4 – Synthesis and Analysis of Complex Waves; Fourier spectra

Complex waves

Many physical systems (such as music instruments) allow existence of a particular set of standing waves with the frequency spectrum consisting of the fundamental (minimal allowed) frequency f_1 and the overtones or harmonics f_n that are multiples of f_1, that is, $f_n = n f_1$, n=2,3,…The amplitudes of the harmonics A_n depend on how exactly the system is excited (plucking a guitar string at the middle or near an end). The sound emitted by the system and registered by our ears is thus a complex wave (or, more precisely, a complex oscillation), or just signal, that has the general form

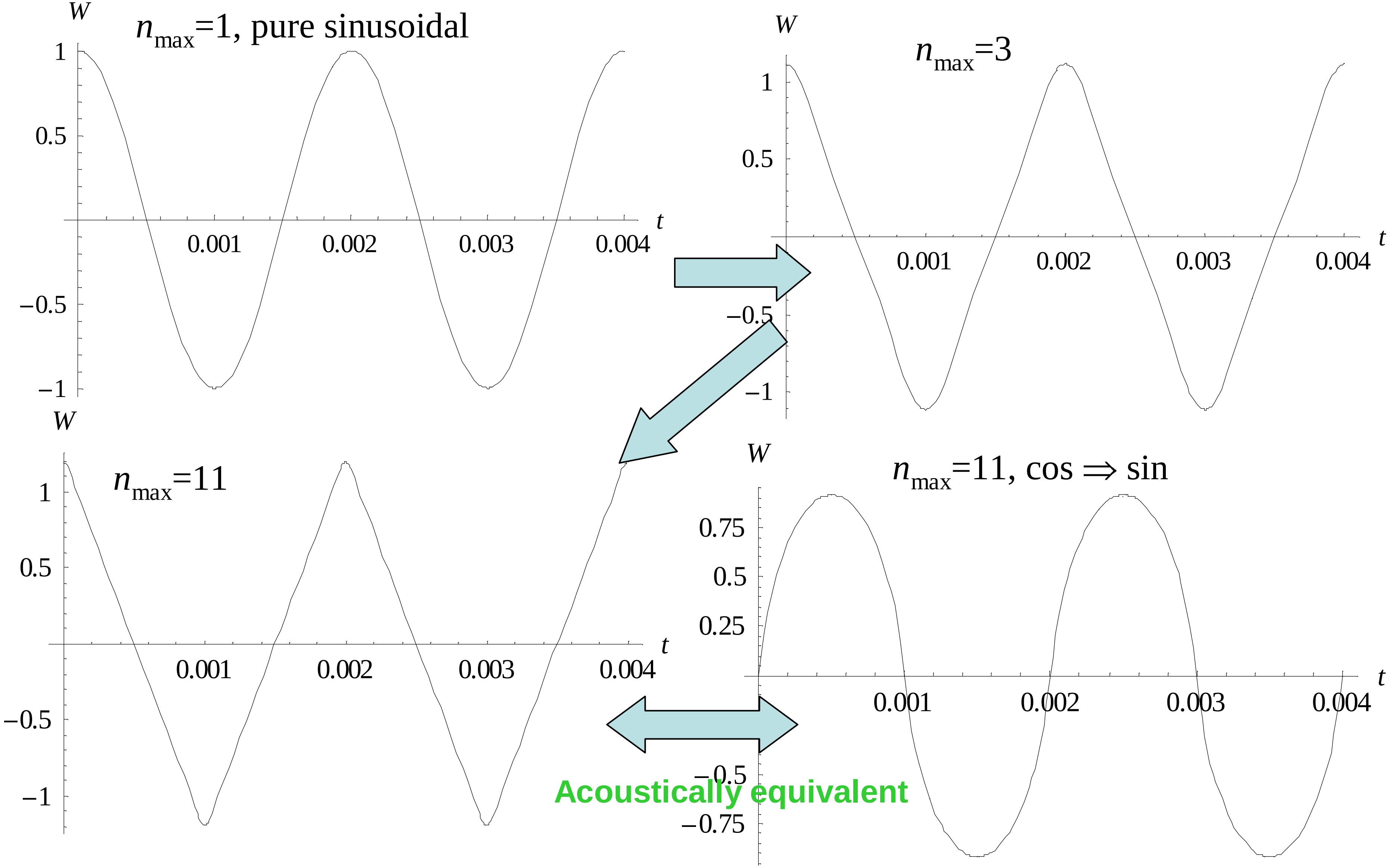
$$W(t) = \sum_{n=1}^{n_{\max}} A_n \sin(2\pi f_n t + \varphi_n), \qquad \text{periodic with period } T = \frac{1}{f_1}$$

Plots of $W(t)$ that can look complicated are called wave forms. The maximal number of harmonics n_{max} is actually infinite but we can take a finite number of harmonics to synthesize a complex wave. The phases φ_n influence the wave form a lot, visually, but the human ear is insensitive to them (the psychoacoustical Ohm's law). What the ear hears is the fundamental frequency (even if it is missing in the wave form, A_1=0!) and the amplitudes A_n that determine the sound quality or timbre.

Examples of synthesis of complex waves

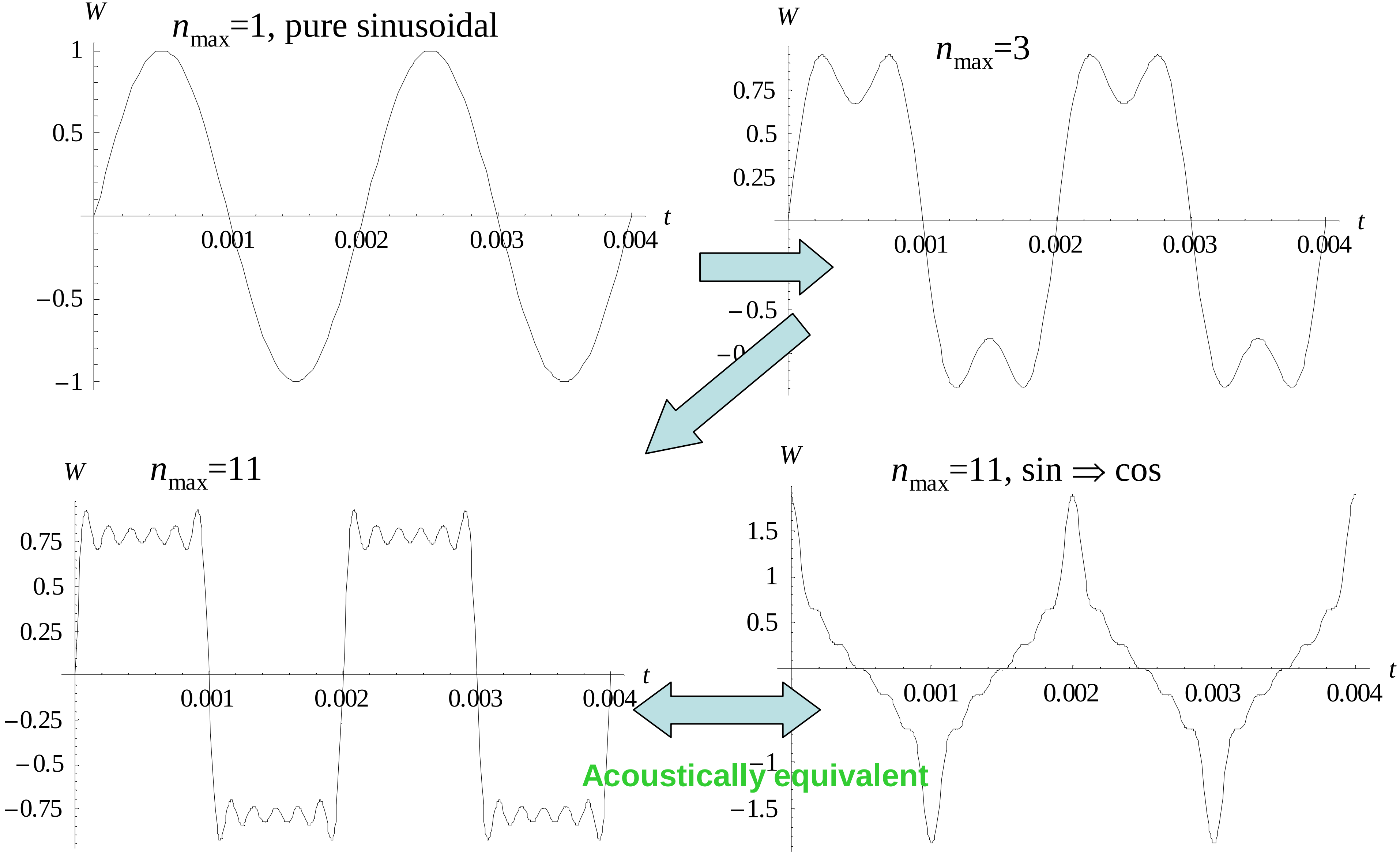
1. Triangular wave: $A_n = \dfrac{1}{n^2}$ for n odd and zero for n even

(plotted for f_1=500)



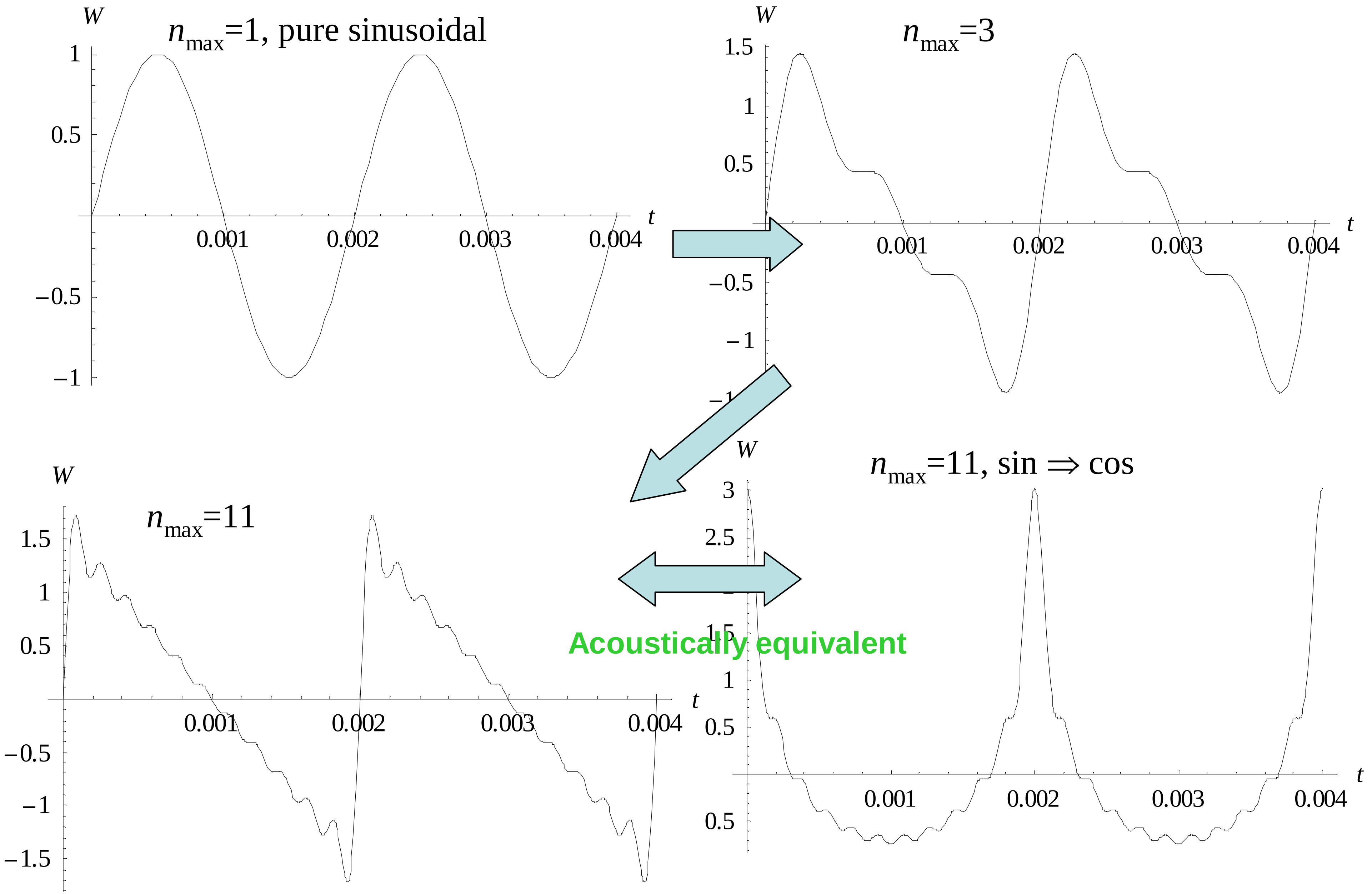
1. Square wave: $A_n = \dfrac{1}{n}$ for n odd and zero for n even

(plotted for f_1=500)



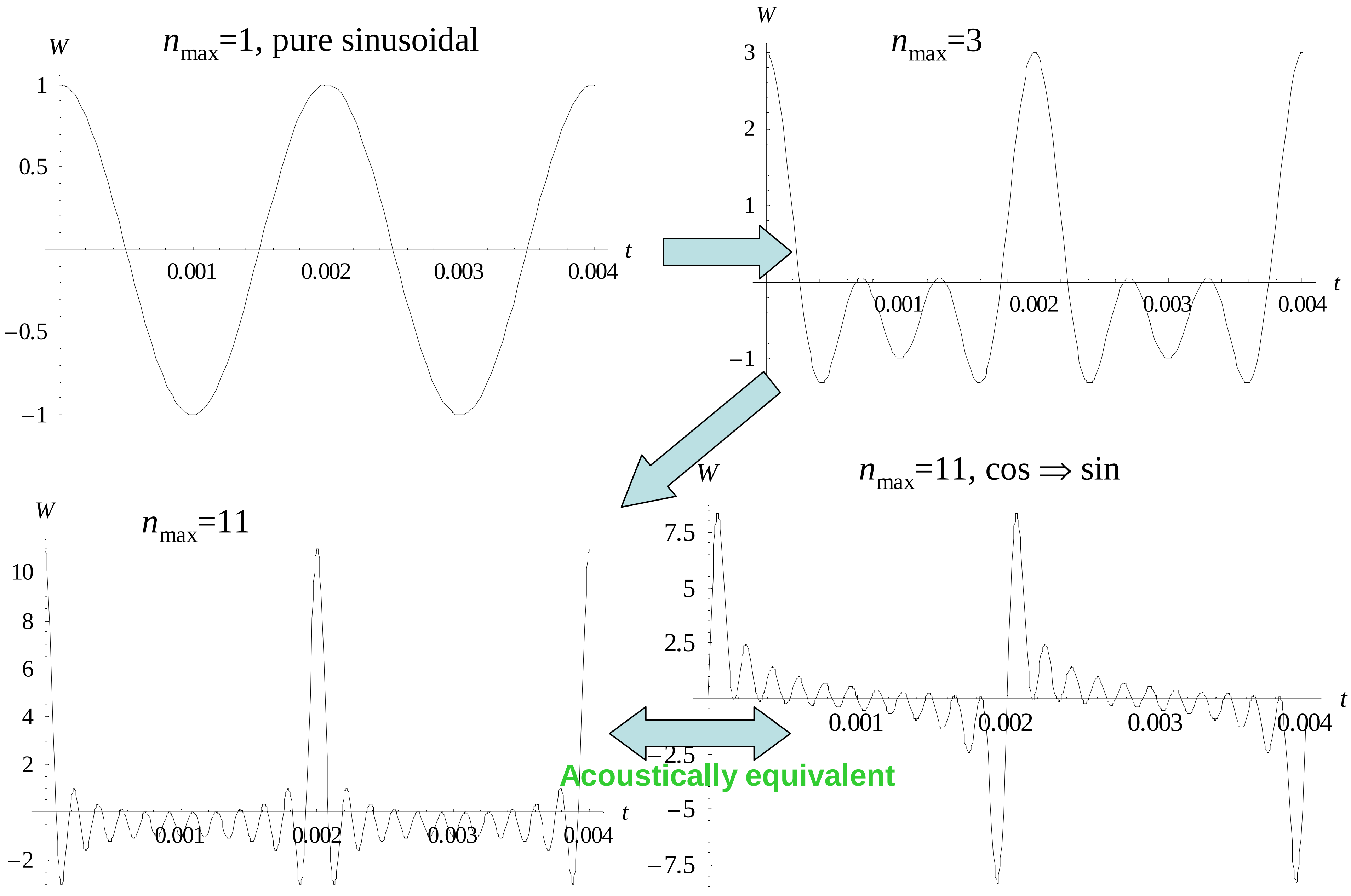
1. Sawtooth wave: $A_n = \dfrac{1}{n}$

(plotted for f_1=500)



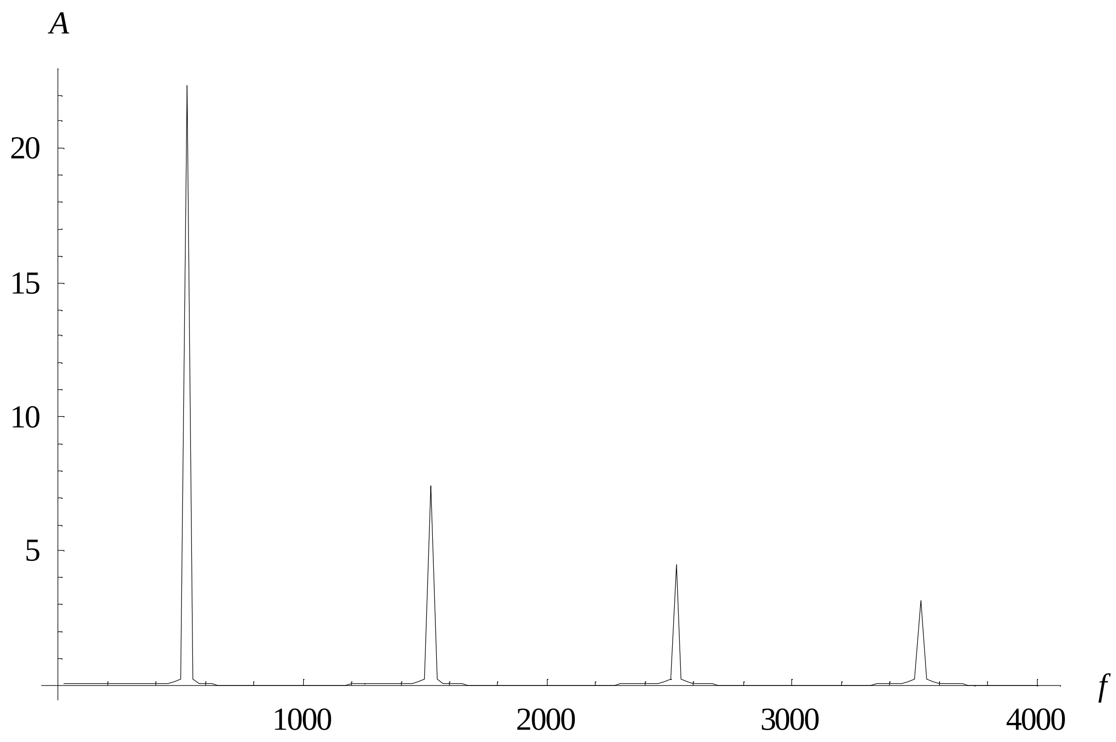
1. Pulse train: $A_n = 1$

(plotted for f_1=500)



Fourier spectrum

The set of the amplitudes A_n of a signal (complex wave) is called the <u>Fourier spectrum</u> of the signal. For a periodic signal the Fourier spectrum can be plotted as a series of spikes at the frequencies f_n with the hights proportional to the corresponding amplitudes A_n.



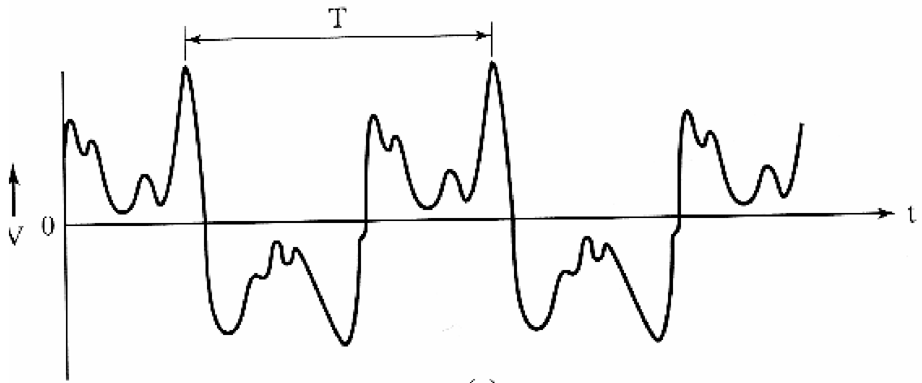
Here is the Fourier spectrum of a square wave. One can see the fundamental frequency f_1=500 Hz and odd overtones at 1500 Hz, 2500 Hz, 3500 Hz, etc. Even overtones at 1000 Hz, etc. are absent for the square wave.

Fourier transformation

If the amplitudes A_n of the complex wave are known, as was the cases above, it is straightforward to plot its Fourier spectrum. In many cases, to the contrary, the structure of a complex wave is unknown from the beginning, and the problem is to find of which simple harmonic waves this complex wave is composed, that is, what are A_n.in the formula on p.1. Mathematics tells us that any periodic wave can be represented as a sum of a fundamental wave and overtones, and the whole wave can be written in a form similar to that on p.1. There is a mathematical method called Fourier transformation to obtain the coefficients A_n, that is, the Fourier spectrum of a signal, from the wave form that can be, say, obtained experimentally, like the recorded sound of a music instrument.

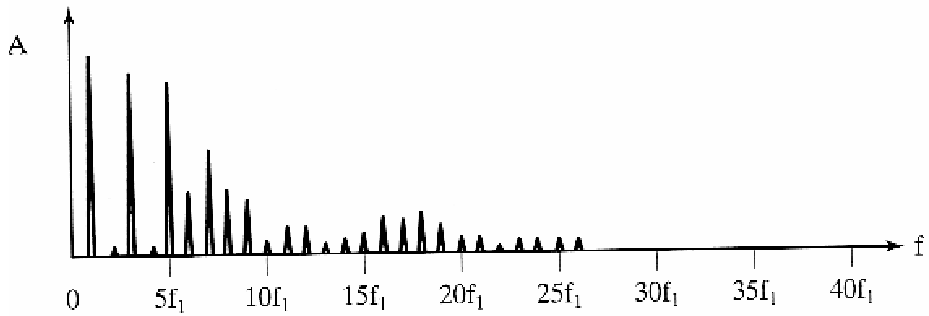
Example:

Wave form of a sound played by a clarinet



(a)

Fourier spectrum of the clarinet sound obtained by the Fourier transformation of this wave form. Note that at least two of the lowest even overtones are missing since clarinet is an air pipe closed at one end and open at the other end, so is has node-antinode boundary conditions.



(b)

Properties of Fourier spectra

• Fourier spectrum of periodic signals consist of regularly-spaced spikes corresponding to the fundamental and overtones. This is a discrete Fourier spectrum

• Fourier spectrum of a superposition of different periodic signals (such as several notes played at the same time) also consists of spikes, that is, it is discrete. However, these spikes are not equally spaced. Still, one can figure out which spikes correspond to each note, and the spikes corresponding to a given note are equidistant. Obviously a superposition of two signals gives a superposition of two Fourier spectra

• If too many different notes sound at the same time, there are too many sets of spikes in the Fourier spectrum so that the spikes merge together and the Fourier spectrum becomes a continuous function. This also happens in the case of nonperiodic (nonmusical) signals, such as noises.

Limited usefulness of the Fourier spectra:

Analyzing Fourier spectra of sounds, one can distinguish different music instruments from each other. However, it is impossible to distinguish a good violinist playing a good violin from a bad violinins playing a bad violin. For this task, human ear is much better suited than the analysis of wave forms and Fourier spectra.

Sound quality

Simple harmonic oscillation sounds plain. Real sounds including musical sounds have more character due to the overtones present in them. These overtones define the sound quality or timbre of the sound. Other factors such as the attack and decay affect the perception of the sound no less than the overtones. If one combines the attack played by one instrument with a subsequent sustained sound played by another instrument, the listener can be easily confused into thinking that also the sustained sound belongs to the first instrument.

Formantes

Formante is a part of a Fourier spectrum (a group of harmonics) that is emphesized with respect to the surrounding parts of the spectrum. That is, the harmonics within a formante have a larger amplitudes than others. An example is the formante in the clarinet‘s sound between $15f_1$ and $20f_1$. Formantes render the sound of a music instrument and voice a special character. In particular, good singers (even bassos) have formantes at high frequencies that make them audible through the orchestral part. Also formantes are responsible for formation of different vowels in the speech. As a characteristic of overtons, formantes are responsible for the sound quality, of course.

Spectrograms

Real sounds, even music sounds, are not strictly periodic. Music sounds have an attack (beginning) and the decay transient, different for different instruments. Also there are superpositions of different sounds, beginning and ending at different moments of time. This makes a problem for defining Fourier spectra. Fourier transformation of strictly periodic signals uses only one period, there is no additional information in other parts of the signal. In the case of signals that are nonperiodic or not strictly periodic, one cannot use just one period. (It is even not defined for a sum of signals with different periods.) Instead, one has to do Fourier transformation using a period of time longer than one period. In particular, for complex signals that are stable over a significant time interval (such as a sustained sound of an orchestra), the longer the time interval used for making the Fourier transformation, the more accurate are the Fourier spectra, the spikes are better resolved from each other.

If a complex signal significantly changes with time (some notes stop, other begin, or different vowels and consonants follow each other in speech), one has to make a compromise choosing the time interval Δt for the Fourier transformation. What is done is that for any time t one makes a Fourier transformation using a time interval Δt centered at t. The resulting Fourier spectrum depends on the frequency f, as the standard Fourier spectrum, but also on the time t. To represent this function of two variables graphically, one uses 3d plots that are called spectrograms. Usually spectrogramms use a gray-scale or color coding of the intensity with t at the horizontal axis and f at the vertical axis (see next page).

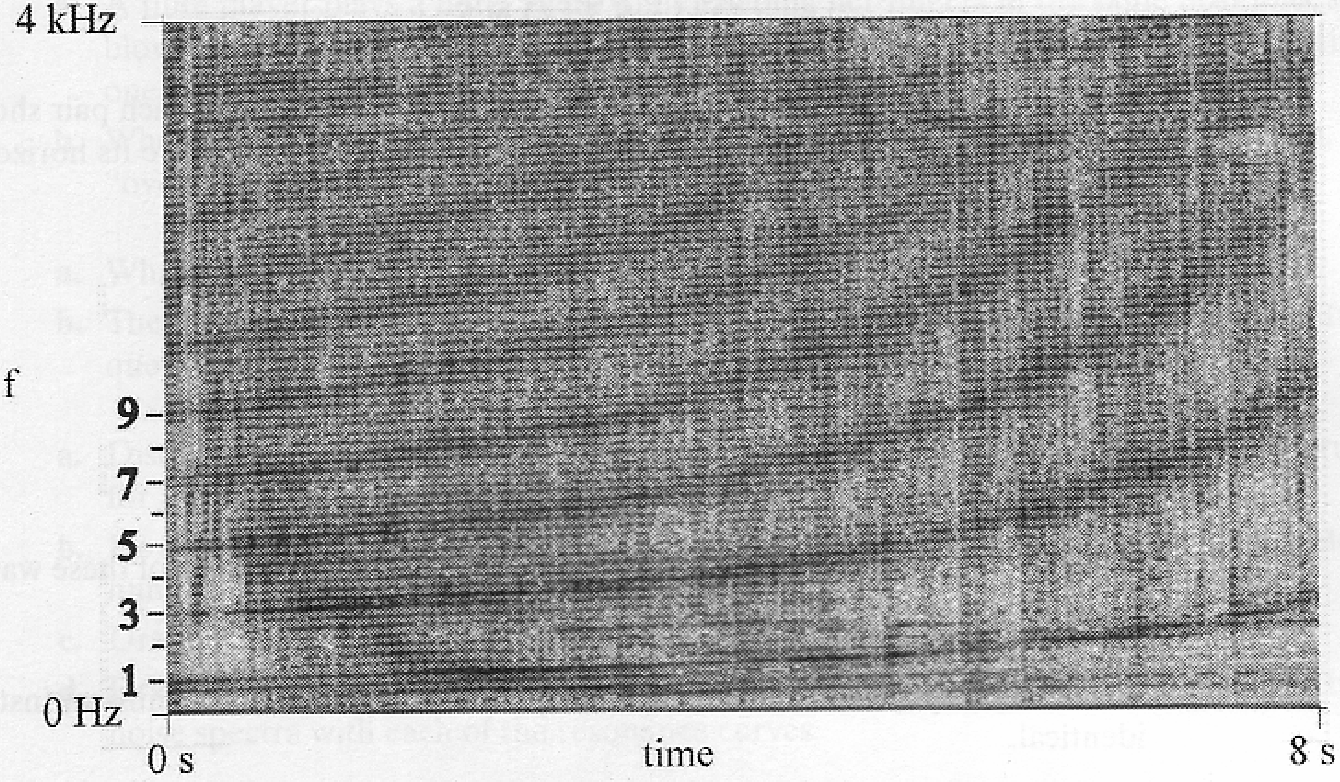


Figure 4-37 Spectrogram of water being poured into a one-liter graduated cylinder. Note the missing even harmonics of the closed tube and the increasing frequencies of the harmonics as the length of the air column decreases. (Luke Keller)

- The so-called <u>narrow-band spectrograms</u> use longer time intervals Δt for the Fourier transformation. They provide a better frequency resolution but a worse time resolution;
- To the contrary, <u>wide-band spectrograms</u> use shorter time intervals Δt. They provide a better time resolution but worse frequency resolution.

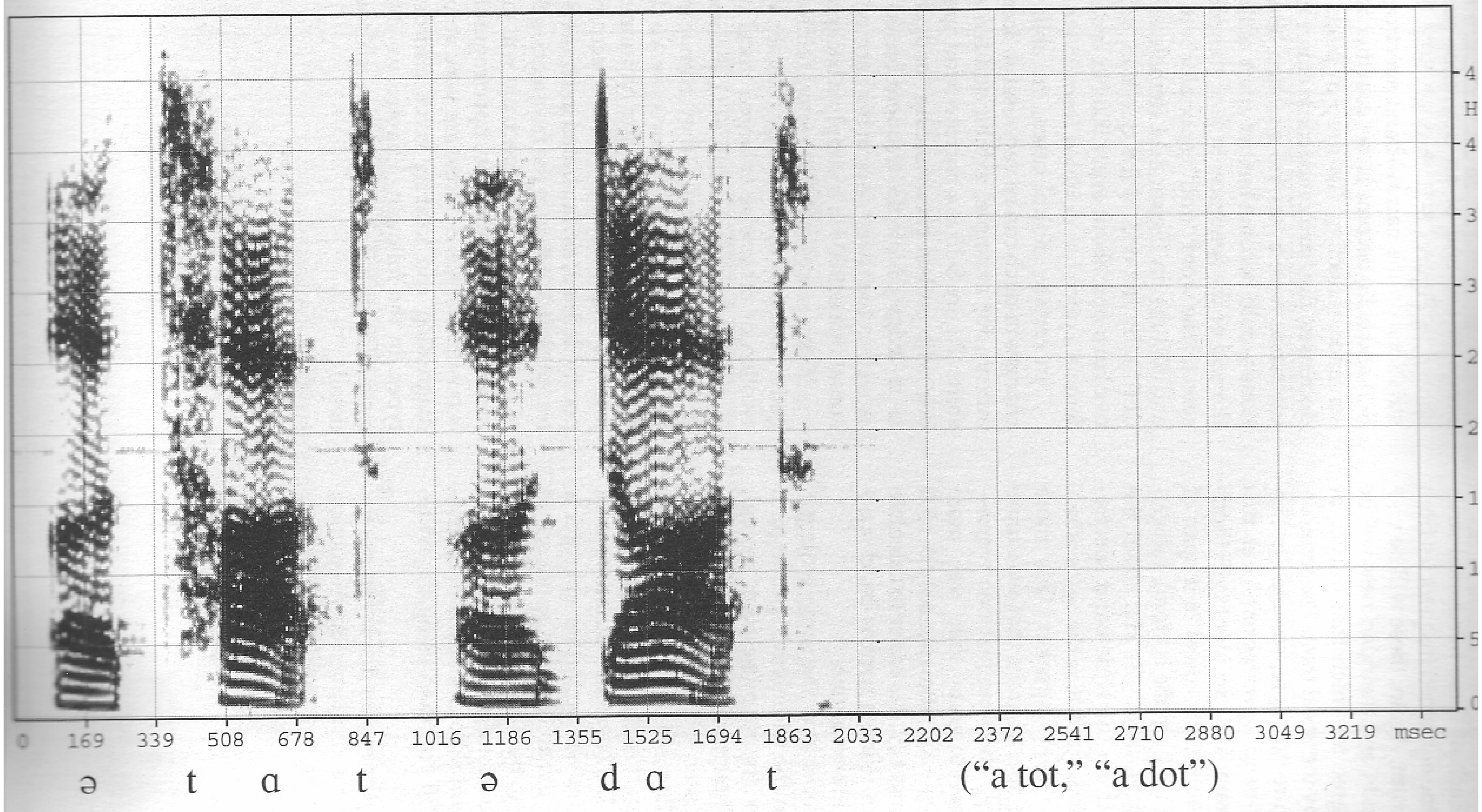
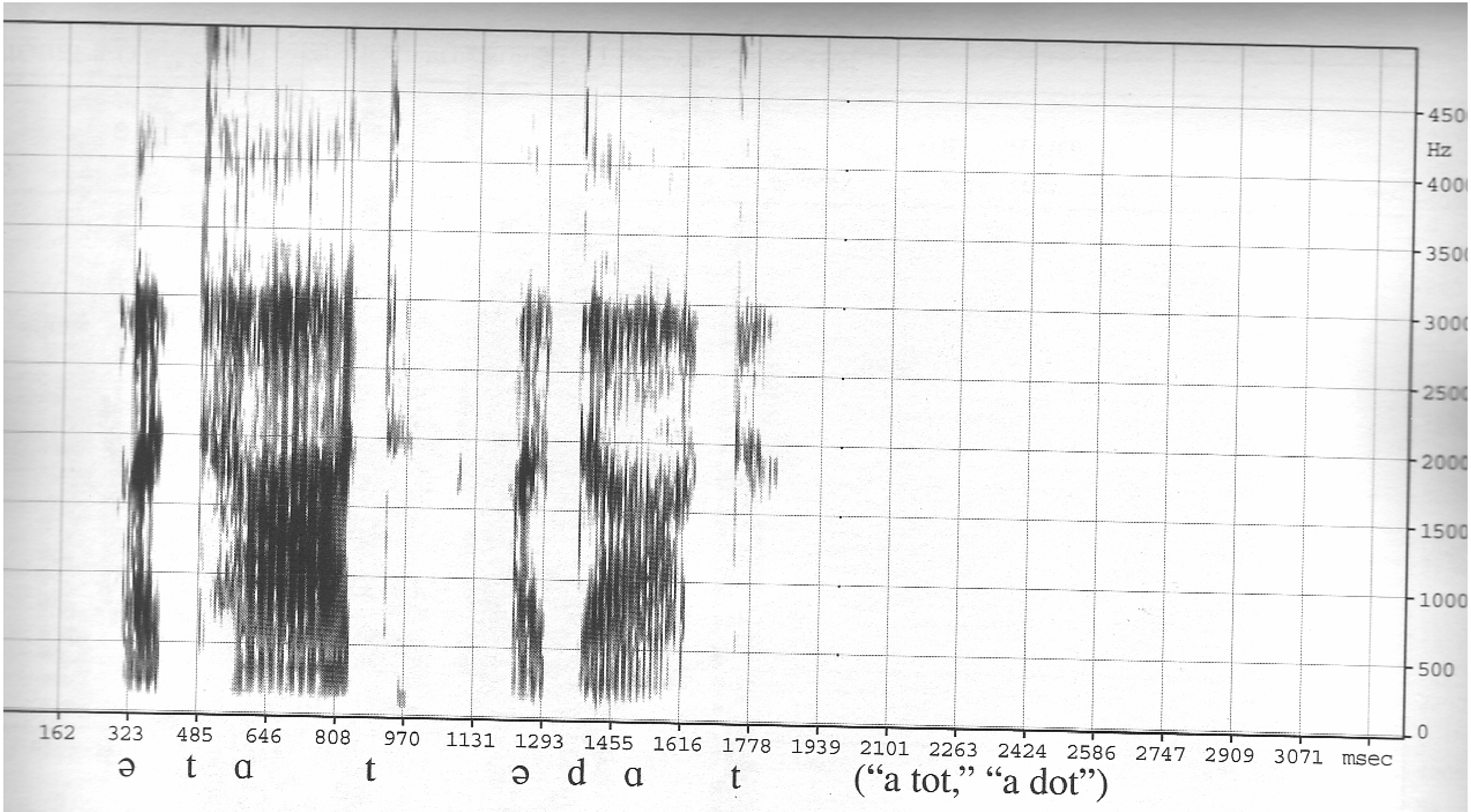


FIGURE 8-11 Narrow-band spectrogram of an adult male saying "A tot, a dot."

A narrow-band spectrogram: One can see fundamentals and overtones (black stripes) along the vertical direction. The frequency of the fundamental and overtones change with time during the speech. Formants are seen as darker regions.



GURE 8-12 Wide-band spectrogram of an adult female saying “A tot, a dot.”

A wide-band spectrogram: One can see formants but the overtones are not resolved. Vertical striations represent individual glottal pulses.