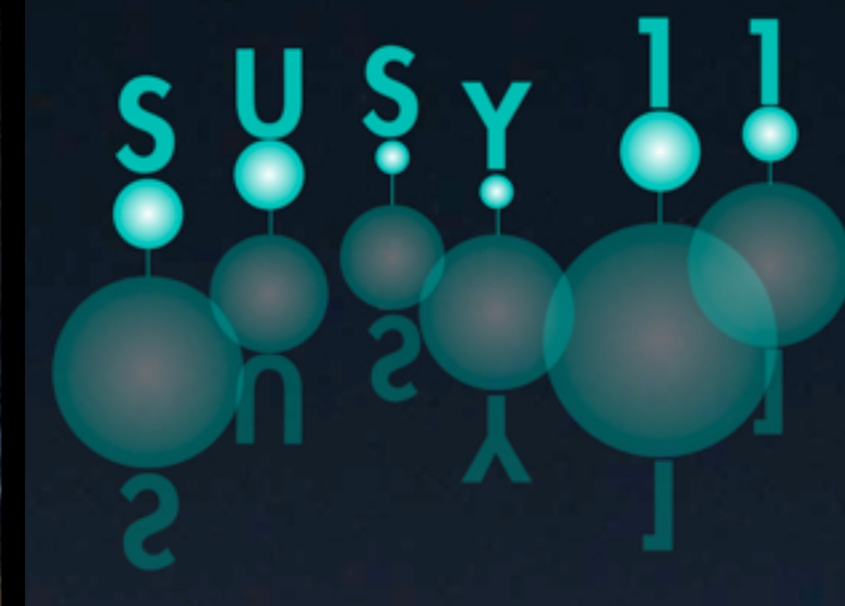
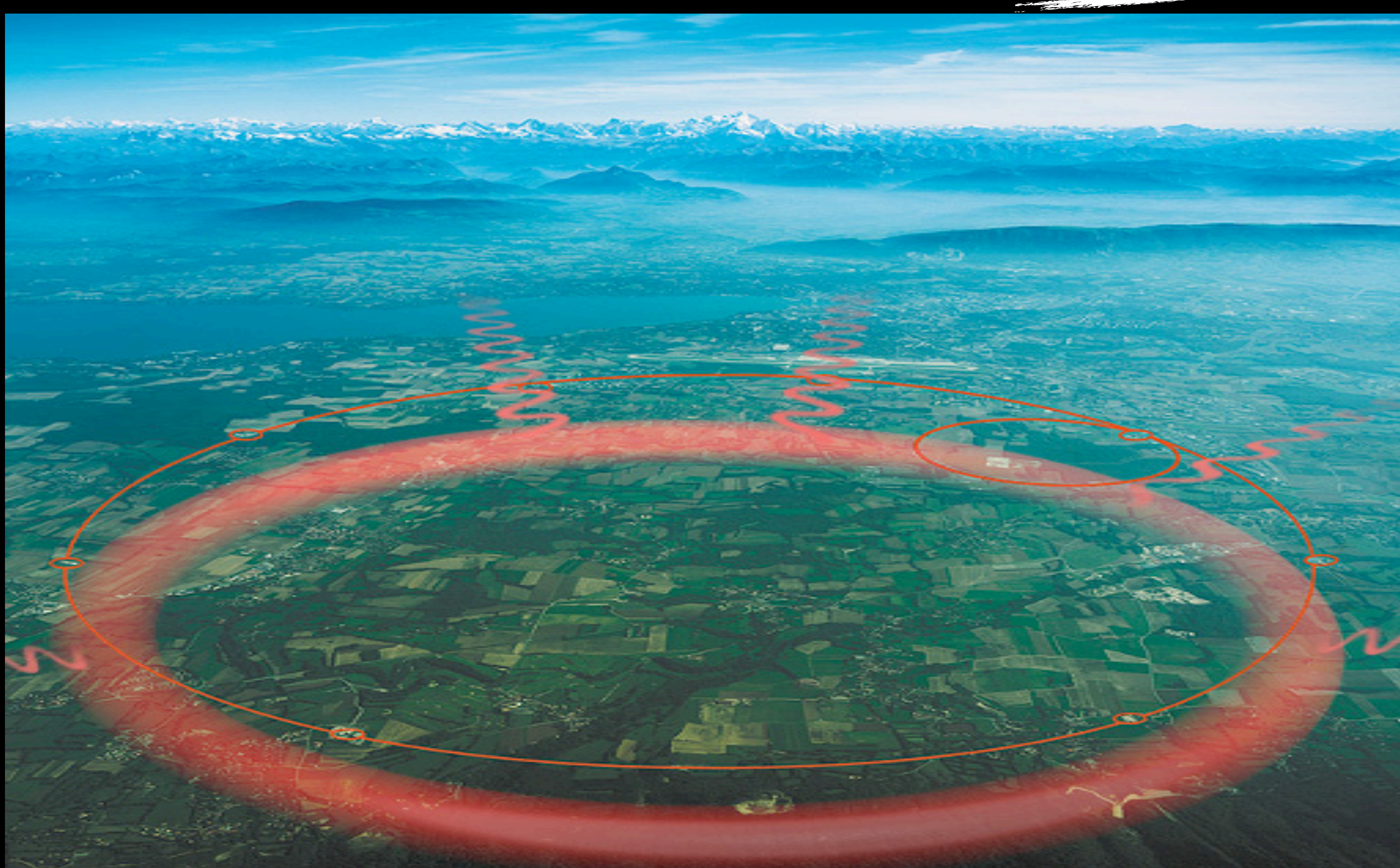




Particle Physics 2011

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Radiative Corrections

As a rule ☛ size of radiative corrections to a given process is determined by discrepancy between various mass & energy scales involved

For a wide class of low-energy and Z-boson observables dominant effects originate entirely in gauge boson propagators
(oblique corrections)
and can be parametrized in terms of 4 electroweak parameters:

$\Delta\alpha, \Delta\rho, \Delta r,$ and $\Delta\kappa$

1. $\Delta\alpha$ determines running fine structure constant at Z boson scale

$$\alpha(m_Z)/\alpha = (1-\Delta\alpha)^{-1}$$

2. $\Delta\rho$ measures quantum corrections of NC/CC amplitudes at low energy

3. Δr embodies non-photonic corrections to muon lifetime

4. $\Delta\kappa$ controls effective weak mixing angle

$$\sin^2\bar{\theta}_w = \sin^2\theta_w(1+\Delta\kappa)$$

that occurs in ratio of $Zf\bar{f}$ vector and axial-vector couplings

i.e. $c_V^f/c_A^f = 1 - 4|Q_f|\sin^2\bar{\theta}_w$

Chess Notation

Today's class ☛ intro to theory of electroweak radiative corrections and its role in testing SM, predicting top mass, constraining Higgs mass, and searching for deviations that may signal presence of new physics

Implementing such a program can be first formulated from point of view of experimentalist

Introducing notation ☛ $\sin^2\theta_w = s^2 = 1 - c^2, \;\; m_W^2 \equiv w, \;\; m_Z^2 \equiv z$

Electroweak theory predicts at Born level that:

$$\frac{\sigma(\nu_\mu e)}{\sigma(\bar{\nu}_\mu e)} = \frac{3 - 12s^2 + 16s^4}{1 - 4s^2 + 16s^4}$$ ♔

$$\frac{w}{z} = 1 - s^2$$ ♖

$$\frac{\pi\alpha}{\sqrt{2}\,G_F}\frac{1}{w} = s^2$$ ♙

$$\frac{\Gamma(Z \to f\bar{f})}{m_Z} = \frac{\alpha}{3} C_F \left((c_V^f)^2 + (c_A^f)^2\right)$$ ♗

$$A_{\rm LR} \simeq A_\tau \simeq \left[\frac{4}{3}A_{\rm FB}\right]^{1/2} \simeq 2(1 - 4s^2)$$ ♕

Chessboard

Chess Eqs. represent incomplete list of experiments capable of measuring $\sin^2\theta_w$

Validity of SM requires that each measurement yields same value of s^2

I Ratio of ν_μ scattering on left- and right-handed electrons which is a function of $\sin^2\theta_w$ only ☛ ♚

II Measurement of weak boson masses ☛ ♜

III Combination of m_W, α, and G_F as determined by muon lifetime ☛ ♟

IV Partial widths of Z into a fermion pair with vector and axial coupling c_V^f and c_A^f and color factor $C_F = 3\ (1)$ for quarks (leptons) ☛ ♝

V Various asymmetries measured at Z-factories ☛ ♛

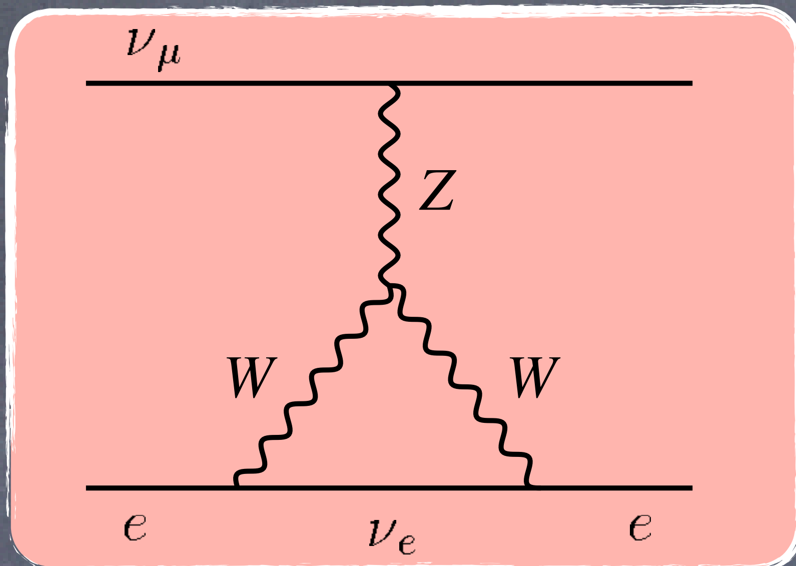
Corrections to Z propagator

After inclusion of $\mathcal{O}(\alpha)$ corrections

$\sin^2\theta_w$ values obtained from different methods will no longer be same because radiative corrections modify chess eqs. in different ways

E.G.

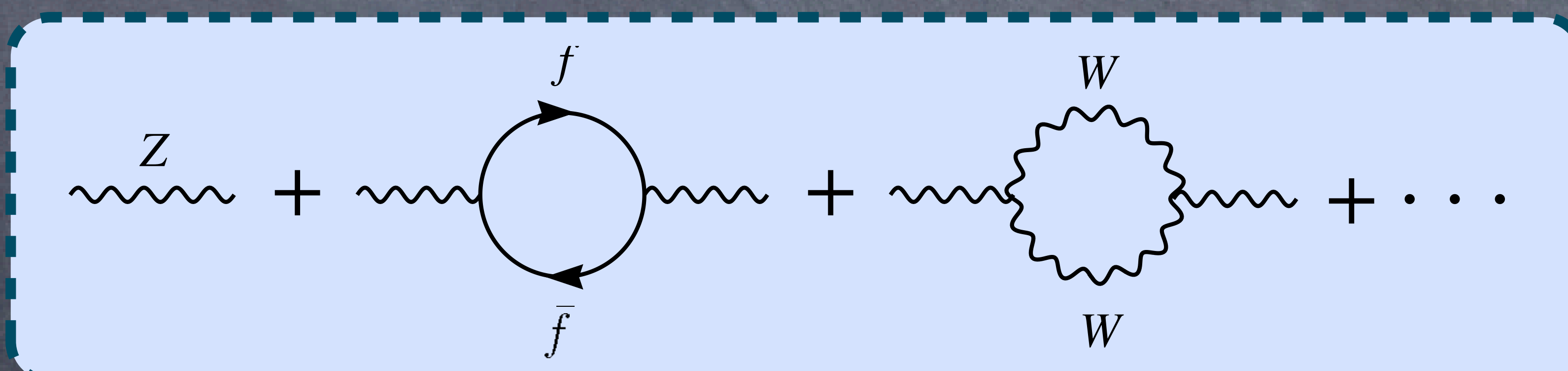
diagram ☛

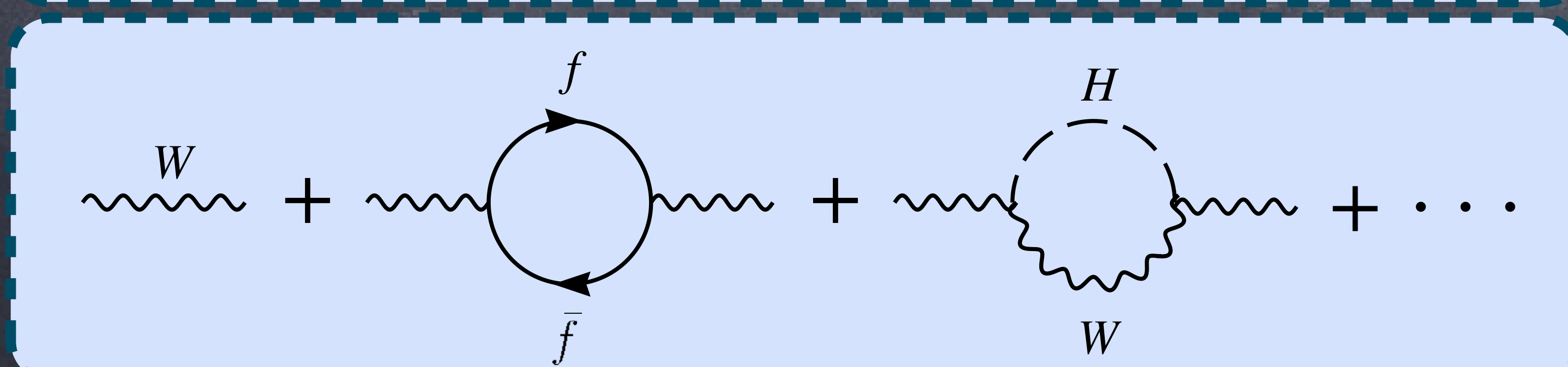


modifies t-channel

Z propagator measured by ♚

It does not -- however -- contribute to $\mathcal{O}(\alpha)$ shifts in W, Z masses





which yield an improved $\sin^2\theta_w$ value via ♜

Weinberg Angle

Experimentalist has to make a choice and define Weinberg angle to $\mathcal{O}(\alpha)$

All other experiments should be reformulated in terms of preferred s^2

What this choice should be is no longer a matter of debate

we will define $\sin^2\theta_w$ in terms of physical masses of weak bosons

i.e. $$\sin^2\theta_w \equiv 1 - \frac{m_W^2}{m_Z^2} = 0.23122(15)$$ ☎

This choice is particularly useful in that one can estimate radiative corrections in terms of renormalization group which has been previously introduced

$\mathcal{O}(\alpha)$ corrections can be qualitatively understood in terms of loop corrections to vector-boson propagators ☂ and ☁

A most straightforward test of theory is now obtained by fixing $\sin^2\theta_w$ in terms of measured weak boson masses and verifying that its value coincides with other measurements of θ_w

Free Parameters

All UV divergences in QED can be absorbed in two parameters: α and m_e

List of parameters to be fixed by experiment in electroweak theory includes

$$\alpha,\ m_W,\ m_Z,\ m_H,\ m_f$$ ♨

weak mixing angle does not appear in list of parameters:

its value is automatically determined by $m_W,\ m_Z$ via ☎

Traditionally ☛ SM Lagrangian is determined in terms of

$$g,\ g',\ \lambda,\ \mu,\ Y_f$$ ☕

There is in fact a direct translation between sets ♨ and ☕

$$g^2 = e^2 \frac{z}{z-w}$$

$$g'^2 = e^2 \frac{z}{w}$$

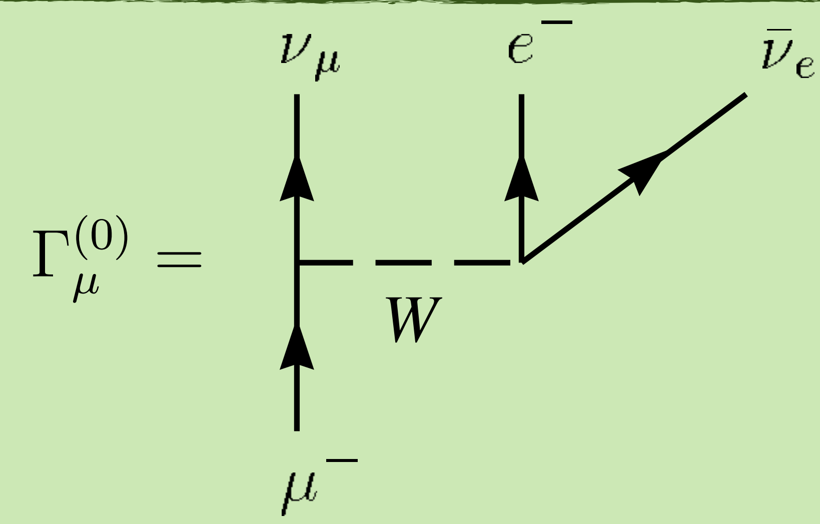
$$\lambda = e^2 \frac{z m_H^2}{8w(z-w)}$$

$$Y_f = e^2 \frac{z m_f^2}{2w(z-w)}$$

Electromagnetic Radiative Corrections

EXAMPLE ☛ Show how relation ♟ is calculated to $\mathcal{O}(\alpha)$ in terms of weak angle θ_w defined by ♜

Origin of relation ♟ is μ lifetime ☛ to leading order is given by diagram



In Fermi theory ☛ electromagnetic radiative corrections must be included to obtain result to $\mathcal{O}(\alpha)$

Symbolically

$$\Gamma_\mu^{(1)} = \frac{G_F}{\sqrt{2}} \left[1 + \text{photonic corrections}\right]$$ ♪

where

photonic corrections =

$+ \cdots$

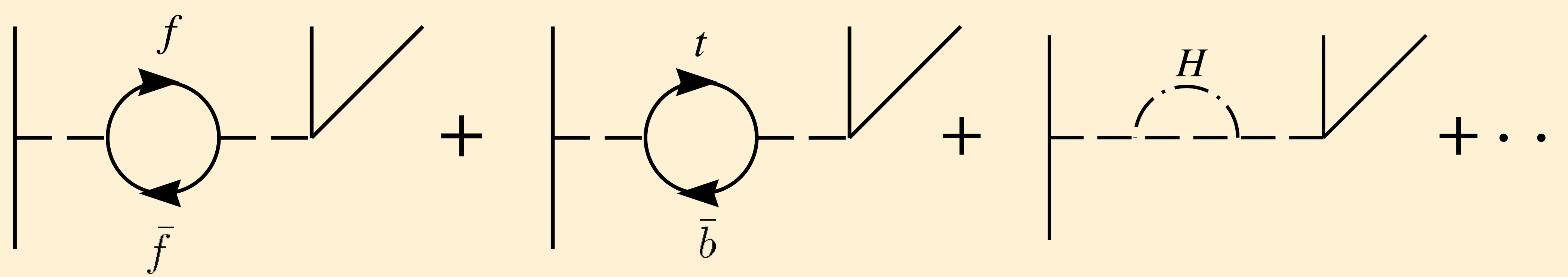
Electroweak Radiative Corrections

In electroweak theory

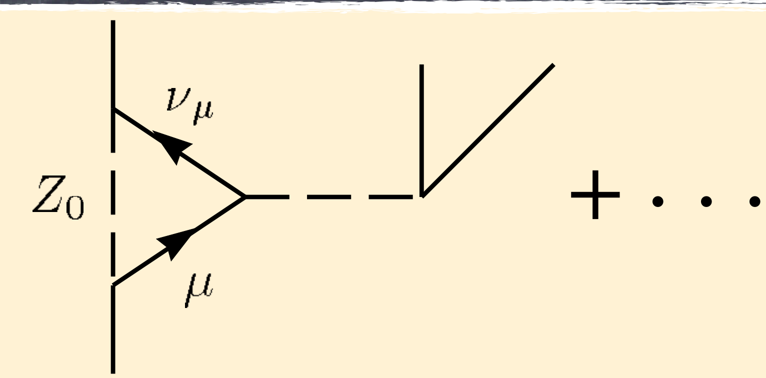
$$\Gamma^{(1)}_\mu = \frac{e^2}{8s^2c^2z}\Big[1 + \text{photonic corrections} + \text{propagator} + \text{vertex} + \text{box}\Big]$$

where

propagator $=$

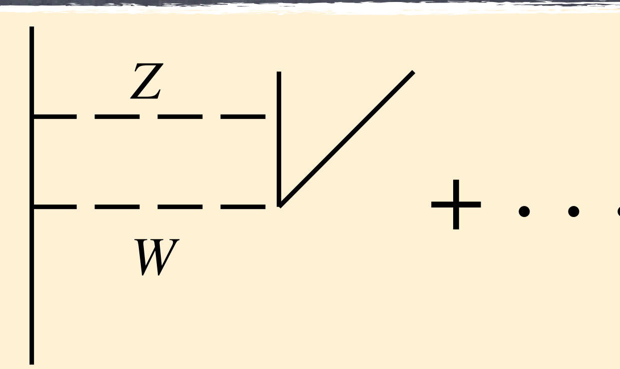


vertex $=$ Z_0



and

box $=$



Electroweak Model at Born Level

Equating ♪ and ♬ we obtain
$$G_F = \frac{\pi\alpha}{\sqrt{2}}\frac{1}{ws^2}(1+\Delta r)$$ ≈

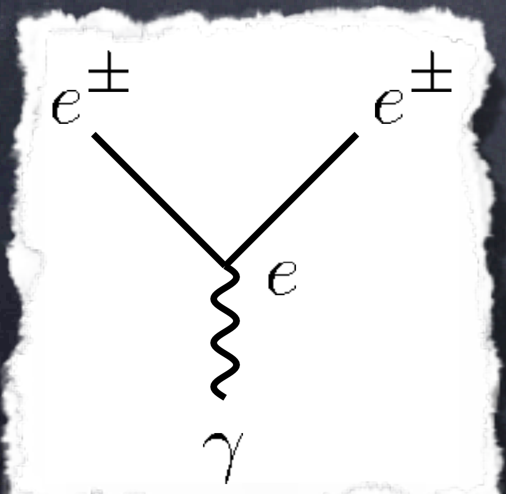
with
$$\Delta r = \Delta\alpha - \frac{c^2}{s^2}\Delta\rho + \Delta_{\rm rem}$$

Before discussing status of measurements of Δr ☛ note that

to leading order ☛ $\Delta r = 0$

using

$$\alpha = \frac{e^2}{4\pi} \qquad m_W = \frac{g\,v}{2} = \frac{g}{2\sqrt{2}\,\lambda}m_H \qquad \text{and} \qquad m_Z = \frac{m_W}{\cos\theta_w}$$

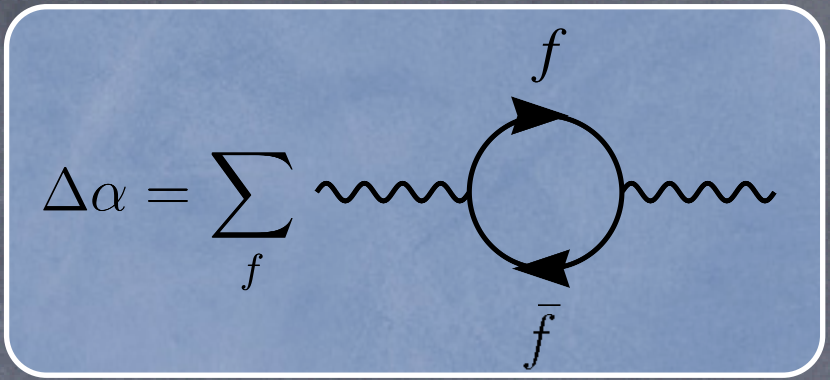


≈ reduces to Born relation ♟

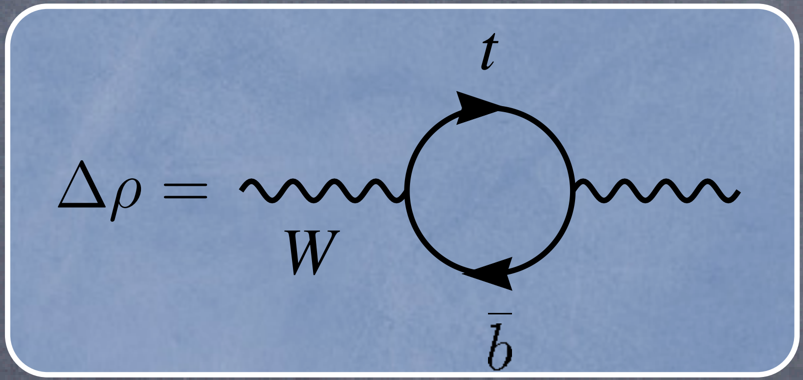
Dominant Terms

Electroweak radiative corrections are gathered in Δr

We specifically isolated fermions which are responsible for running of

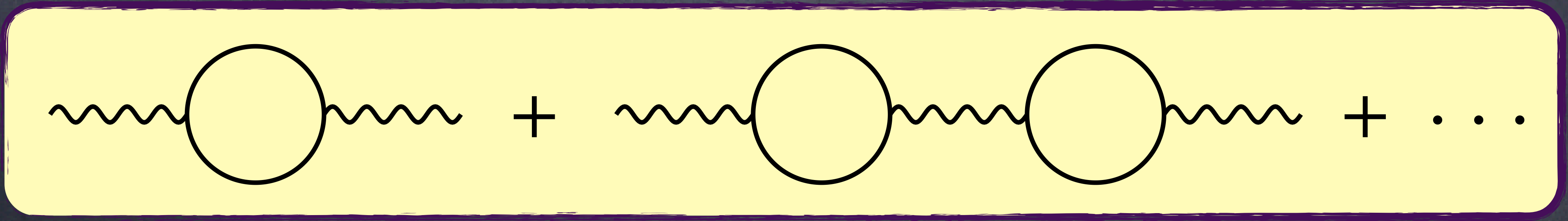


as well as third generation heavy quark diagram



Other contributions are small and are grouped in remainder Δ_{rem}

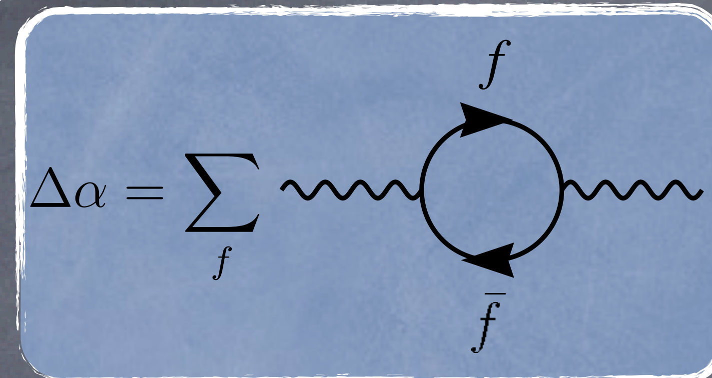
To the extent that Δ_{rem} is small ☛ one can imagine summing series



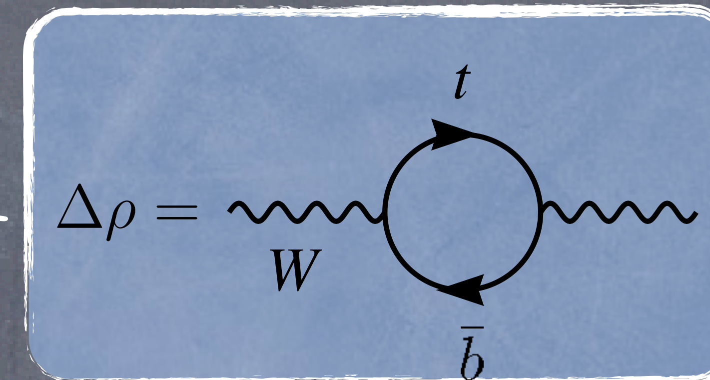
by replacement $(1+\Delta r) \rightarrow (1-\Delta r)^{-1}$ in ≋

$\Delta\rho$

We already discussed running of α from small lepton masses to m_Z



Other large contribution $\Delta\rho$ which represents loop ☛



is our primary focus here

Its value is given by

$$\Delta\rho = \frac{\alpha}{4\pi}\frac{z}{\omega(z-\omega)}N_C|U_{tb}|^2\left[m_t^2F(m_t^2,m_b^2)+m_b^2F(m_b^2,m_t^2)\right]$$ ♻

with

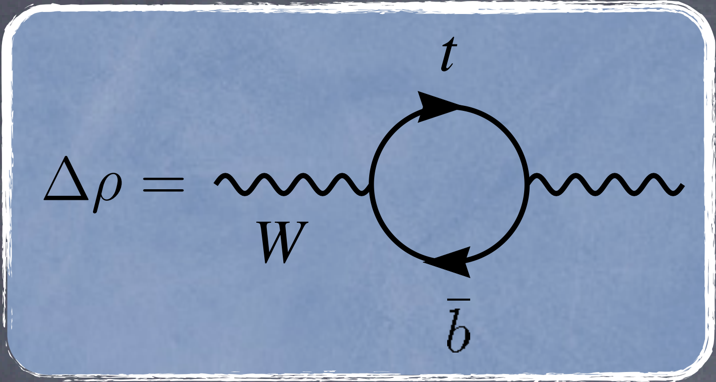
$$F(m_1^2,m_2^2) = \int_0^1 dx\, x\, \ln\left[m_1^2(1-x)+m_2^2x\right]$$ ☢

where $N_C = 3$ is number of colors and U_{tb} is CKM matrix element

$$|U_{tb}|^2 \simeq 1$$

FINGEPRINTS OF ELECTROWEAK THEORY

Diagram



has important property that

defining $m_t = m_b + \epsilon$ ☛

$$\Delta\rho \simeq \frac{G_F}{3\pi^2}\,\epsilon$$ ⌚

In QED

only equal mass fermions and antifermions appear in neutral photon loops and diagrams of this type are not possible

Rewrite ♻ and ☢ in form

$$\Delta\rho = \frac{G_F}{4\pi}\left[m_t^2 + m_b^2 - \frac{2m_b^2 m_t^2}{m_t^2 - m_b^2}\ln\frac{m_t^2}{m_b^2}\right]$$

$$\simeq \frac{G_F}{4\pi}m_t^2 \simeq \frac{3\alpha}{16\pi}\frac{1}{c^2 s^2}\frac{m_t^2}{z}$$ ❋

m_t^2/z contribution to an observable is forbidden in QED and QCD where virtual particle effects are suppressed by inverse powers of their masses

⌚ embodies this requirement because $\epsilon = 0$ for photon loops

Conversely ☛ appearance of m_t^2/z term

is a characteristic feature of electroweak theory

WE ARE NOW READY TO ILLUSTRATE THAT $\Delta\rho \neq 0$...

- We first determine experimental value of Δr from ≋

Using $\dfrac{G_F}{\sqrt{2}} = \dfrac{g^2}{8m_W^2}$ and $e = g\sin\theta_w$

$$\Delta r_{\rm exp} \simeq 1 - (37.281\ {\rm GeV})^2 \frac{z}{\omega(z-\omega)} \simeq 0.035 \qquad \triangle$$

- We next recall

$$\Delta\alpha \simeq 1 - \frac{\alpha(0)}{\alpha(m_Z^2)} \simeq 1 - \frac{128}{137} \simeq 0.066 \qquad ⌛$$

- Crucial point is that $\Delta r_{\rm exp} \neq \Delta\alpha$ ($\triangle \neq$ ⌛)

$\mathcal{O}(\alpha)$ SM relation ◒ requires a non-vanishing value of $\Delta\rho$

- Using ✻ we obtain that $\Delta\rho = 0.0086$ and ◒ yields

$$(\Delta r)_{\rm calculated} = \Delta\alpha - \frac{c^2}{s^2}\Delta\rho = 0.037$$

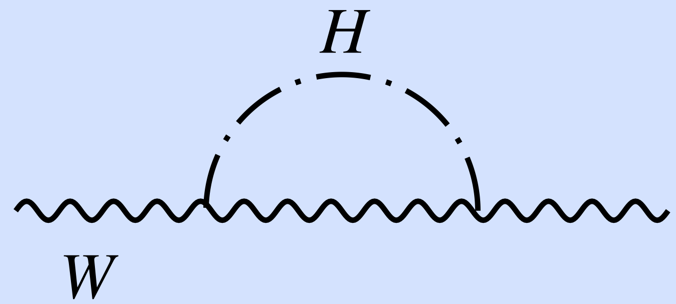
in agreement with experimental value

- We leave it as an exercise to insert errors into calculation and show that argument survives a straightforward statistical analysis

Constrains on Higgs Mass

Higgs particle makes a contribution to Δr

$\Delta h =$



$$= \frac{11\alpha}{48\pi}\frac{1}{c^2}\ln\frac{m_H^2}{z}$$

From $114.4 \text{ GeV} < m_H \lesssim 1 \text{ TeV}$ we obtain that $\Delta h < 0.0006$

contribution too small to be sensed by simple analysis presented above

Quantity Δr is in principle sensitive to Higgs mass

Radiative corrections predicted by SM have successfully confronted experiment

program is however far from complete

problem can be quantified by rewriting ◬ and ◒ as

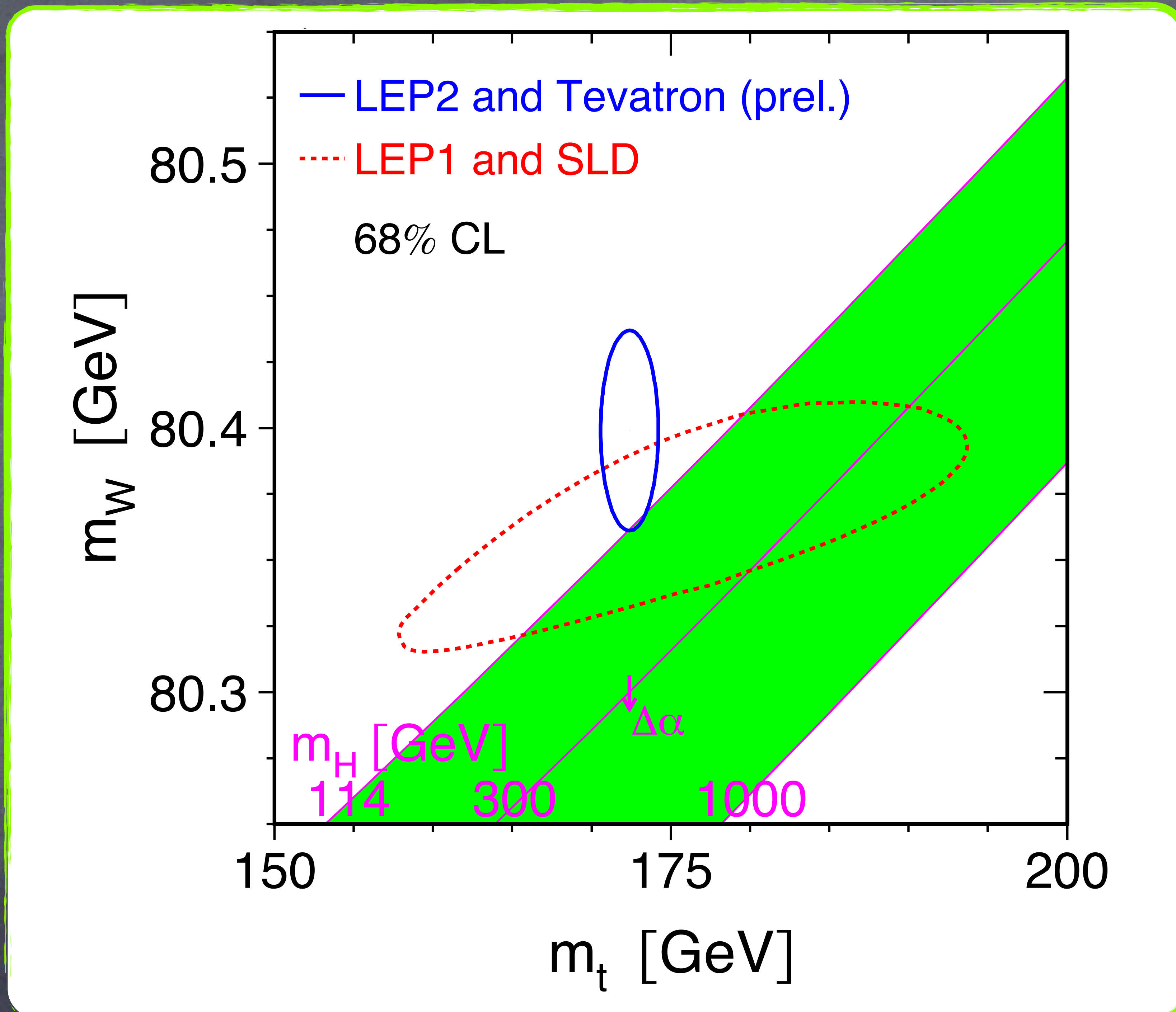
$$\Delta r_{\text{exp}} = F(m_W, m_t, m_H)$$

Using Z-pole measurements of SLD and LEP1

electroweak radiative corrections are evaluated

to predict masses of top quark and W-boson

(m_W, m_t) PLANE



Contour curves of 68% CL in (m_t, m_W) plane
for direct measurements and indirect determinations
Band shows correlation between m_W and m_t expected in SM

Neutrino Oscillation

Convincing experimental evidence exists
for oscillatory transitions $\nu_\alpha \rightleftharpoons \nu_\beta$ between different neutrino flavors

Simplest and most direct interpretation of atmospheric data
is that of muon neutrino oscillations

Evidence of atmospheric ν_μ disappearing is now at $> 15\sigma$
most likely converting to ν_τ

Angular distribution of contained events shows that
for $E_\nu \sim 1\ \mathrm{GeV}$ deficit comes mainly from $L_{\mathrm{atm}} \sim 10^2 - 10^4\ \mathrm{km}$

These results have been confirmed by KEK-to-Kamioka (K2K) experiment
which observes disappearance of accelerator ν_μ at a distance of $250\ \mathrm{km}$

Data collected by the Sudbury Neutrino Observatory (SNO)
in conjunction with data from Super-Kamiokande (SK)
show that solar ν_e convert to ν_μ or ν_τ with CL of more than 7σ

KamLAND Collaboration has measured flux of $\overline{\nu}_e$ from distant reactors
and find that $\overline{\nu}_e$ disappear over distances of about $180\ \mathrm{km}$

All these data suggest that neutrino eigenstates that travel through space
are not flavor states that we measured through weak force
but rather mass eigenstates

Lepton Flavor Mixing

Flavor eigenstates $|\nu_\alpha\rangle$ and mass eigenstates $|\nu_i\rangle$
are related by a unitary transformation U (i.e. mixing matrix)

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}|\nu_i\rangle \Leftrightarrow |\nu_i\rangle = \sum_\alpha (U^\dagger)_{i\alpha}|\nu_\alpha\rangle = \sum_\alpha U^*_{\alpha i}|\nu_\alpha\rangle$$

with $U^\dagger U = \mathbb{I}$, i.e., $\sum_i U_{\alpha i}U^*_{\beta i} = \delta_{\alpha\beta}$ and $\sum_i U_{\alpha i}U^*_{\alpha j} = \delta_{ij}$

For antineutrinos one has to replace $U_{\alpha i}$ by $U^*_{\alpha i}$

i.e. $$|\bar{\nu}_\alpha\rangle = \sum_i U^*_{\alpha i}|\bar{\nu}_i\rangle$$

Number of parameters of an $n \times n$ unitary matrix is $2n$
It is easy to see that $2n-1$ relative phases of n^2 neutrino states can be redefined such that $(n-1)^2$ independent parameters are left

For these it is convenient to take:

$\frac{1}{2}n(n-1)$ weak mixing angles of an n-dimensional rotation

and $\frac{1}{2}(n-1)(n-2)$ CP-violating phases

Hamiltonian Mechanics

- Being eigenstates of mass matrix ☛ states $|\nu_i\rangle$ are stationary states (i.e. they have the time dependence)

$$|\nu_i(t)\rangle = e^{-iE_i t}|\nu_i\rangle$$

with
$$E_i = \sqrt{p^2 + m_i^2} \approx p + \frac{m_i^2}{2p} \approx E + \frac{m_i^2}{2E}$$

where $E \approx p$ is total neutrino energy

☛ here it is assumed that neutrinos are stable

- A pure flavor state $|\nu_\alpha\rangle = \sum_i U_{\alpha i}|\nu_i\rangle$ present at $t = 0$

develops with time into state

$$|\nu(t)\rangle = \sum_i U_{\alpha i} e^{-iE_i t}|\nu_i\rangle = \sum_{i,\beta} U_{\alpha i} U^*_{\beta i} e^{-iE_i t}|\nu_\beta\rangle$$

Transition Amplitude

Time dependent transition amplitude for transition from flavor ν_α to flavor ν_β is

$$\begin{aligned}\mathfrak{A}(\nu_\alpha \to \nu_\beta) \equiv \langle \nu_\beta | \nu(t) \rangle &= \sum_i U_{\alpha i} U^*_{\beta i} e^{-iE_i t} \\ &= \sum_{i,j} U_{\alpha i} \delta_{ij} e^{-iE_i t} (U^\dagger)_{j\beta} \\ &= (UDU^\dagger)_{\alpha\beta}\end{aligned}$$

with $D_{ij} = \delta_{ij} e^{-iE_i t}$ (diagonal matrix)

An equivalent expression for transition amplitude is obtained by inserting into and extracting an overall phase factor e^{-iEt}

$$\begin{aligned}\mathfrak{A}(\nu_\alpha \to \nu_\beta, t) &= \sum_i U_{\alpha i} U^*_{\beta i}\, e^{-\frac{i m_i^2 t}{2E}} \\ &= \sum_i U_{\alpha i} U^*_{\beta i}\, e^{-\frac{i m_i^2 L}{2E}}\end{aligned}$$

where $L = ct$ (recall $c = 1$) is distance of detector in which ν_β is observed from ν_α source

$i \longrightarrow j$ Transition Amplitude

For an arbitrary chosen fixed j transition amplitude becomes

$$\begin{aligned}\widetilde{\mathfrak{A}}(\nu_\alpha \to \nu_\beta, t) &= e^{iE_j t}\mathfrak{A}(\nu_\alpha \to \nu_\beta, t) \\ &= \sum_i U_{\alpha i} U^*_{\beta i}\, e^{-i(E_i - E_j)}t \\ &= \delta_{\alpha\beta} + \sum_i U_{\alpha i} U^*_{\beta i} \left[e^{-i(E_i - E_j)t} - 1\right] \\ &= \delta_{\alpha\beta} + \sum_{i \neq j} U_{\alpha i} U^*_{\beta i} \left[e^{-i\Delta_{ij}} - 1\right]\end{aligned}$$

☤

with $$\Delta_{ij} = (E_i - E_j) = 1.27\,\frac{\delta m^2_{ij} L}{E}$$ when

L is measured in km E in GeV and $\delta m^2_{ij} = m^2_i - m^2_j$ in eV^2

(in ☤ unitarity relation ⊕ has been used)

Properties of Transition Amplitude

✓ Transition amplitudes are thus given by $n(n-1)$ real parameters

$(n-1)^2$ independent parameters of unitary matrix

(which determines sizes of oscillations)

and $n-1$ mass square differences

(which determine frequencies of oscillations)

✓ If CP is conserved in neutrino oscillations

all CP-violating phases vanish and $U_{\alpha i}$ are real

i.e. U is an orthogonal matrix $U^{-1} = U^T$ with $\frac{1}{2}n(n-1)$ parameters

Number of parameters for transition amplitude is then $\frac{1}{2}(n-1)(n+2)$

✓ Using ÷ we obtain amplitudes for transitions between antineutrinos

$$\mathfrak{A}(\bar{\nu}_\alpha \to \bar{\nu}_\beta; t) = \sum_i U^*_{\alpha i} U_{\beta i} e^{-iE_i t}$$

CPT

Comparing ⊠ and ☀ following relation holds for transformations between neutrinos and antineutrinos

$$\mathfrak{A}(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = \mathfrak{A}(\nu_\beta \to \nu_\alpha) \neq \mathfrak{A}(\nu_\alpha \to \nu_\beta)$$

This follows directly from CPT theorem:

C changes particle into antiparticle

P provides necessary flip from left-handed neutrino to right-handed antineutrino and vice versa

and T reverses arrow indicating transition

If CP is conserved ☛ $U_{\alpha i}$ and $U_{\beta i}$ are real in ⊠ and ☀

That is ☛ if time reversal invariance holds one has

$$\mathfrak{A}(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = \mathfrak{A}(\nu_\alpha \to \nu_\beta) = \mathfrak{A}(\bar{\nu}_\beta \to \bar{\nu}_\alpha) = \mathfrak{A}(\nu_\beta \to \nu_\alpha)$$

Therefore ☛ CP violation can be searched for e.g. by comparing oscillations $\nu_\alpha \to \nu_\beta$ and $\nu_\beta \to \nu_\alpha$

Transition Probability

Transition probabilities are obtained by squaring moduli of amplitudes ⊠

$$
\begin{aligned}
P_{\nu_\alpha \to \nu_\beta} &= \left| \sum_i U_{\alpha i} U^*_{\beta i} e^{-iE_i t} \right|^2 \\
&= \delta_{\alpha\beta} - 4 \sum_{i>j} \Re\mathrm{e} \left(U^*_{\alpha i} \, U_{\beta i} \, U_{\alpha j} \, U^*_{\beta j} \right) \sin^2 \Delta_{ij} \\
&+ 2 \sum_{i>j} \Im\mathrm{m} \left(U^*_{\alpha i} \, U_{\beta i} \, U_{\alpha j} \, U^*_{\beta j} \right) \sin 2\Delta_{ij}
\end{aligned}
$$

⌘

Matching Experimental data

- In standard treatment of neutrino oscillations

  flavor eigenstates $|\nu_\alpha\rangle (\alpha = e,\, \mu,\, \tau)$

  are expanded in terms of 3 mass eigenstates $|\nu_i\rangle (i = 1,\, 2,\, 3)$

- Atmospheric neutrino data suggest corresponding

  corresponding oscillation phase must be maximal $\Delta_{\rm atm} \sim 1$

  which requires $\delta m^2_{\rm atm} \sim 10^{-4} - 10^{-2}\ {\rm eV}^2$

- Assuming all upgoing multi-GeV ν_μ oscillate into different flavor

  while none of downgoing ones do

  observed up-down asymmetry

  leads to a mixing angle very close to maximal $\sin^2 2\theta_{\rm atm} > 0.85$

- Combined analysis of atmospheric neutrinos with K2K

  leads to a best fit-point and 1σ ranges

  $\delta m^2_{\rm atm} = 2.2^{+0.6}_{-0.4} \times 10^{-3}\ {\rm eV}^2$ and $\tan^2 \theta_{\rm atm} = 1^{+0.35}_{-0.26}$

- On the other hand ☛ reactor data suggest $|U_{e3}|^2 \ll 1$

Mass eigenstates

To simplify...

we use fact that $|U_{e3}|^2$ is nearly zero to ignore possible CP violation and assume that elements of U are real

With this in mind ☛ we can define a mass basis as follows

$$|\nu_1\rangle = \sin\theta_\odot|\nu^\star\rangle + \cos\theta_\odot|\nu_e\rangle$$

$$|\nu_2\rangle = \cos\theta_\odot|\nu^\star\rangle - \sin\theta_\odot|\nu_e\rangle$$

where $\theta_\odot$ is solar mixing angle

$$|\nu_3\rangle = \frac{1}{\sqrt{2}}(|\nu_\mu\rangle + |\nu_\tau\rangle)$$

$$|\nu^\star\rangle = \frac{1}{\sqrt{2}}(|\nu_\mu\rangle - |\nu_\tau\rangle)$$

is eigenstate orthogonal to $|\nu_3\rangle$

Flavor Eigenstates

Inversion of neutrino mass-to-flavor mixing matrix leads to

$$|\nu_e\rangle = \cos\theta_\odot |\nu_1\rangle - \sin\theta_\odot |\nu_2\rangle$$ & $$|\nu^\star\rangle = \sin\theta_\odot |\nu_1\rangle + \cos\theta_\odot |\nu_2\rangle$$

by adding 𝔹 and ⏏ one obtains ν_μ flavor eigenstate

$$|\nu_\mu\rangle = \frac{1}{\sqrt{2}}\left[|\nu_3\rangle + \sin\theta_\odot |\nu_1\rangle + \cos\theta_\odot |\nu_2\rangle\right]$$

and by substracting these same equations ν_τ eigenstate

Combined analysis of Solar neutrino data and KamLAND data are consistent at 3σ CL with best-fit point and 1σ ranges:

$$\delta m^2_\odot = 8.2^{+0.3}_{-0.3} \times 10^{-5}\ \mathrm{eV}^2$$

$$\tan^2\theta_\odot = 0.39^{+0.05}_{-0.04}$$

Far-out neutrinos

For $\Delta_{ij} \gg 1$ phases will be erased by uncertainties in L and E (as would be case for neutrinos propagating over cosmic distances),

averaging over $\sin^2 \Delta_{ij}$ in ⌘ we obtain

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 2 \sum_{i>j} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} \quad ⌬$$

using $2 \sum_{1>j} = \sum_{i,j} - \sum_{i=j}$, ⌬ can be re-written as

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta) &= \delta_{\alpha\beta} - \sum_{i,j} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} + \sum_i U_{\alpha i} U_{\beta i} U_{\alpha i} U_{\beta i} \\ &= \delta_{\alpha\beta} - \left(\sum_i U_{\alpha i} U_{\beta i} \right)^2 + \sum_i U_{\alpha i}^2 U_{\beta i}^2 \end{aligned} \quad ☋$$

Since $\delta_{\alpha\beta} = \delta_{\alpha\beta}^2$ first and second terms in ☋ cancel each other

$$P(\nu_\alpha \to \nu_\beta) = \sum_i U_{\alpha i}^2 \, U_{\beta i}^2$$

Average Probability

Probabilities for flavor oscillation are then

$$P(\nu_\mu \to \nu_\mu) = P(\nu_\mu \to \nu_\tau) = \frac{1}{4}\,(\cos^4\theta_\odot + \sin^4\theta_\odot + 1)$$

$$P(\nu_\mu \to \nu_e) = P(\nu_e \to \nu_\mu) = P(\nu_e \to \nu_\tau) = \sin^2\theta_\odot \;\cos^2\theta_\odot$$

$$P(\nu_e \to \nu_e) = \cos^4\theta_\odot + \sin^4\theta_\odot$$

Let ratios of neutrino flavors at production in cosmic sources

be written as $w_e : w_\mu : w_\tau$ with $\sum_\alpha w_\alpha = 1$

so that relative fluxes of each mass eigenstate are $w_j = \sum_\alpha \omega_\alpha \, U^2_{\alpha j}$

We conclude that probability of measuring on Earth a flavor α is

$$P_{\nu_\alpha \text{ detected}} = \sum_j U^2_{\alpha j} \; \sum_\beta w_\beta \, U^2_{\beta j}$$

Earhtly Ratios

- Any initial flavor ratio that contains $w_e = 1/3$
  will arrive at Earth with equipartition on 3 flavors
- Cosmic neutrinos arise dominantly from decay of charged pions
  their initial flavor ratios of nearly $1:2:0$
  should arrive at Earth democratically distributed
- Fairly robust prediction of $1:1:1$ cosmic neutrino flavor ratios
- Prediction for pure $\bar{\nu}_e$ source originating via neutron β-decay
  has different implications for flavor ratios
  $w_e = 1$ yields Earthly ratios $\sim 5:2:2$
- Such a unique ratio would appear above $1:1:1$ background
  in direction of neutron source
- Flavor identification of neutrinos on a statistical basis
  becomes possible at IceCube
  opening up a window for discoveries in particle physics
  not otherwise accessible to experiment

the Vampire

- In SM masses arise from Yukawa interactions which couple right-handed fermion with its left-handed doublet after spontaneous symmetry breaking
- Because no right-handed neutrinos exist in SM Yukawa interactions leave neutrinos massless
- One may wonder if neutrino masses could arise from loop corrections or even by non-perturbative effects but this cannot happen because any neutrino mass term that can be constructed with SM fields would violate total lepton symmetry
- To introduce a neutrino mass term we must either extend particle content or else abandon gauge invariance and/or renormalizability

How to kill a Vampire

- We keep gauge symmetry and introduce arbitrary number m of additional right-handed neutrino states (singlets under hypercharge) $\nu_{Rs}(1,1)_0$
- With particle contents of SM

  and addition of an arbitrary m number of right-handed neutrinos one can construct two types of mass terms

  that arise from gauge invariant renormalizable operators

$$-\mathcal{L}_{M_\nu} = \sum_{\alpha=e,\mu,\tau} \sum_{i=1}^{m} {M_D}^{i\alpha}\ \bar{\nu}_{Ri}\ \nu_{L\alpha} + \frac{1}{2} {M_N}^{ij}\ \bar{\nu}_{Ri}\ \nu^c_{Rj} + \text{h.c.}$$

- ν^C indicates a charge conjugated field $(\nu^c = C\bar{\nu}^T)$
- M_D is a complex $m \times 3$ matrix and M_N is a symmetric matrix of dimension $m \times m$

Dirac Neutrinos

- Forcing $M_N = 0$ leads to a Dirac mass term which is generated from Yukawa interactions after spontaneous electroweak symmetry breaking

$$Y_\nu{}^{i\alpha} \; \bar{\nu}_{Ri} \; \bar{\phi}^\dagger \; L_{L\alpha} \Rightarrow M_D{}^{i\alpha} = Y_\nu{}^{i\alpha} \frac{v}{\sqrt{2}}$$

similarly to charged fermion masses

- Such a mass term conserves total lepton number but it breaks the lepton flavor number symmetries
- For $m = 3$ we can identify the hypercharge singlets with right-handed component of 4-spinor neutrino field
- Since matrix Y is (in general) a complex 3×3 matrix flavor neutrino fields ν_e, ν_μ and ν_τ do not have a definite mass
- Massive neutrino fields are obtained via diagonalization of $\mathcal{L}_{M_\nu}$

MAJORANA NEUTRINOS

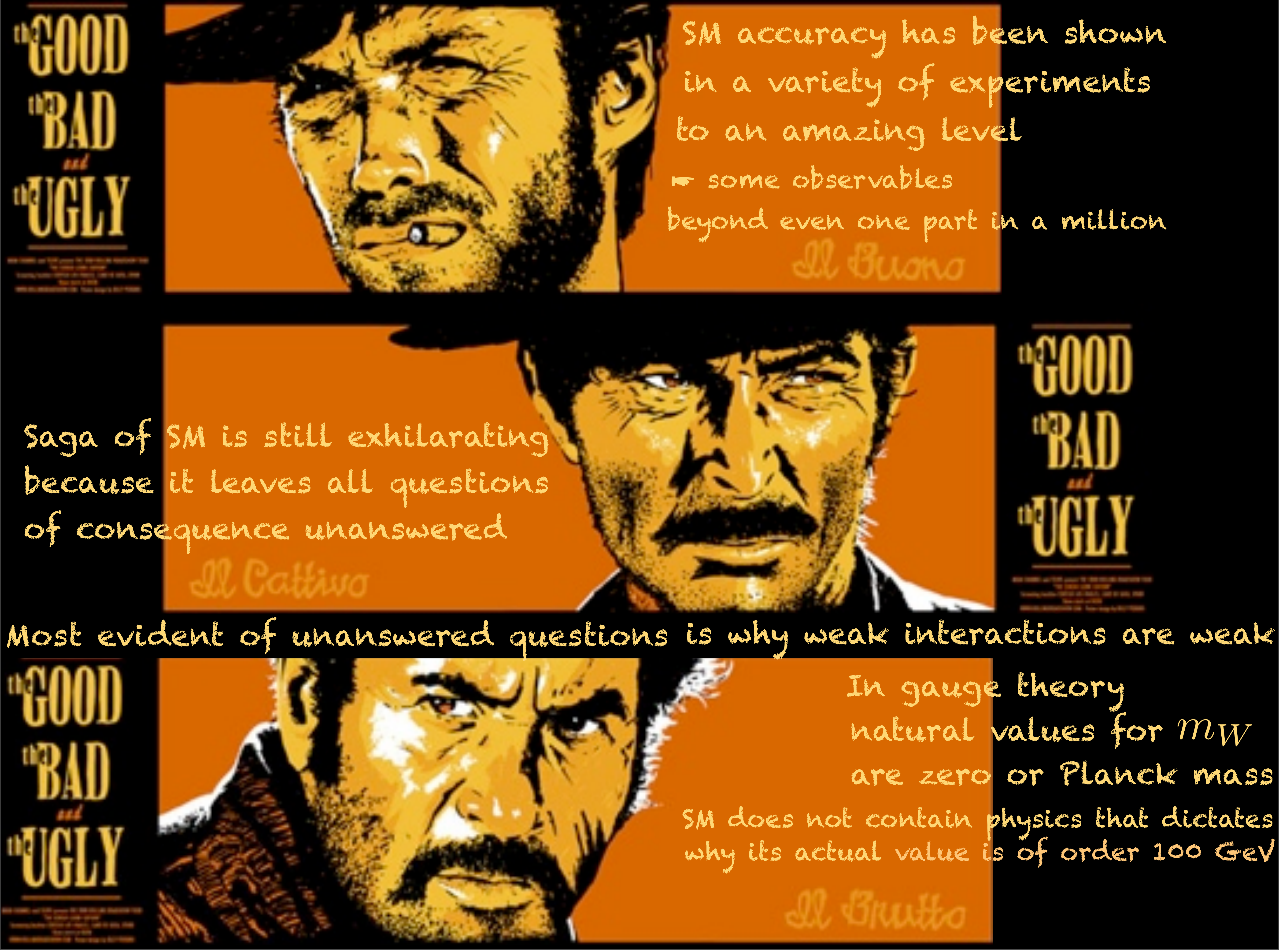
- If $M_N \neq 0$ ☛ neutrino masses
  receive important contribution from Majorana mass term
- Such a term is singlet of SM gauge group
- Therefore ☛ it can appear as bare mass term
- Furthermore ☛ since it involves two neutrino fields
  it breaks lepton number by two units
- More generally ☛ such a term is allowed
  only if neutrinos carry no additive conserved charge
- This is reason that such terms are not allowed
  for any charged fermions which (by definition) carry $U(1)_{\rm EM}$

Majorana neutrinos (cont'd)

Dirac mass terms are protected by symmetries of SM
(neutrino masses $\propto$ EW symmetry-breaking scale)

Majorana mass term is SM singlet
elements of M_N are not protected by SM symmetries

It is plausible that Majorana mass term
is generated by new physics beyond SM
and right-handed chiral neutrino fields ν_R
belong to nontrivial multiplets of symmetries of high energy theory



SM accuracy has been shown in a variety of experiments to an amazing level

☛ some observables beyond even one part in a million

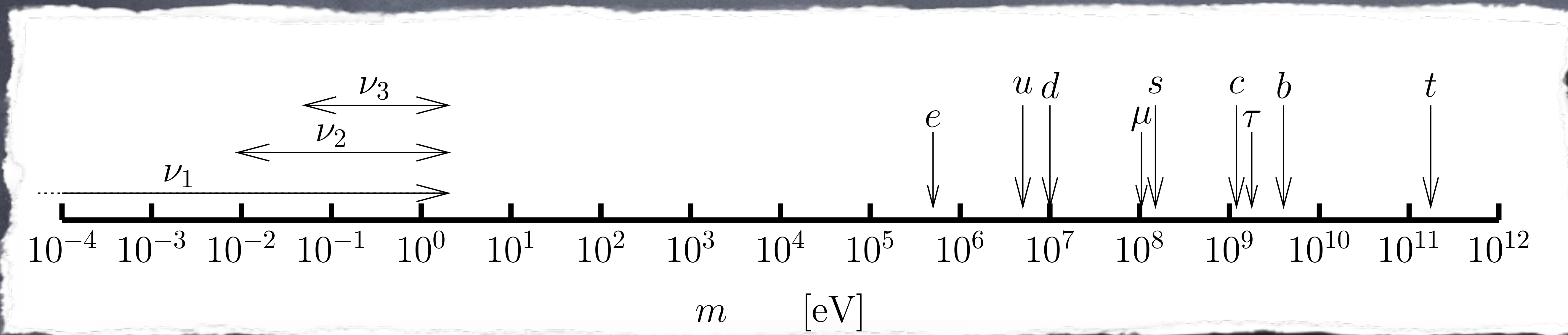
Saga of SM is still exhilarating because it leaves all questions of consequence unanswered

Most evident of unanswered questions is why weak interactions are weak

In gauge theory natural values for m_W are zero or Planck mass

SM does not contain physics that dictates why its actual value is of order 100 GeV

The Ugly...



Order of magnitude of masses of quarks & leptons

The Good...

Already in 1934 Fermi provided an answer
with a theory that prescribed a quantitative relation
between fine structure constant and weak coupling $G_F \sim \alpha / m_W^2$
Although Fermi adjusted m_W
to accommodate strength and range of nuclear radioactive decays
one can readily obtain a value of m_W of 40 GeV
from observed μ decay rate for which proportionality factor is $\pi/\sqrt{2}$
Answer is off by a factor of 2
because discovery of parity violation and neutral currents
was in future and introduces an additional factor $1 - m_W^2/m_Z^2$

$$G_F = \left[\frac{\pi\alpha}{\sqrt{2} m_W^2} \right] \left[\frac{1}{1 - m_W^2/m_Z^2} \right] (1 + \Delta r)$$

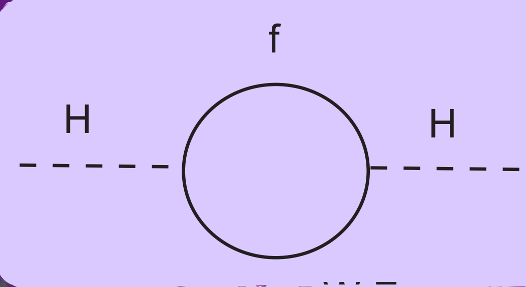
Fermi could certainly not have anticipated
that we now have a renormalizable gauge theory
that allows us to calculate radiative correction Δr to his formula

Higgs mass Radiative Corrections

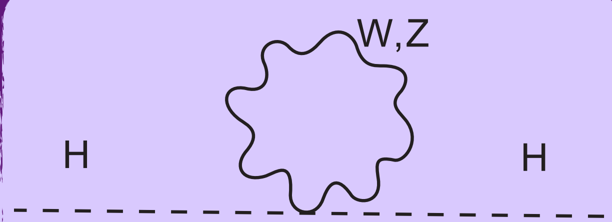
One of most acute problems connected with ultraviolet divergences concerns radiative corrections to mass appearing in Higgs potential

$$V = \mu^2 \Phi\Phi^\dagger + \lambda(\Phi^\dagger\Phi)^2$$

$f, W,$ and H self-coupling radiative corrections to Higgs mass are



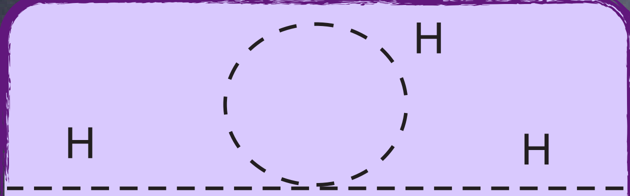
$$i\frac{Y_f^2}{2}\int^\Lambda \frac{d^4k}{(2\pi)^4}\,\mathrm{tr}\left(\frac{i}{\not k - m_f}\,\frac{i}{\not k + \not p - m_f}\right) \sim -\Lambda^2\,\mathrm{tr}(\mathbb{I})\,\frac{Y_f^2}{32\pi^2}$$



$$i\frac{g^2}{4}\int^\Lambda \frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2 - m_W^2} \sim \Lambda^2\,\frac{g^2}{64\pi^2}$$

$$m_W \xrightarrow{Z} m_Z$$

$$g^2 \to g^2 + g'^2$$



$$i6\lambda\int^\Lambda \frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2 - m_H^2} \sim \Lambda^2\,\frac{3\lambda}{8\pi^2}$$

$$m_W^2 = \frac{1}{4}g^2v^2,\quad v = 246\ \mathrm{GeV},\quad m_Z^2 = \frac{1}{4}(g^2 + g'^2)v^2,\quad m_f^2 = \frac{1}{2}Y_f^2v^2$$

Y_f is Yukawa coupling

$m_H^2 = 2\lambda v^2$, g and g' are $SU(2)_L \times U(1)_Y$ gauge couplings

λ is quartic Higgs coupling and Λ is a cutoff

Veltman Condition

Difference between bare and renormalized masses is

$$\Delta\mu^2 = \frac{1}{64\pi^2}\left(9g^2 + 3g'^2 + 24\,\lambda - 8\sum_f N_f\, Y_f^2\right)\Lambda^2$$
$$\simeq \frac{3}{16\pi^2 v^2}\,(2m_W^2 + m_Z^2 + m_H^2 - 4m_t^2)\,\Lambda^2$$

SM works amazingly well by fixing Λ at electroweak scale

generally assumed ☛ this indicates existence of new physics beyond SM

Following Weinberg

$$\mathcal{L}(m_{\rm W}) = |\mu^2|\, H^\dagger H + \frac{1}{4}\,\lambda(H^\dagger H)^2 + \mathcal{L}_{\rm SM}^{\rm gauge} + \mathcal{L}_{\rm SM}^{\rm Yukawa} + \frac{1}{\Lambda}\,\mathcal{L}^5 + \frac{1}{\Lambda^2}\mathcal{L}^6 + \ldots$$

where operators of higher dimension parametrize physics beyond SM

Some have resorted to rather extreme lengths by proposing that factor multiplying unruly quadratic correction $(2m_W^2 + m_Z^2 + m_H^2 - 4m_t^2)$ must vanish EXACTLY!

By most conservative estimates this new physics is within our reach

New Physics at the TeV Scale?

Avoiding fine tuning requires $\Lambda \lesssim 2-3$ to be revealed by CERN LHC

E.G. for $m_H = 115 - 200$ GeV,

$$\left|\frac{\Delta\mu^2}{\mu^2}\right| = \frac{\delta v^2}{v^2} \leq 10 \Rightarrow \Lambda = 2-3 \text{ TeV}$$

we have implicity used $v^2 = -\mu^2/\lambda$

[valid in approximation

disregarding terms beyond $\mathcal{O}(H^4)$ in Higgs potential]

Let's contemplate possibilities

- Veltman condition happens to be satisfied
  and this would leave particle physics with an ugly fine tuning problem
- This is very unlikely ☛ LHC must reveal Higgs physics
  already observed via radiative correction
  or at least discover physics that implements Veltman condition
- It must appear at $2-3$TeV
  even though higher scales can be rationalized
  when accommodating selected experiments

SUSY

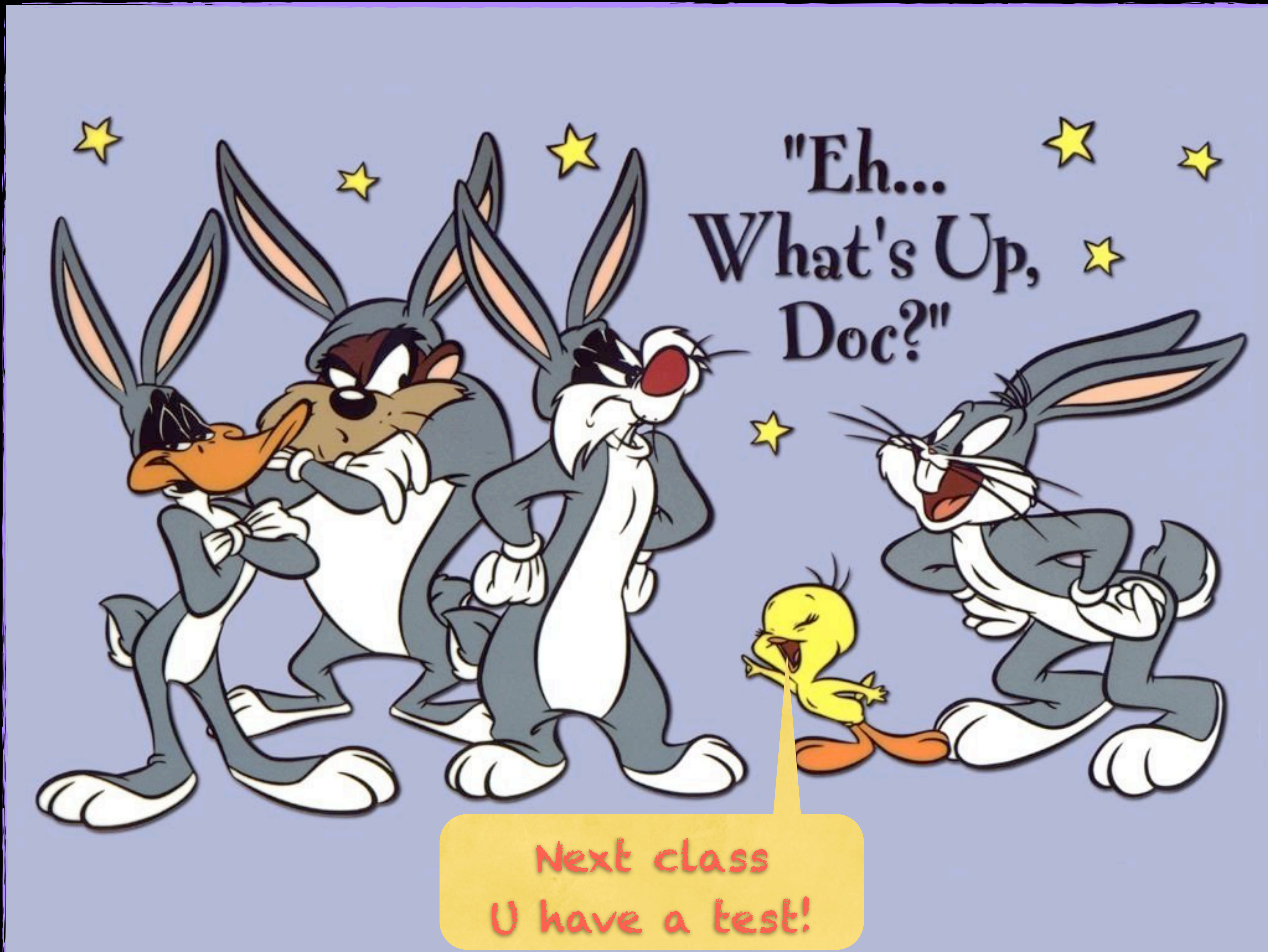
Supersymmetry predicts that interactions exist
that would change fermions into bosons and vice versa
and that all known fermions (bosons) have SUSY boson (fermion) partner

SUSY offers solution of bad behavior of radiative corrections in SM

As for every boson there is a companion fermion
bad divergence associated with Higgs loop
cancelled by a fermion loop with opposite sign

even though it elegantly controls quadratic divergence
by cancellation of boson and fermion contributions
it is already fine-tuned at a scale of $2 - 3\mathrm{TeV}$

THE BAD...



Happy thanksgiving