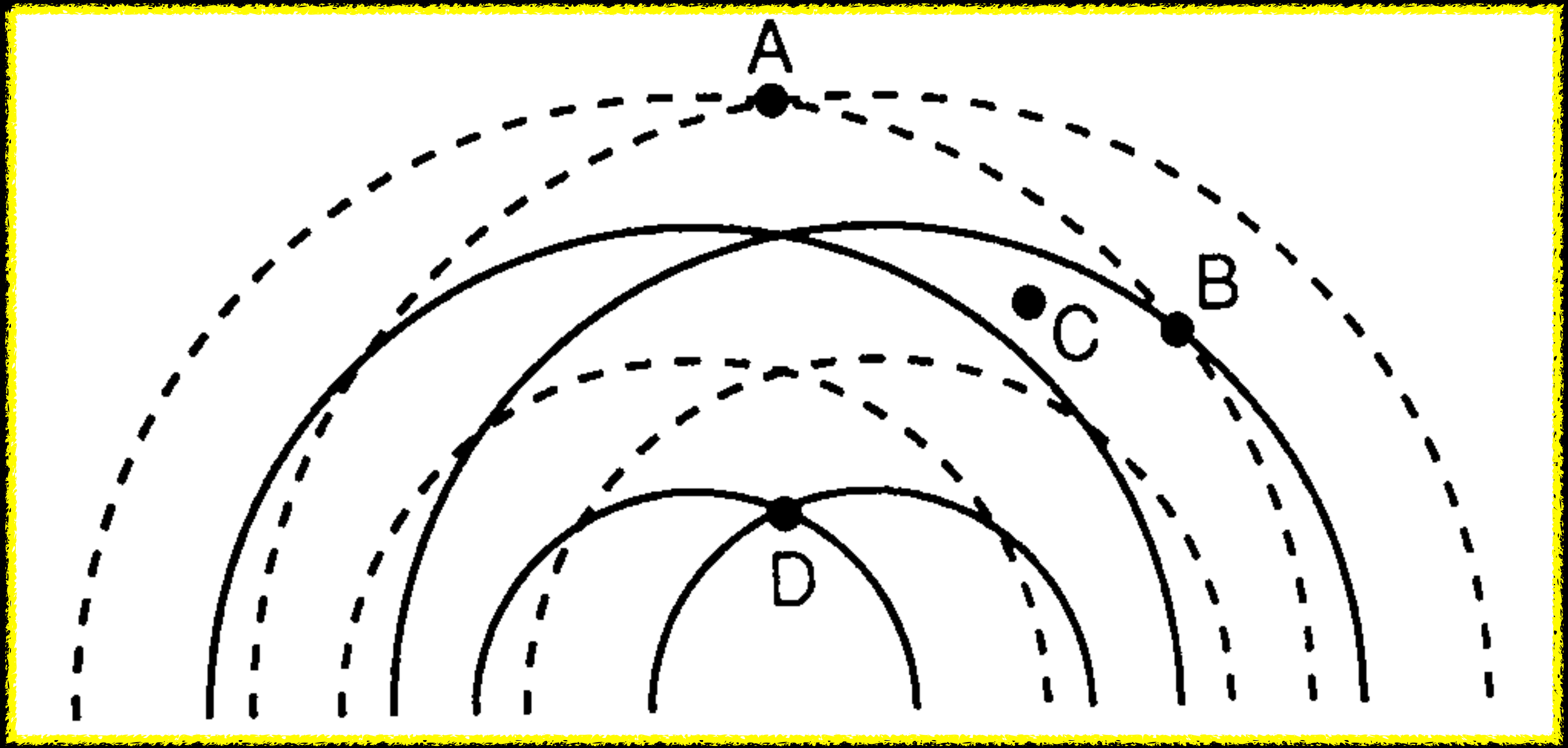
The solid lines represent wave crests and the dashed lines represent wave troughs

A. A

B. B

C. C

D. D



Which point is at a position of maximum destructive interference?

Two wave sources operating in phase in the same medium produce the circular wave patterns shown in the diagram below

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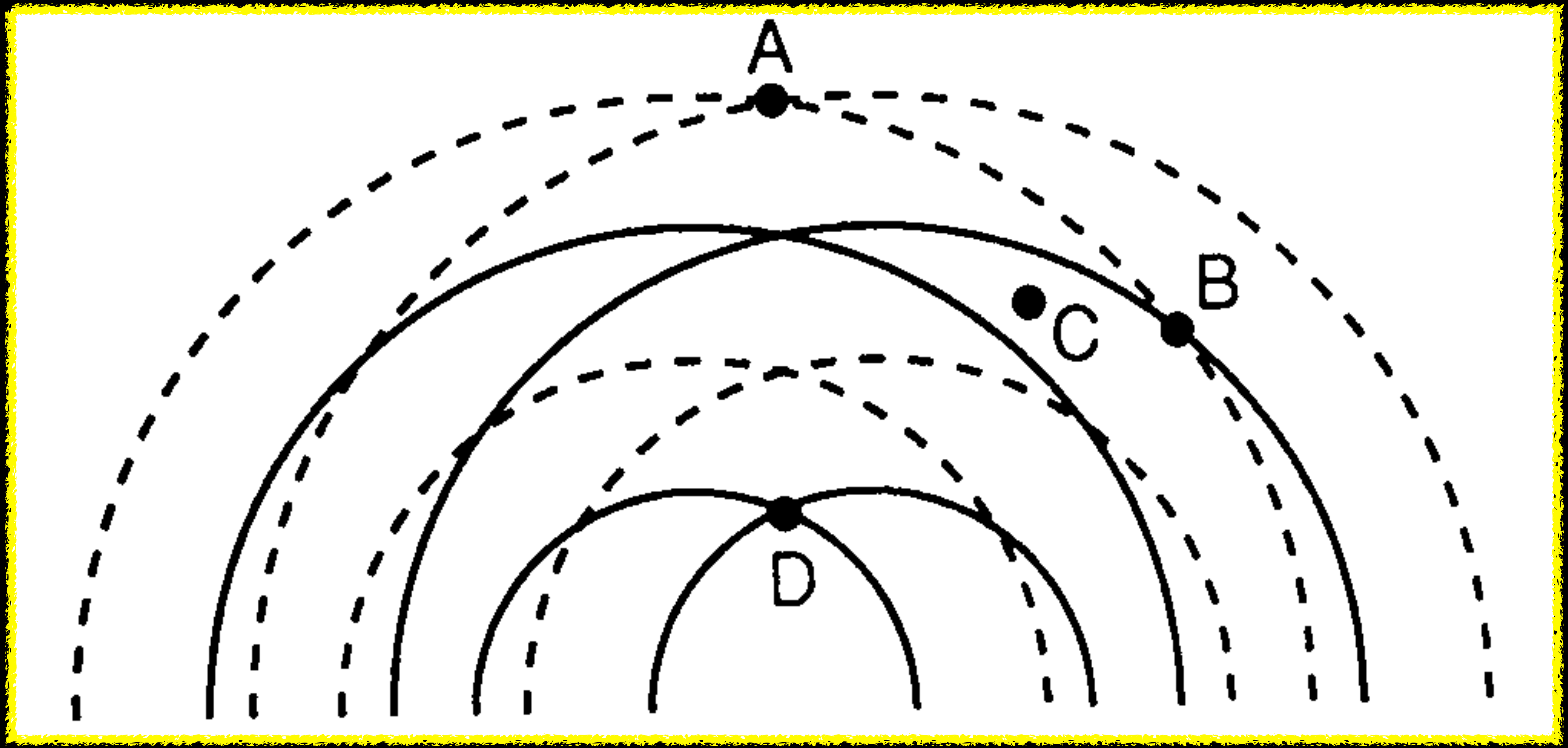
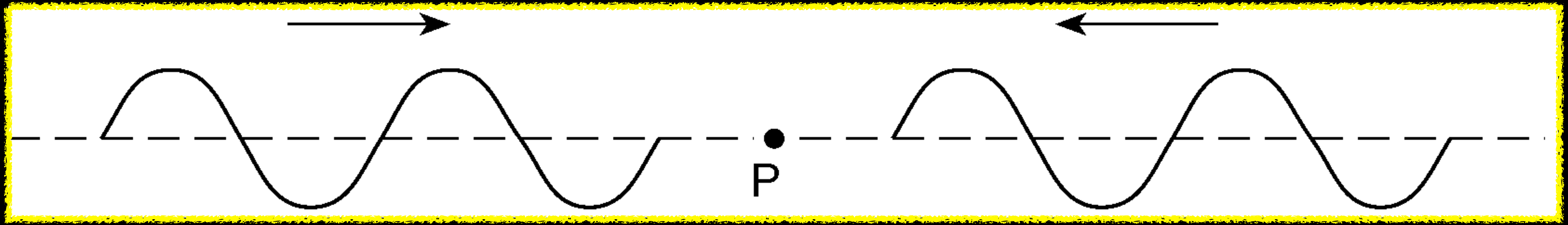


Which point is at a position of maximum destructive interference?

The diagram below represents two waves of equal amplitude and frequency approaching point P as they move through the same medium



As the two waves pass through each other, the medium at point P will

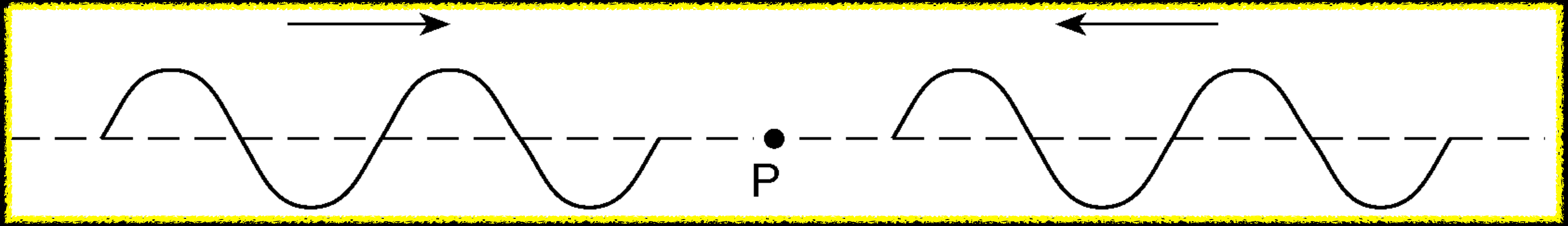
A. vibrate up and down

B. vibrate left and right

C. vibrate into and out of the page

D. remain stationary

The diagram below represents two waves of equal amplitude and frequency approaching point P as they move through the same medium



As the two waves pass through each other, the medium at point P will

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B. vibrate left and right

C. vibrate into and out of the page

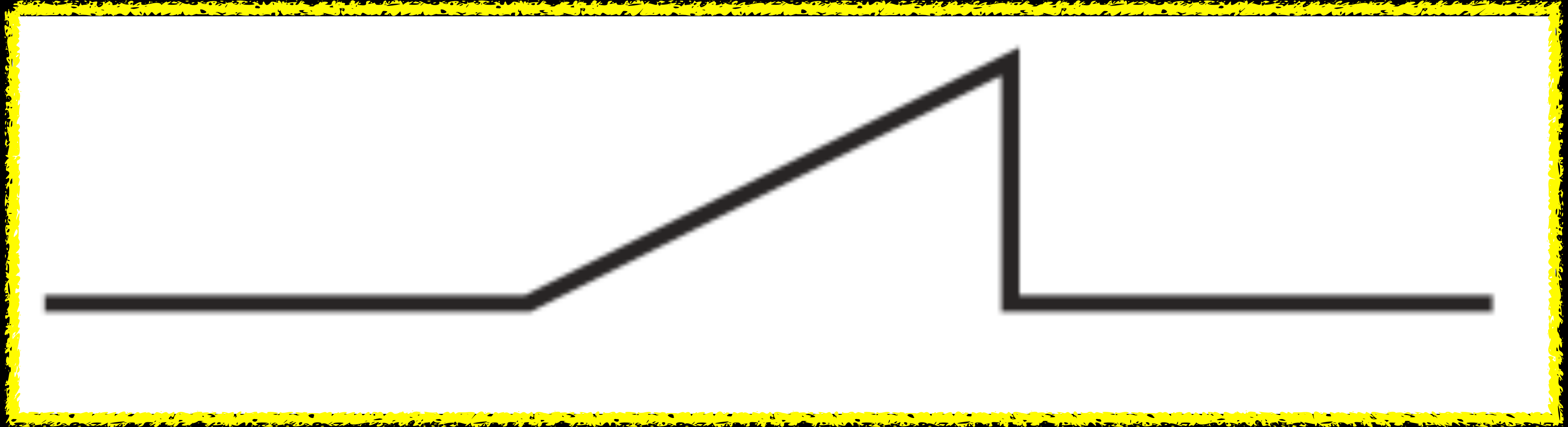
D. remain stationary

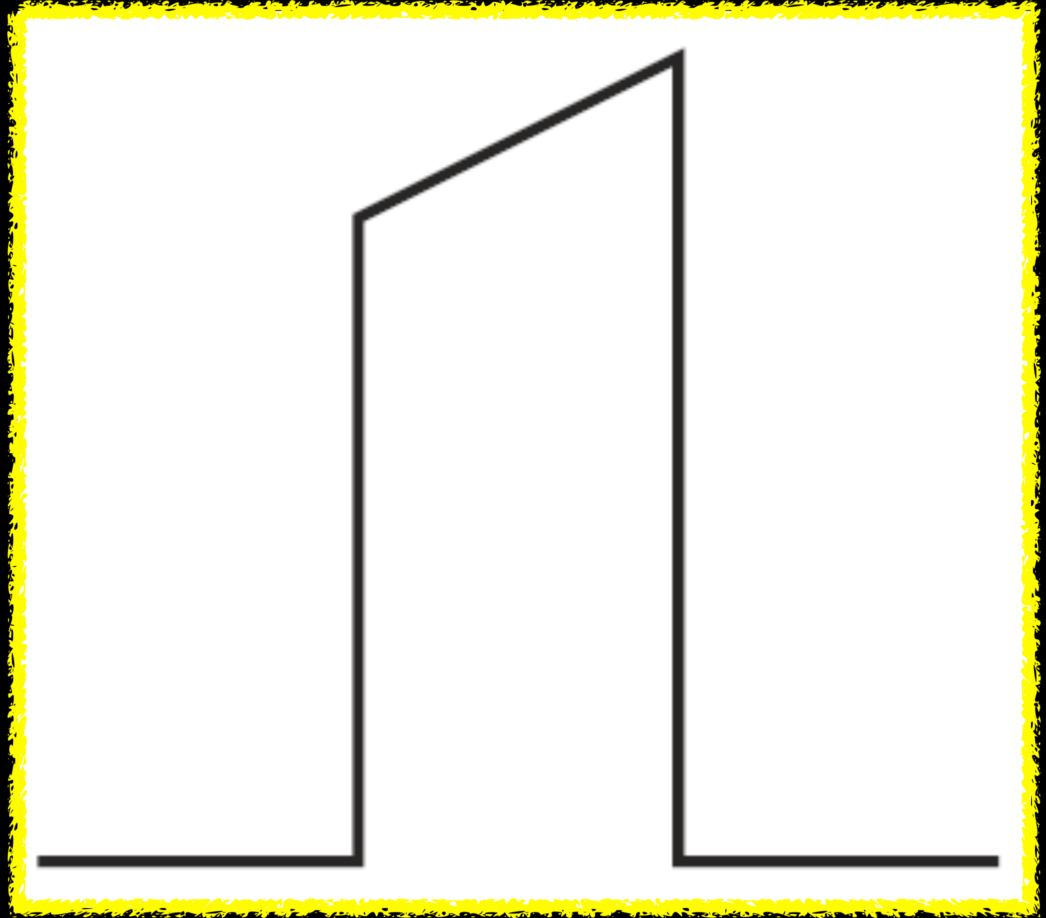
Two pulses approach each other in the same medium

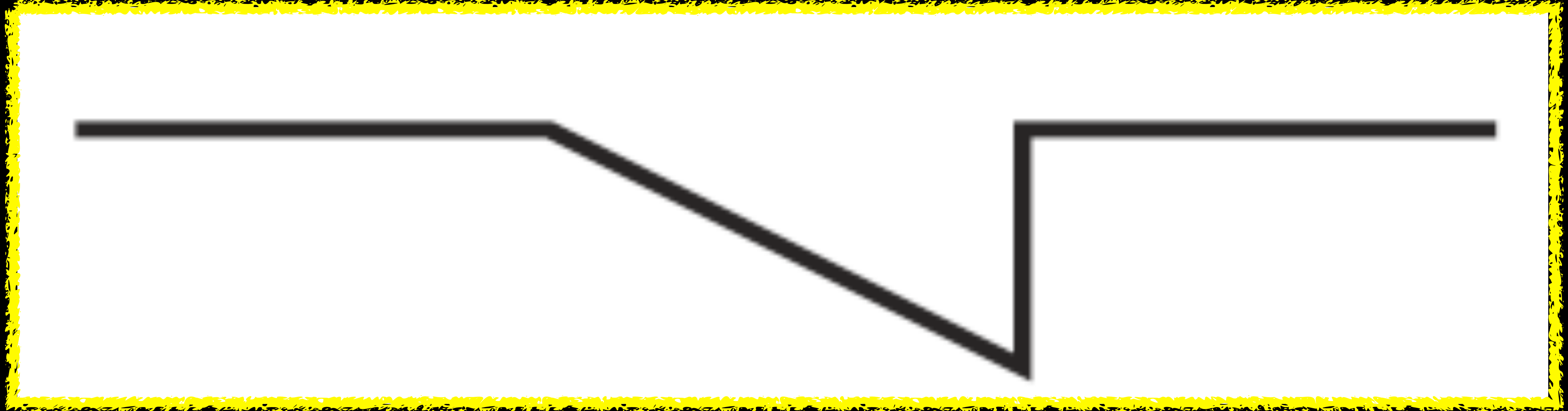
The diagram below represents the displacements caused by each pulse

Which diagram best represents the resultant displacement of the medium as the pulses pass through each other?

A. 

B. 

C. 

D. 

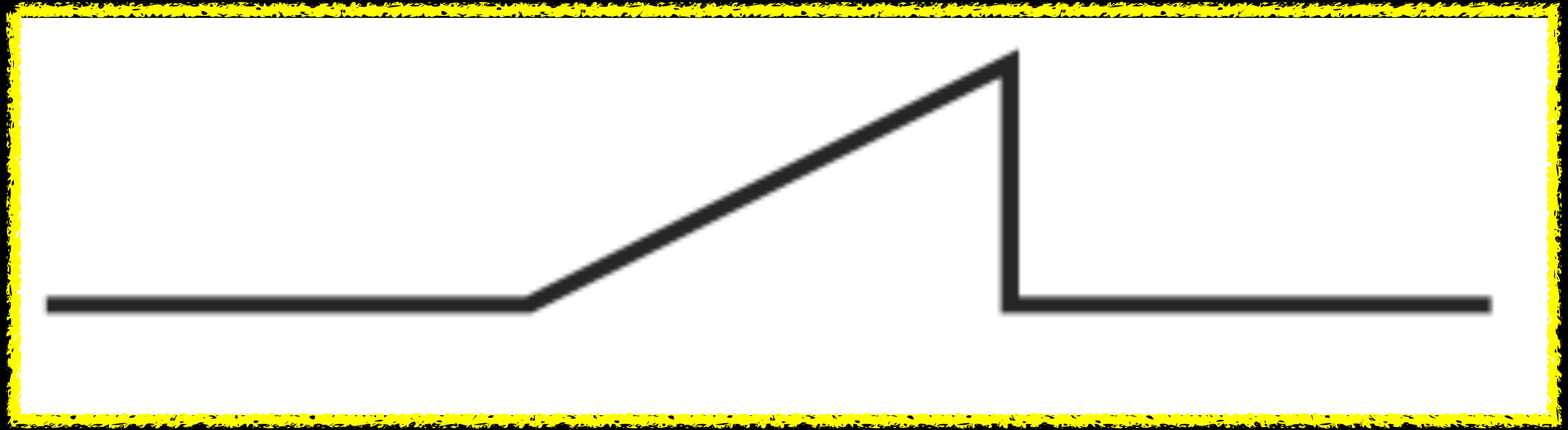
Two pulses approach each other in the same medium

The diagram below represents the displacements caused by each pulse

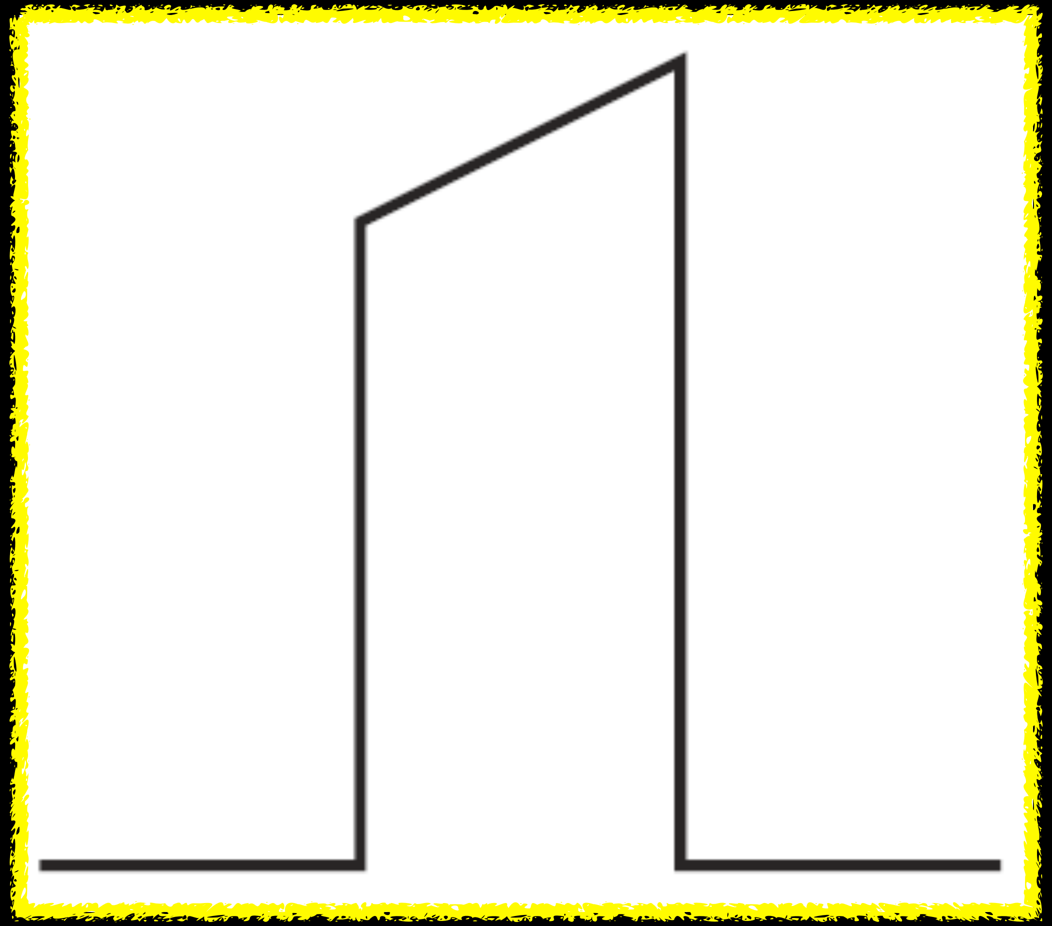
Which diagram best represents the resultant displacement of the medium as the pulses pass through each other?

A.

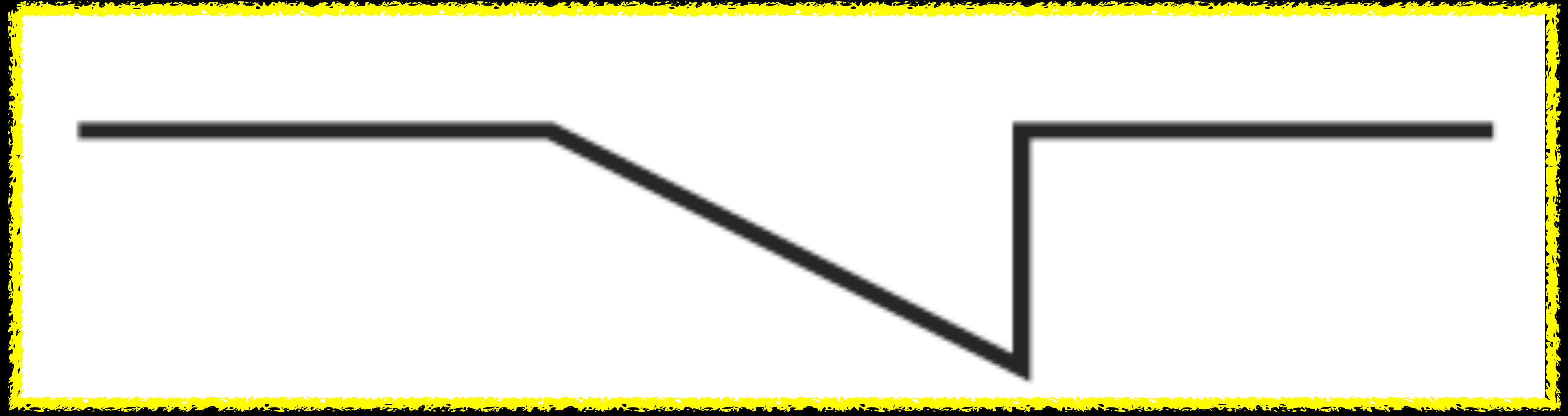
B.



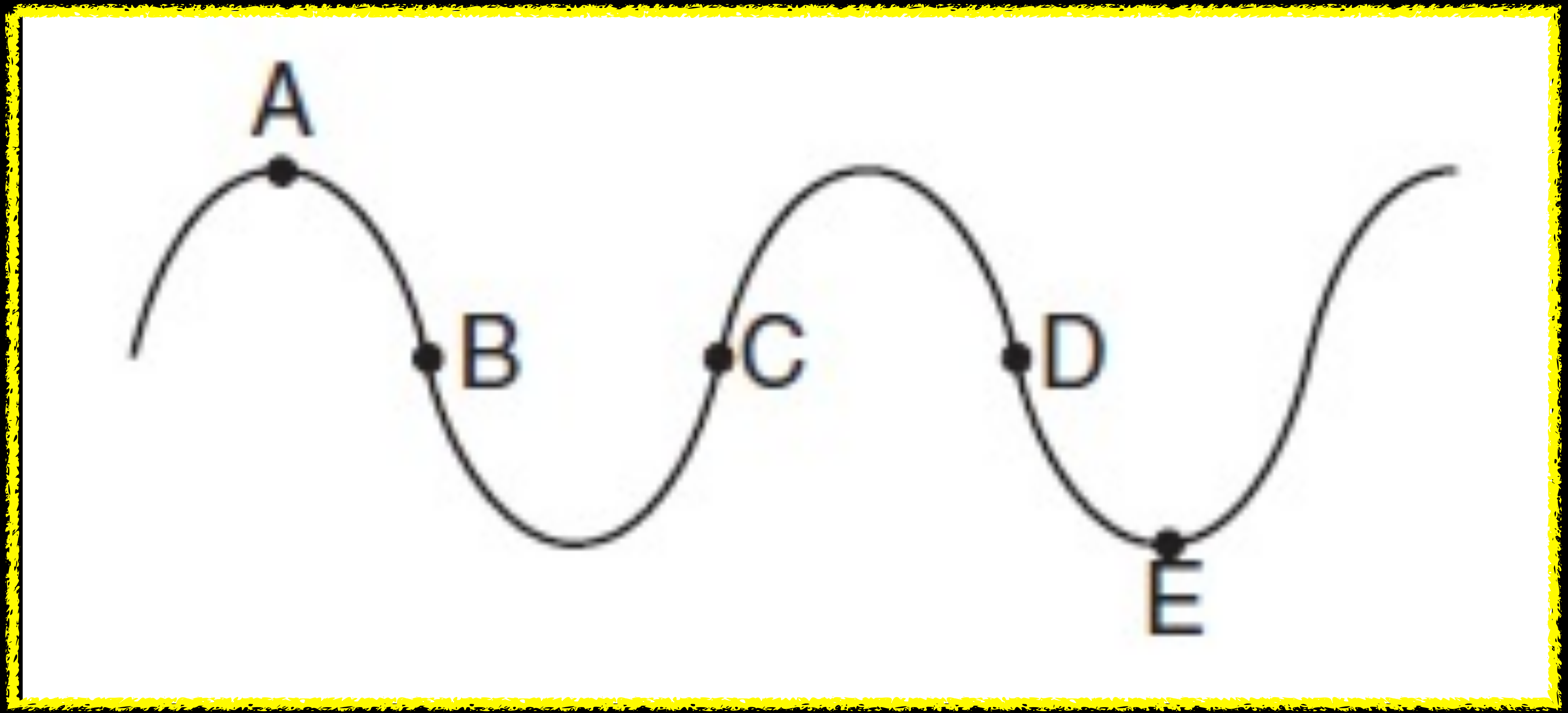
C.



D.



Which two points on the wave shown in the diagram below are in phase with each other?



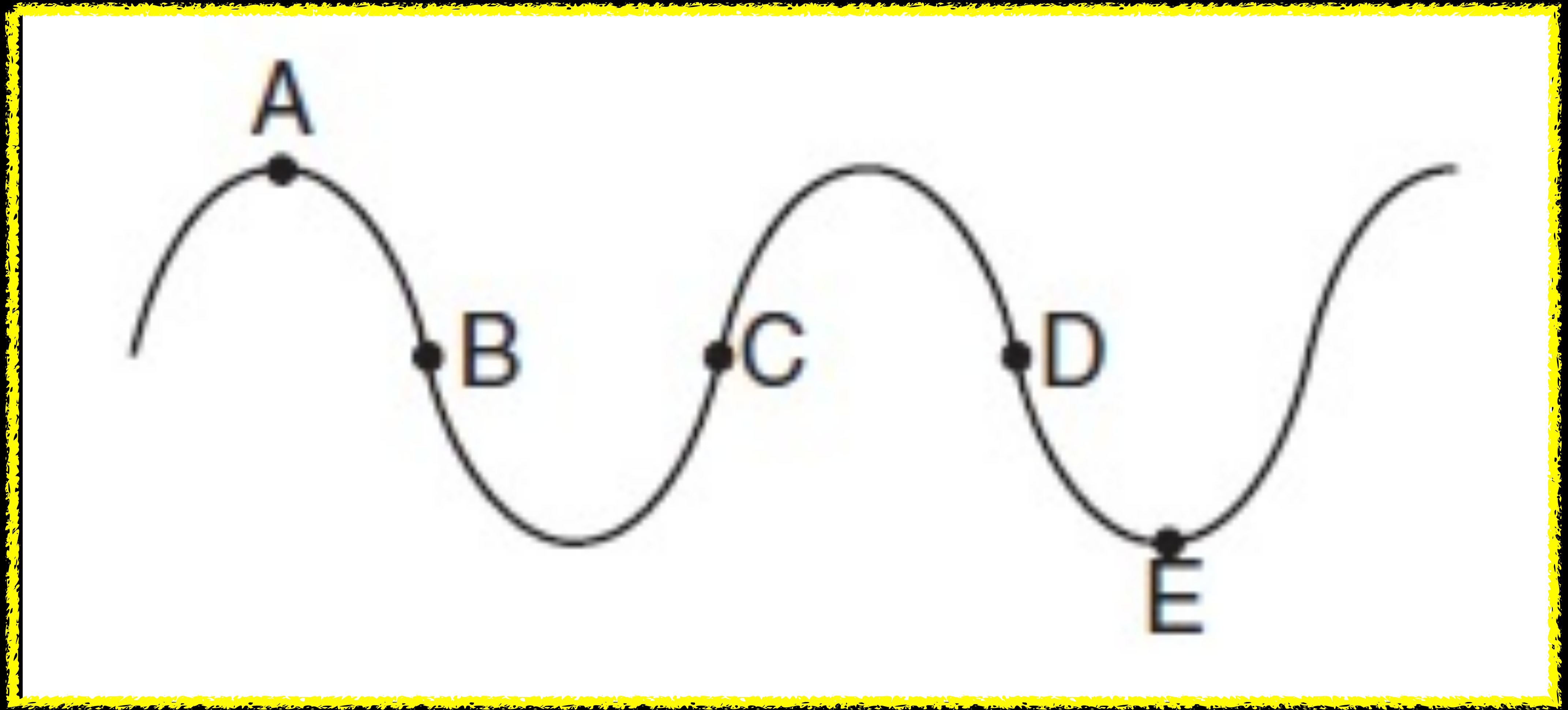
A. A and B

B. A and E

C. B and C

D. B and D

Which two points on the wave shown in the diagram below are in phase with each other?



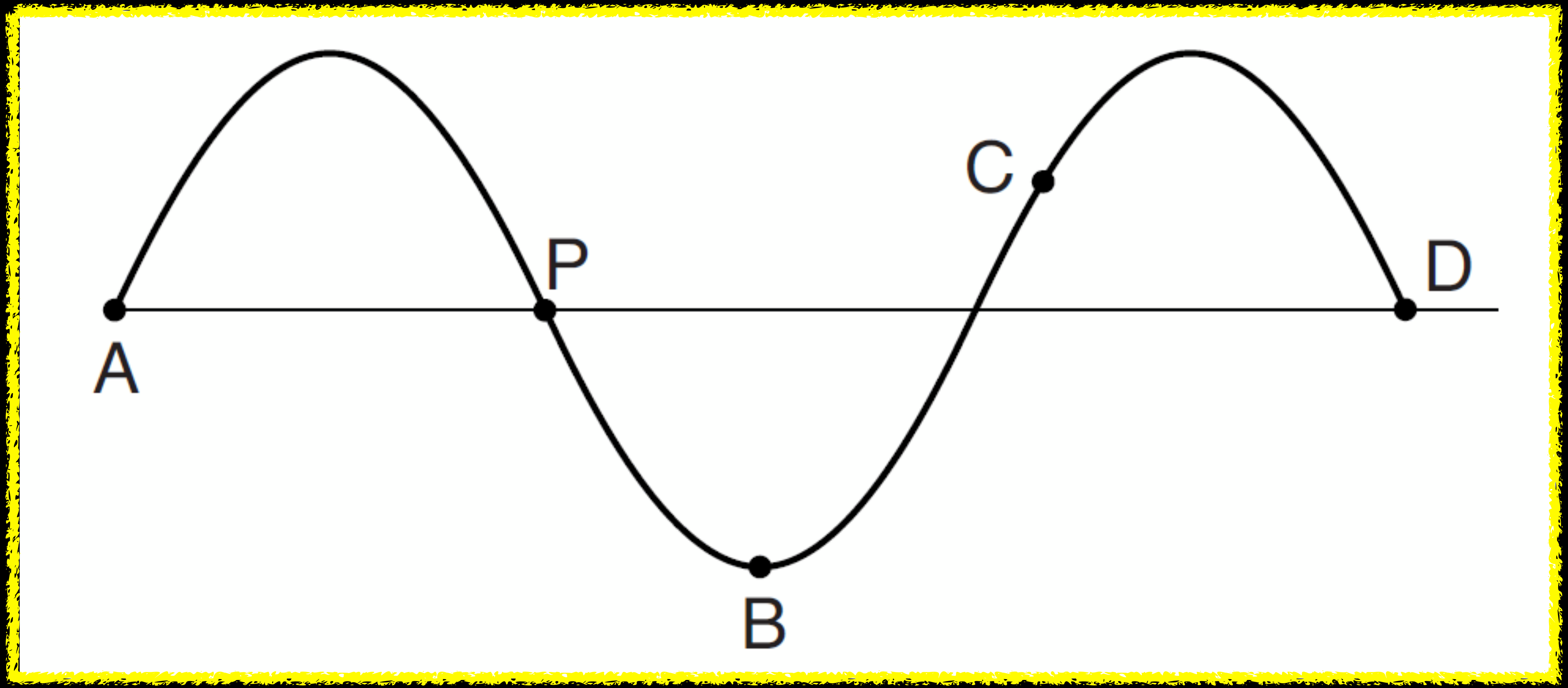
A. A and B

B. A and E

C. B and C

D. B and D

The diagram below represents a periodic wave



Which point on the wave is 90° out of phase with point P?

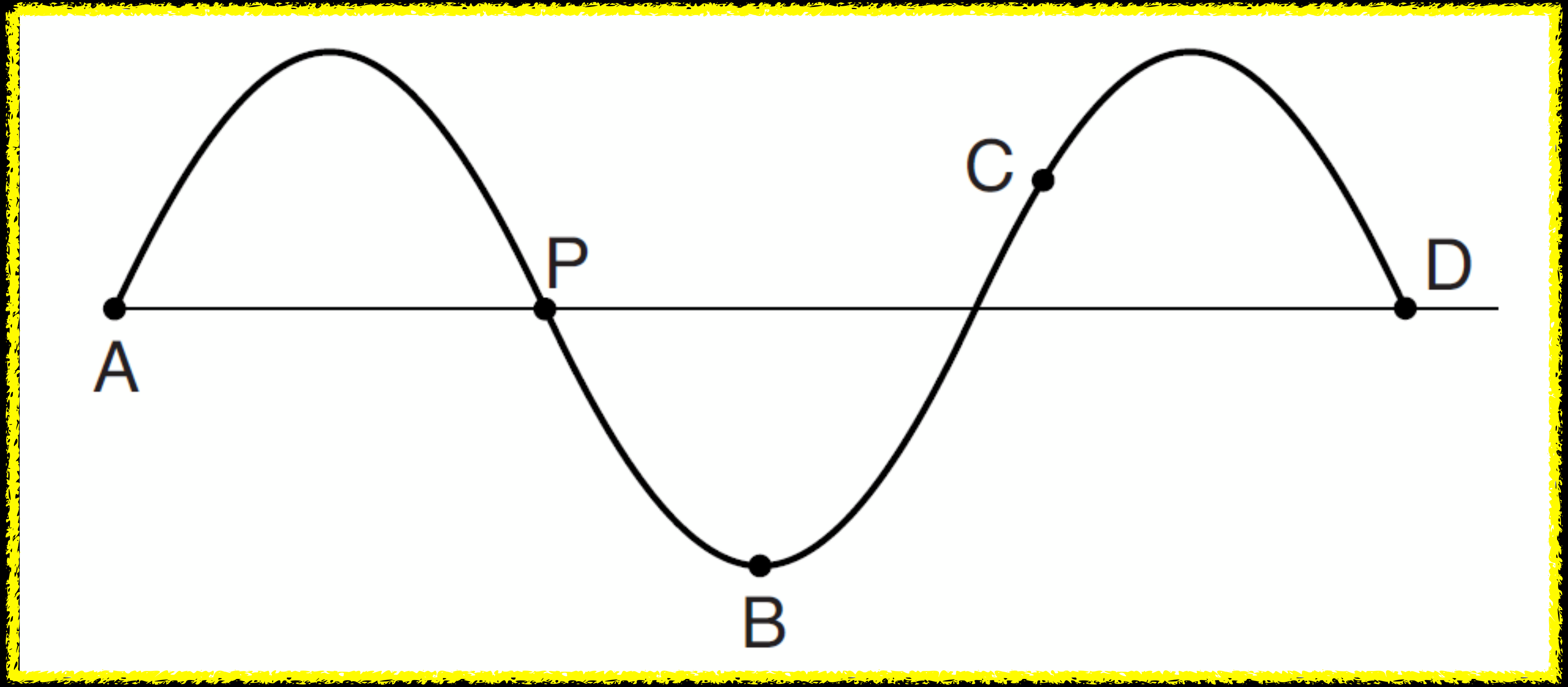
A. A

B. B

C. C

D. D

The diagram below represents a periodic wave



Which point on the wave is 90° out of phase with point P?

A. A

B. B

C. C

D. D

Maximum constructive interference will occur at points where the phase difference between two waves is

A. 0°

B. 90°

C. 180°

D. 270°

Maximum constructive interference will occur at points where the phase difference between two waves is

A. 0°

B. 90°

C. 180°

D. 270°