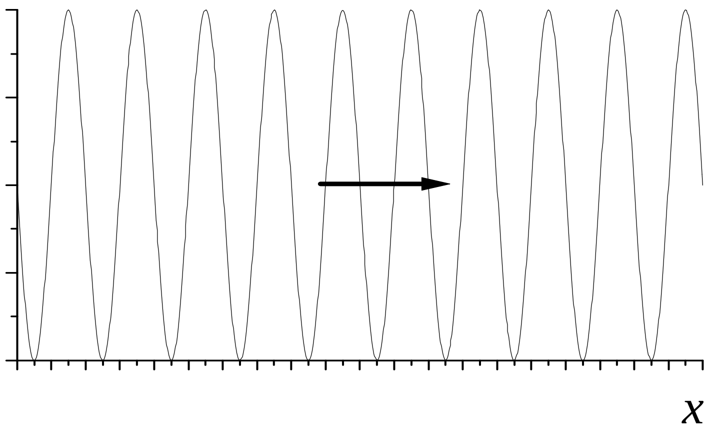
2 – Waves

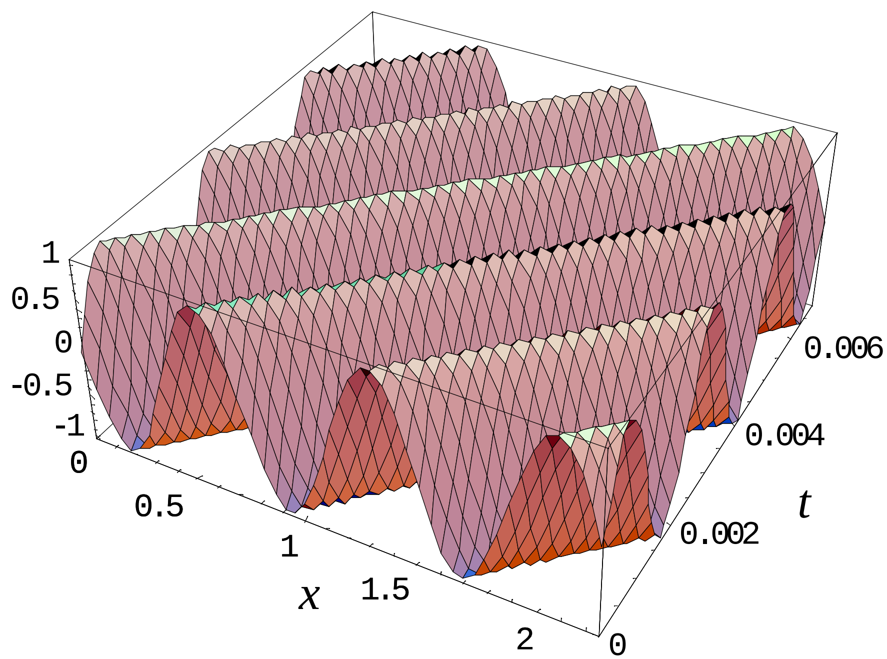
A wave is a disturbance within some media (like change of the pressure P in the air or displacement u of atoms in a solid) that propagates (or travels) with some velocity, which depends on the medium.

Plane waves: Periodic waves, solitary waves, wave packets

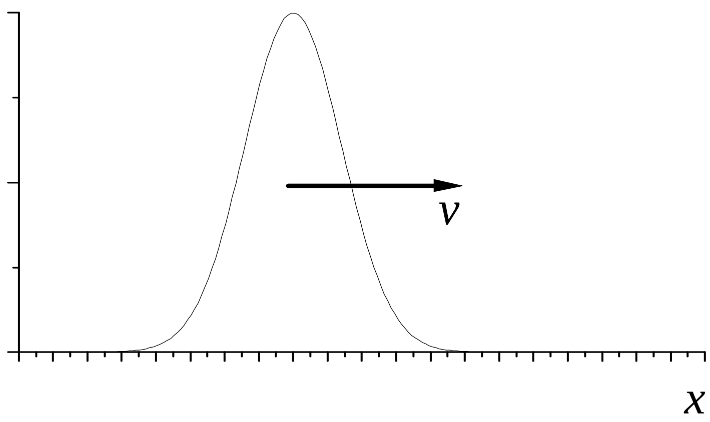
Waves can be periodic waves (like a steady sound wave) or solitary waves that are nonperiodic localized perturbations travelling through the media. In between these two types of waves are so-called wave packets that are pieces of periodic waves localized in space.

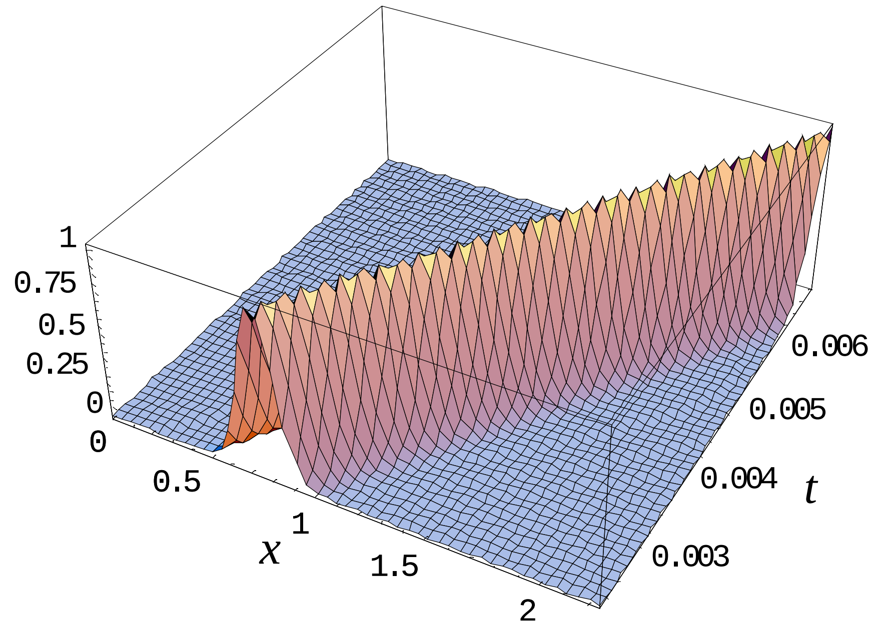
• Plane periodic wave: a) Coordinate x dependence for t=const)   b) Full x-t dependence



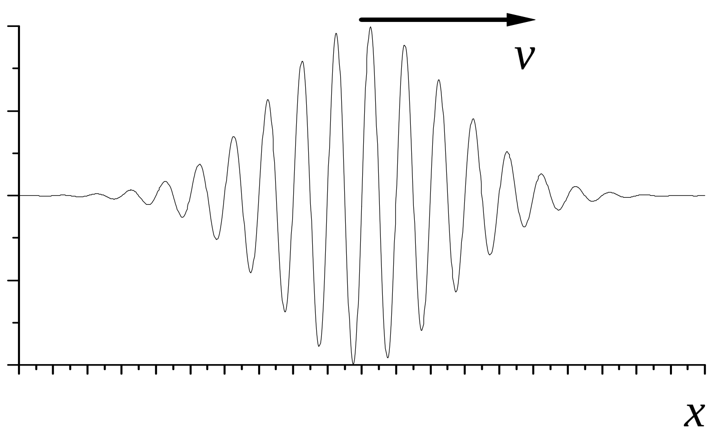


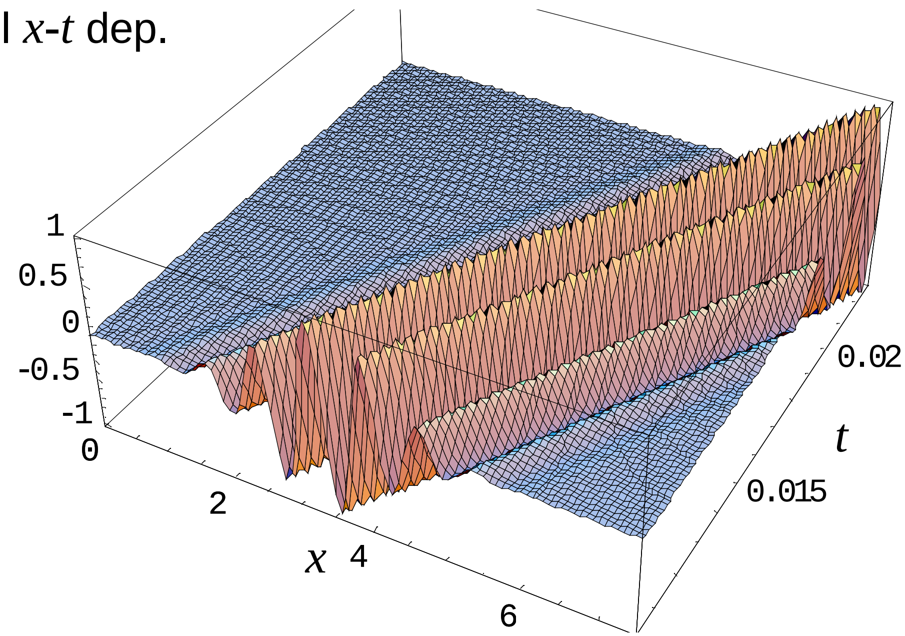
• Solitary wave: a) Coordinate *x* dependence for *t*=const b) Full *x*-*t* dependence





• Wave packet: a) *x* dependence, *t*=const b) Full *x*-*t* dep.



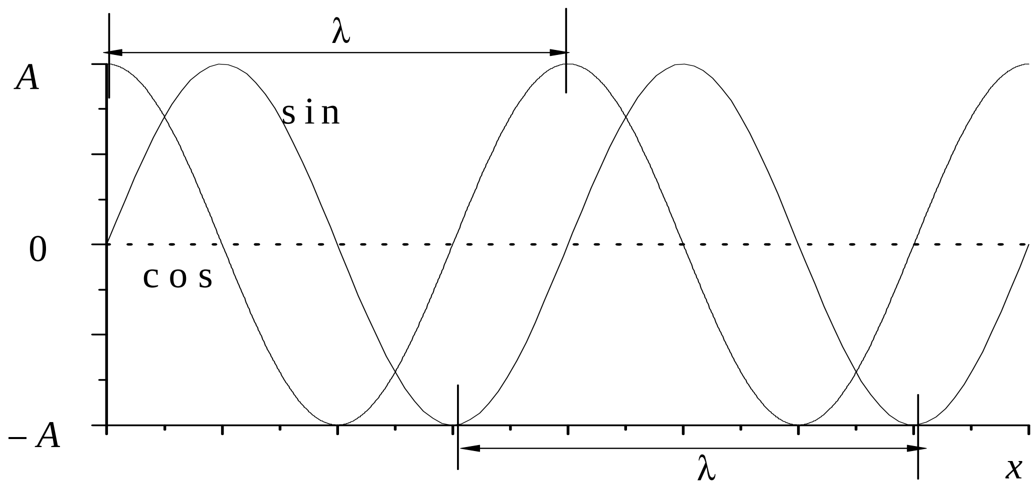


Formula for the periodic plane wave: Wave length, sound velocity

Mathematical formula for the plane periodic wave has the form

$$Q(x,t) = A\sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right)$$

Here λ is the wave length that is defined similarly to the period T. One also can call λ spatial period.



(t=const)

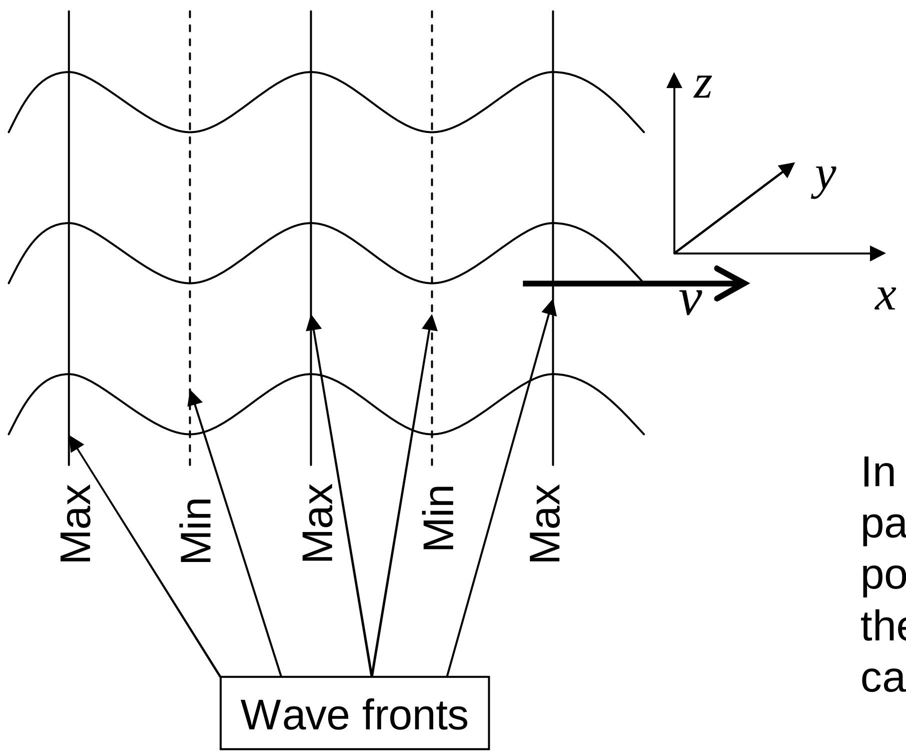
Crests of the periodic plane waves move with the speed v that satisfies

$$v = \frac{\lambda}{T} = \lambda f \quad \Leftrightarrow \quad \lambda = vT$$

The wave above is propagating to the right. The wave propagating to the left is described by a similar formula with + instead of -.

Wave fronts; plane waves

In our three-dimensional world, any physical quantity Q in a wave depends on three Descarte's coordinates x,y,z and on time t: $Q=Q(x,y,z,t)$. Above we have considered the simplest case of plane waves that propagate (move) everywhere along the x axis and nothing depends on y and z. Wave fronts in a plane wave are planes parallel to y,z.

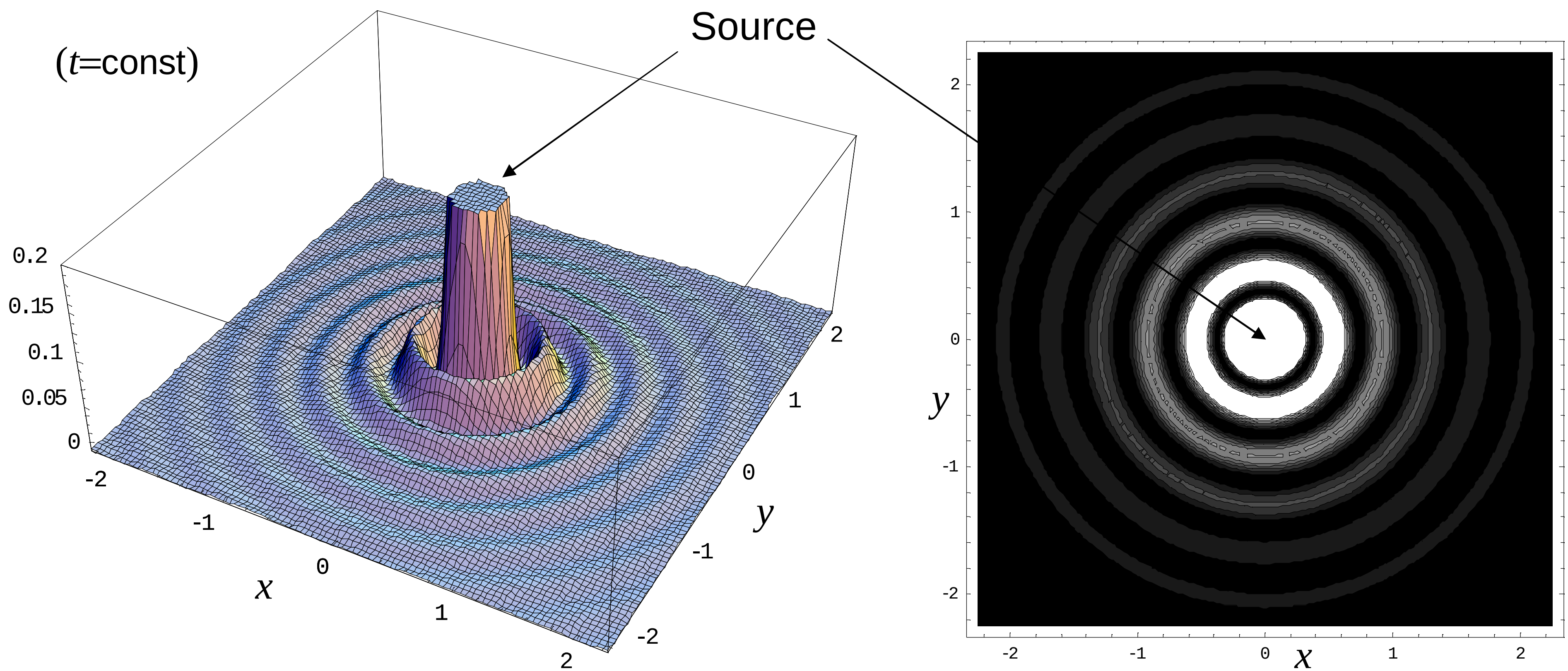


In general, wave fronts are not planes. In particular, if the wave is propagating out of a point source, the fronts are spheres around the source (see next page). Such a wave is called spherical wave.

Spherical and cylindrical waves

The waves considered above are called <u>plane wave</u> because they propagate in the direction x and not in the directions y and z. Crests of plane waves are planes parallel to the y and z axes. If a wave propagates in all directions from of a point source, ist shape is spherically symmetric and the amplitude A <u>decreases</u> with the distance from the source. This wave is called <u>spherical wave</u> since ist crests are spheres centered in the source. If the source is a line, the resulting wave is <u>cylindrical</u>. Waves on the <u>surface</u> of the water are cylindrical, too.

Spherical/cylindrical waves. 3d-plot; Contour plot



Wave intensity; Inverse square law etc.

Waves carry energy in the direction of their propagation. The energy flow is, by definition, the intensity of the wave I. The unit of sound intensity is W/m^2 (Watt per m^2). For the periodic waves I is proportional to the square of the sound amplitude A and to the speed of the wave v, that is, $I \sim A^2 v$. For undamped waves, intensity of the plane wave remains the same everywhere. For spherical and cylindrical waves, I decreases with the distance from the source r according to the law

$$Ir^2 = const, \quad \text{spherical waves (the inverse square law)}$$

$$Ir = const, \quad \text{cylindrical waves}$$

In particular, for the spherical wave one obtains $I_1 r_1^2 = I_2 r_2^2 \quad \Rightarrow \quad I_2 = I_1 \left(\frac{r_1}{r_2} \right)^2$

This amplitude A decrease with the distance r is seen on the previous page.

Remember that the waves on the surface (of the water) generated by a point source are cylindrical.

The law above is the consequence of the energy conservation for undamped waves. For the spherical waves, the energy flow through a sphere of any radius around the source is the same. For cylindrical waves on a surface, the flow of energy through any circle surrounding the source is the same.

The intensity decrease with the distance makes singing in the open air very unrewarding. To the contrary, in rooms and halls the sound partially reflects from the walls. As a result, in a well-built concert hall one can hear well from any point.

The two basic types of waves are <u>longitudinal</u> and <u>transverse</u> waves.

Longitudinal waves

• In longitudinal waves, the displacement of the media occurs in the <u>same</u> direction in which the wave is propagating. The amplitude of this displacement is much smaller than the wave length. The velocities of the media particles (that depend on time periodically, changing their sign) are much smaller than the speed of the wave (speed of sound).

Here is an animation of the longitudinal wave. The crests of this wave correspond to the places where the concentration of the media particles is maximal (minimal). These crests are “running” to the right. Places where the density (concentration) of the particles is maximal, correspond to the maxima of the pressure P. The rough mechanism of the longitudinal wave is the following. Particles, moving to the right, push particles to the right of them and set them in motion. Themselves they rebound and move back. But then the particles to the left of them push them to the right and everything repeats many times.

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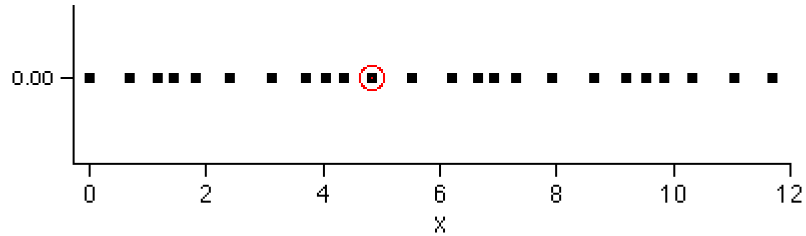
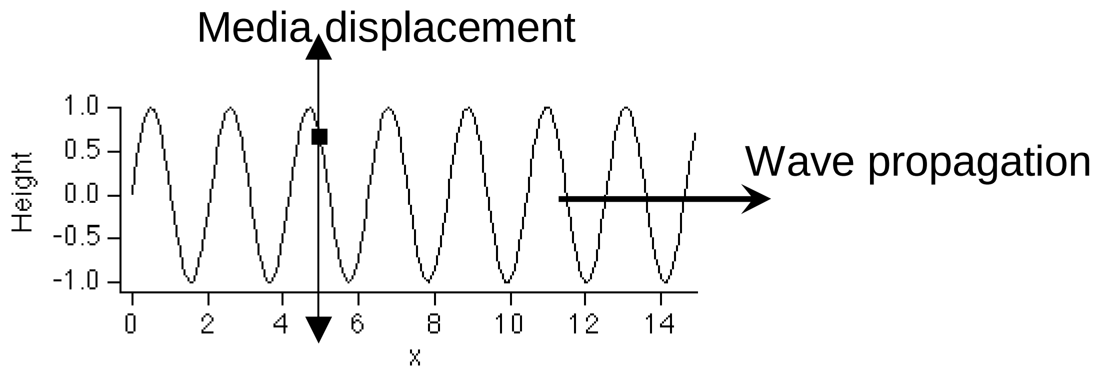


Longitudinal waves is the only kind of waves that exist in liquids and gases.

Transverse waves

• In transverse waves, the displacement of the media occurs in the direction perpendicular (or transverse) to the direction of the wave propagation. Again, the amplitude of this displacement is much smaller than the wave length, and the velocities of the media particles are much smaller than the speed of the wave (speed of sound).



Similarly to oscillations, waves require a restoring force. There is always an elastic restoring force for the longitudinal waves, since compressing the media leads to the increase of the pressure. Thus longitudinal waves can exist in all substances. For transverse waves, the elastic restoring force exists only in solids that retain their shape, apart from small deformations. In liquids and gases, there is no elastic restoring force for shifting layers with respect to each other. Thus there are no transverse elasic waves in the liquids and gases.

- Solids: Transverse and longitudinal elastic waves
- Liquids and Gases: Only longitudinal elastic waves

TABLE 12–1 Speed of Sound in Various Materials (20°C and 1 atm)

Material	Speed (m/s)
Air	343
Air (0°C)	331
Helium	1005
Hydrogen	1300
Water	1440
Sea water	1560
Iron and steel	≈5000
Glass	≈4500
Aluminum	≈5100
Hardwood	≈4000
Concrete	≈3000

Increases with temperature

External links:

- **Applet demonstrating longitudinal and transverse waves** **(http://www.phy.ntnu.edu.tw/java/waveType/waveType.html)**
- **Medium motion in a transverse traveling wave** **(http://www.phy.ntnu.edu.tw/java/wave/wave.html)**

Polarization of transverse waves

There are two mutially perpendicular directions that are perpendicular to the direction of the wave propagation, *t*1 and *t*2. Thus one can speak of two kinds (or two modes) of transverse waves chacterized by different polarizations specified by the directions *t*1 and *t*2. Thus in solids there are totally three elastic wave modes: One longitudinal mode (*l*) and two transverse modes, *t*1 and *t*2..

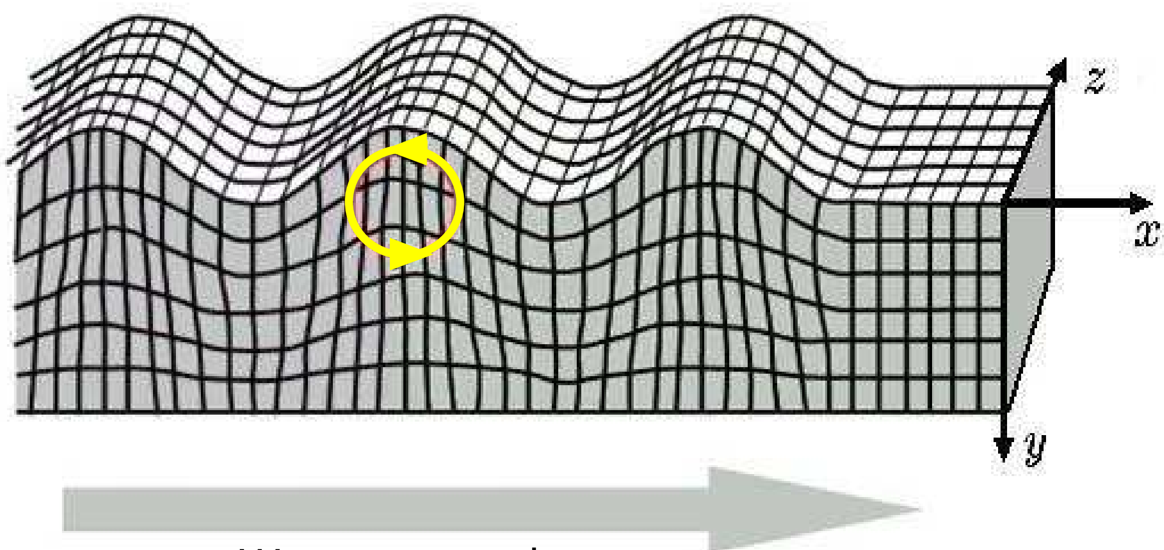
In isotropic solids, that is, in solids properties of which are the same in all directions, both transverse waves are similar and they have the same speed. In fact, one can choose the directions *t*1 and *t*2 in many different ways. One can prove that the speed of longitudinal sound is always greater than the speed of transverse sound.

Properties of solids that form a crystal lattice are not the same in different directions. Such materials are called anisotropic. Properties of anisotropic materials are pretty complicated. There are still three different elastic wave modes but neither of them is purely longitudinal or transverse. In general, in each wave modes the displacements have components along and perpendicular to the direction of the wave propagation.

Surface acoustic waves

A special type of waves are surface waves localized at the boundary of two different medias. An example is surface acoustic waves (SAW) in solids, discovered in 1887 by Lord Rayleigh. Saw or Rayleigh waves find application in electronics, in particular, in cell phones. The common feature of all surface waves is that their amplitude decreases away from the surface. That is, their energy is concentrated is a small region of space that is good for applications. SAWs are neither longitudinal nor transverse, the particles of the media making circles.

Surface acoustic wave

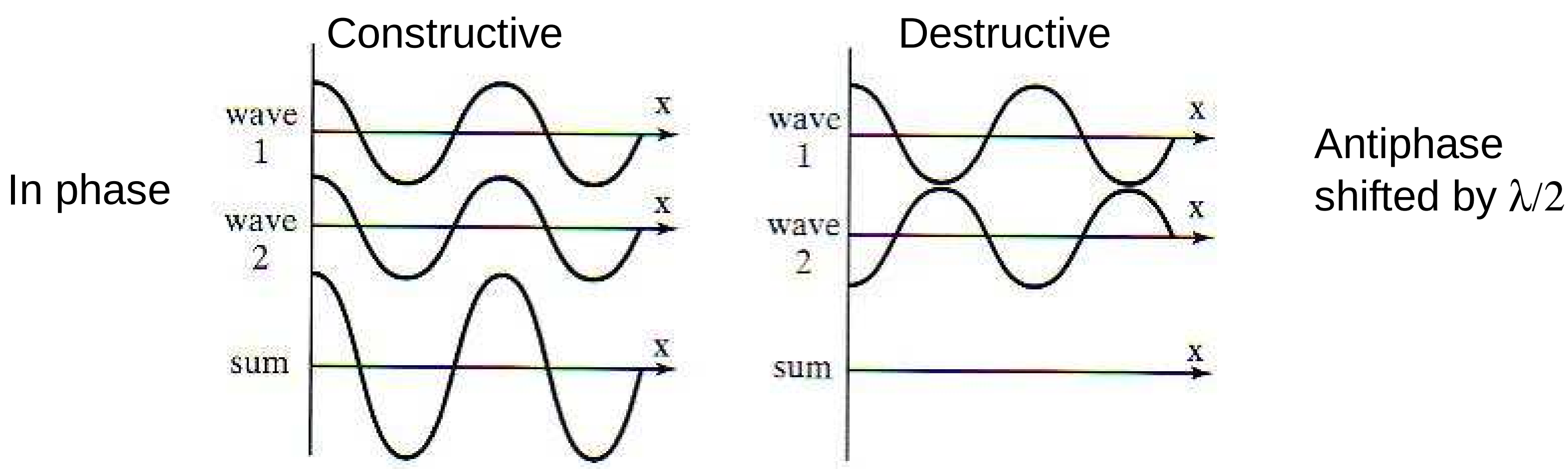


See animation here (http://www.kettering.edu/~drussell/Demos/waves/wavemotion.html)

Other examples of surface waves are waves on the surface of the water and the so-called L-waves in the Earth‘s ground that are responsible for damage during earthquakes.

Interference of plane waves

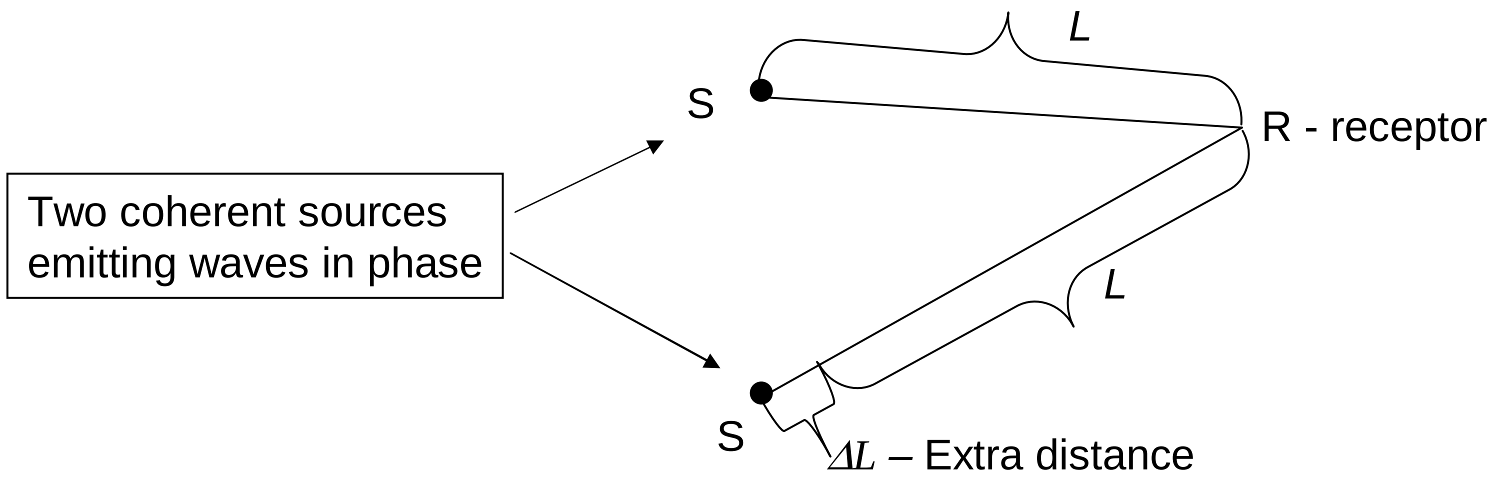
Two periodic plane waves propagating in the same direction can add constructively or destructively, depending on their phases. In the first case the intensity of the resulting wave is large, whereas in the second case it is small or zero, the two waves cancel each other. This is called <u>constructive</u> and <u>destructive</u> <u>interference</u>.



Experimentally constructive and destructive interference can be observed by splitting the sound going in a pipe into two parts, each in its own pipe, and then joining the two split pipes into one pipe again. If the split parts of the pipes have different lengths, the sound will be adding at the end not necessarily in phase. This leads to interference that can be either constructive or destructive.

Interference of two plane waves propagating in different directions will be considered later.

Interference and extra distance

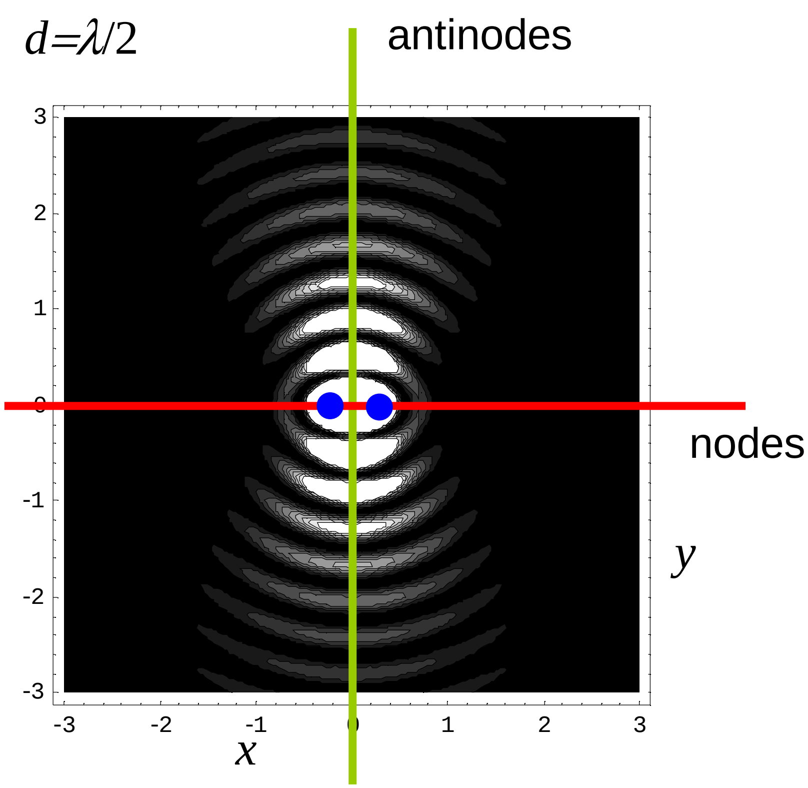


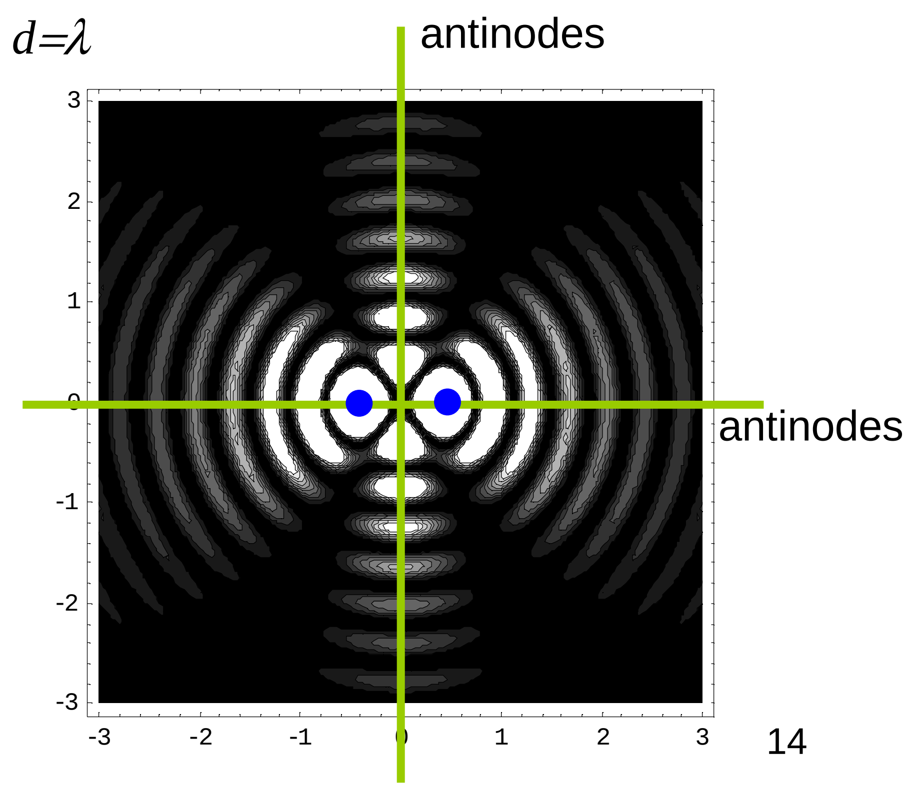
Constructive interference : $\Delta L = \lambda m$, m = 0, ± 1, ± 2, …

Destructive interference : $\Delta L = \lambda/2 + \lambda m$, m = 0, ± 1, ± 2,

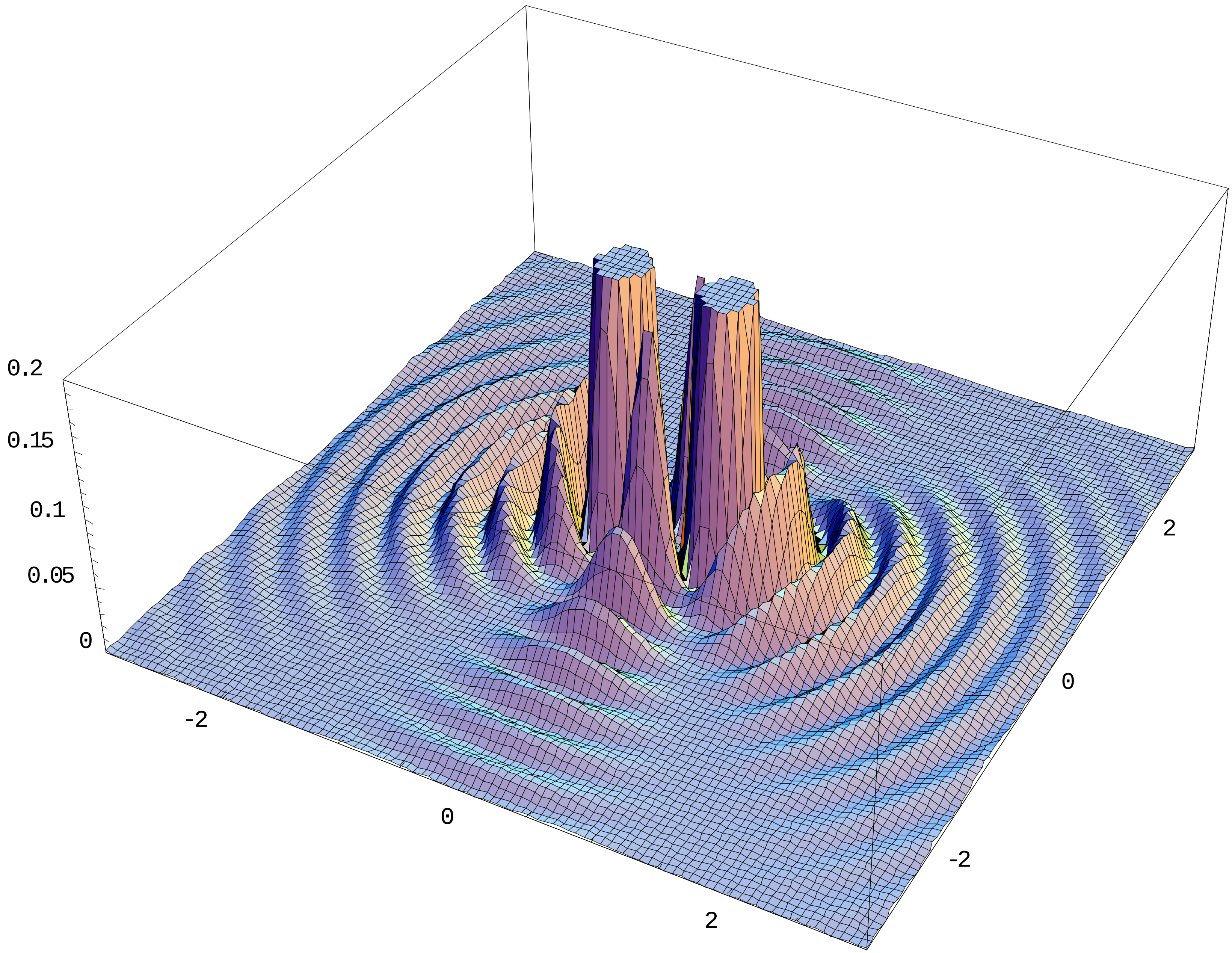
Interference of spherical waves

Interference of two spherical waves emitted by two sources located at some distance from each other leads to a very interesting effect. Unlike the case of a single source considered above, the intensity of the waves propagating in different directions from the two sources is different. There are so-called lines of nodes, along which the interference is destructive, and lines of antinodes, along which the interference is constructive. That is, the waves are emitted by the system of two sources in antinodal directions, whereas nothing goes in the nodal directions. Below are some results for different distances d between the sources shifted along the x axis.

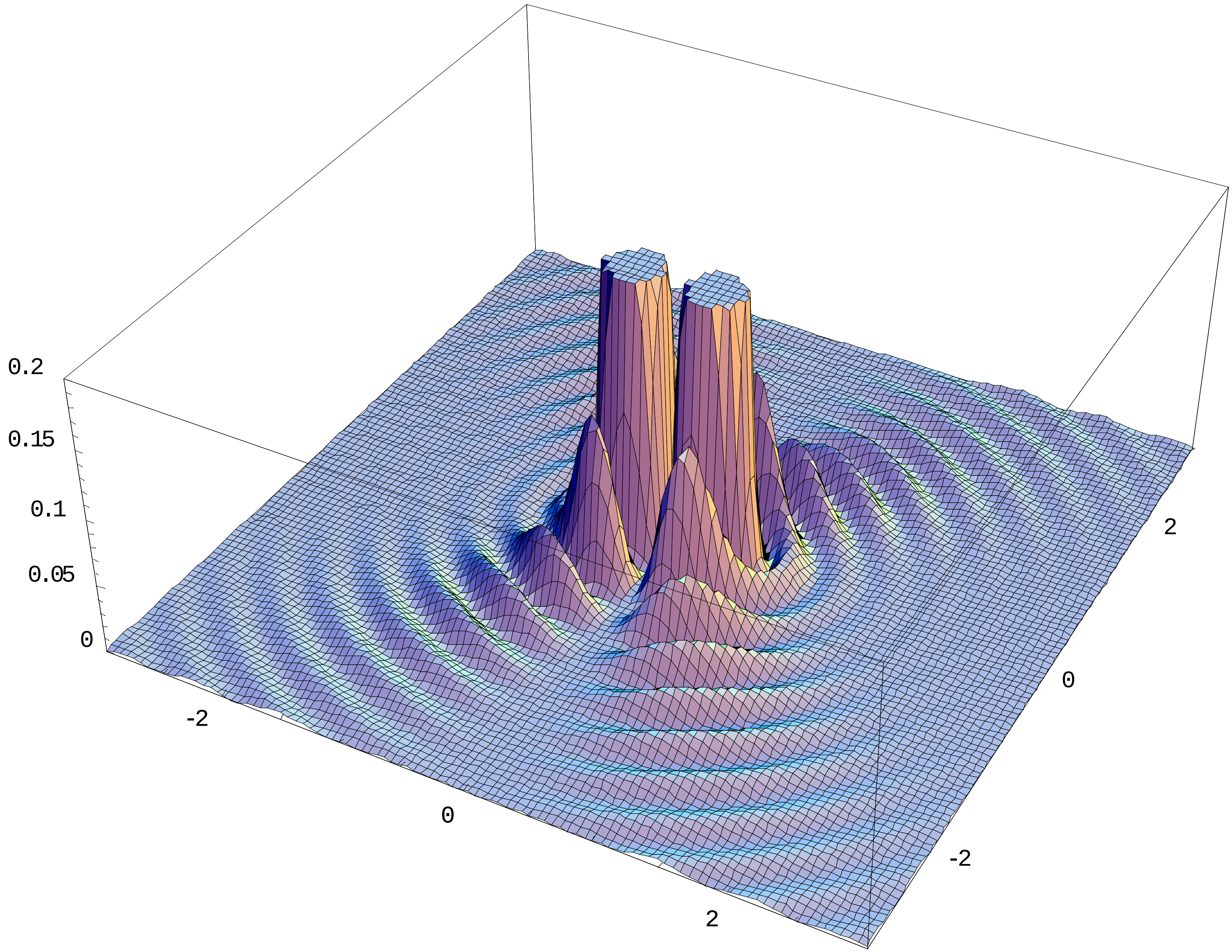




3d-view for $d=\lambda$, in-phase sources

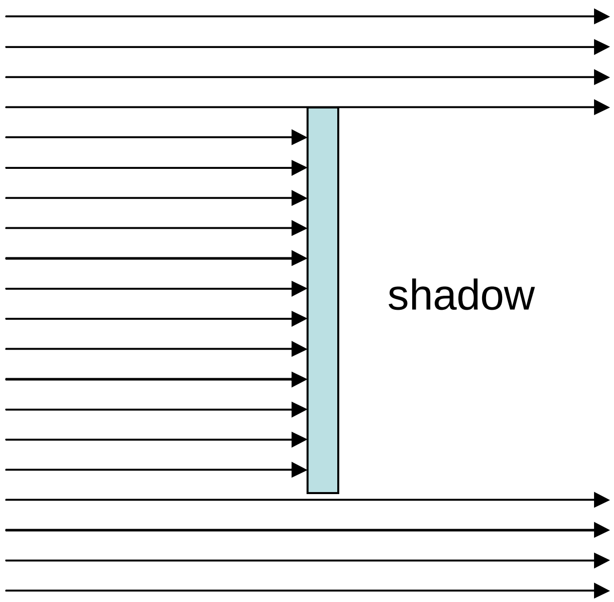


3d-view for $d=\lambda$, antiphase sources

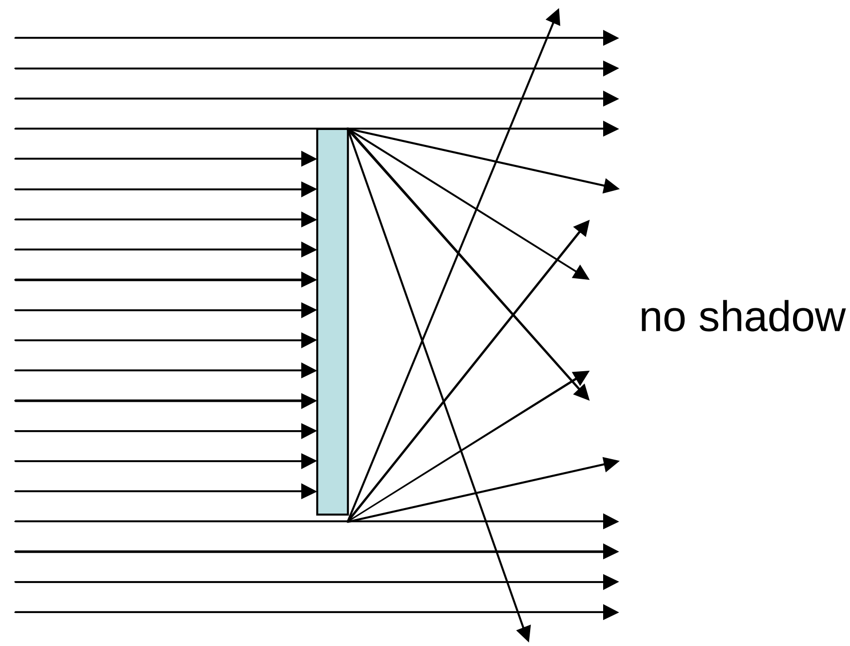


Diffraction of waves

Wave propagating through a medium with obstacles (such as columns in a concert hall) can change their direction going into the shadow regions behind the obstacles. This phenomenon is called <u>diffraction</u>. If the size of the obstacle is L and the wave length is λ, diffraction is strong for $\lambda \geq L$ and weak for $\lambda \leq L$.



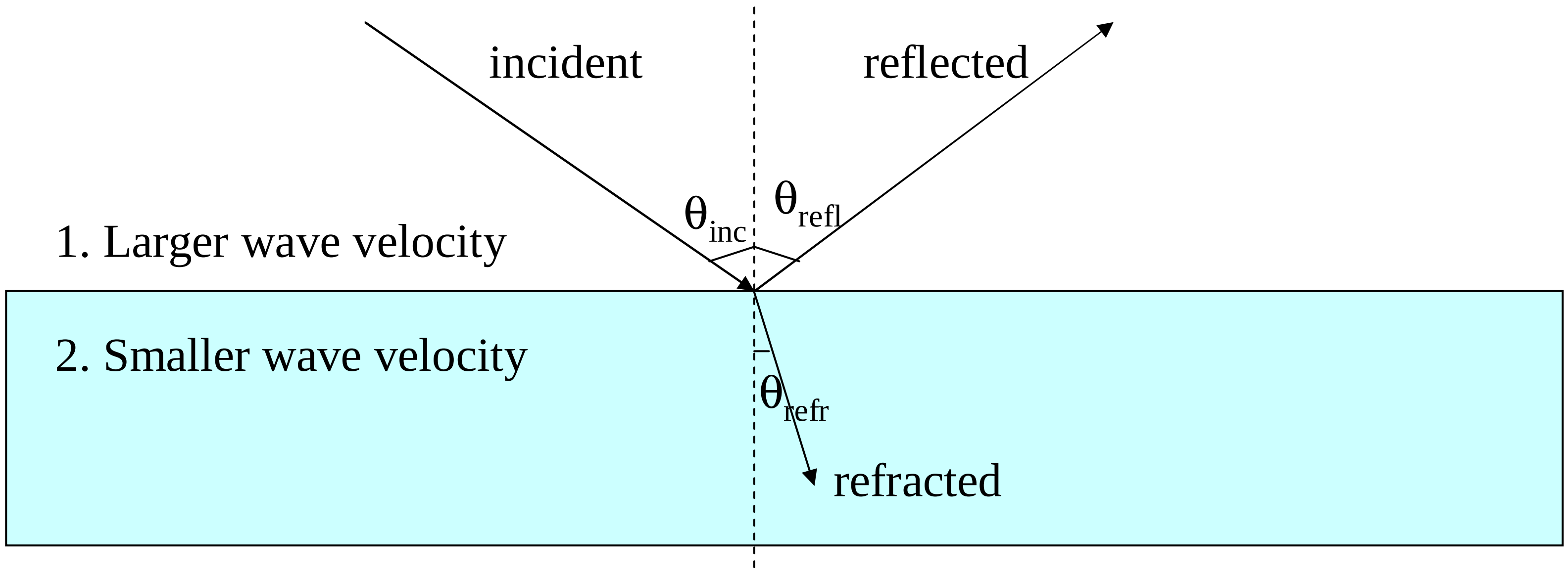
No diffraction



Diffraction

Reflection and refraction of waves

There is <u>reflection</u> and <u>refraction</u> of waves at the boundary of two medias with different speeds of waves:



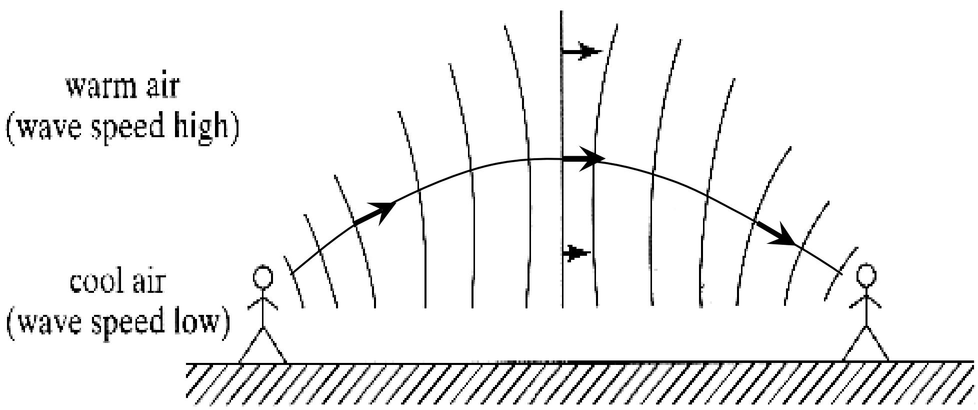
Reflection angle is equal to the incidence angle. Refraction angle is smaller than the incidence angle if the velocity of the waves in the second media is smaller than in the first media.

Refraction of sound in the atmosphere

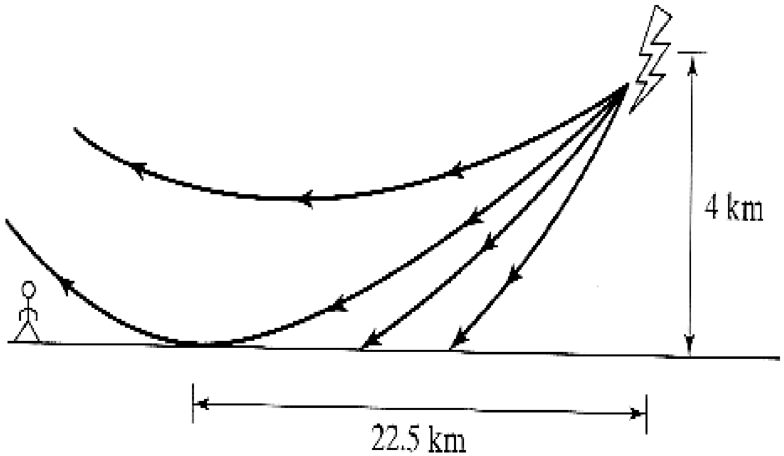
The speed of sound in the air depends on the temperature. Hot air has a greater speed of sound than the cold air. Of course, there are no sharp boundaries between the regions of the warmer and colder air. The temperature and thus the speed of sound change gradually in space. This also leads to reflection and refraction of sound, in a modified form but qualitatively similar.

For instance, during the day usually there are the so-called normal conditions: The sun heats the ground and the temperature closer to the ground is higher. During the cloudy weather or at night the so-called inversion can occur, so that the temperature closer to the ground is lower. Refraction of sound leads to at least the two interesting effects.

Good audibility along the ground during the inversion



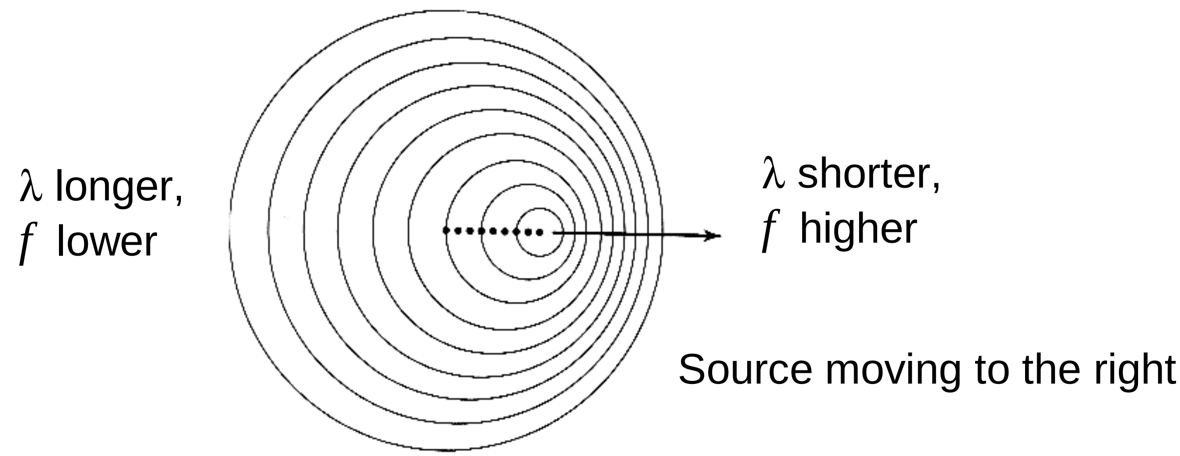
Thunder cannot be heard farther than ~22.5 km away at normal conditions



Berg and Stork, *The Physics of Sound*

Doppler effect

If the source of periodic waves is moving, the frequency of the sound heard by a fixed receptor depends on the direction of the motion of the source. If the source is moving to the observer, the wave length λ becomes shorter and thus wave frequency $f = v/\lambda$ become higher (wave speed v remains the same). If the source is moving away from the receptor, the wave length becomes longer and the frequency becomes lower. This is called Doppler effect.



See animation here (http://www.kettering.edu/~drussell/Demos/doppler/doppler.html)

Sonic booms and shock waves

- just another implication of the Doppler effect

If the speed of the source exceed the speed of sound, the shock wave is formed that separates the regions where no sound exists (before the source) and where the receptor hears a sound at reduced frequency due to the Doppler effect. In the shock wave, waves emitted by the source at different moments of time add up and form a great wave intensity. One can easily observe this effects produced by boats. The shock wave produced by supersonic planes can be very loud. Another word for shock wave is sonic boom.

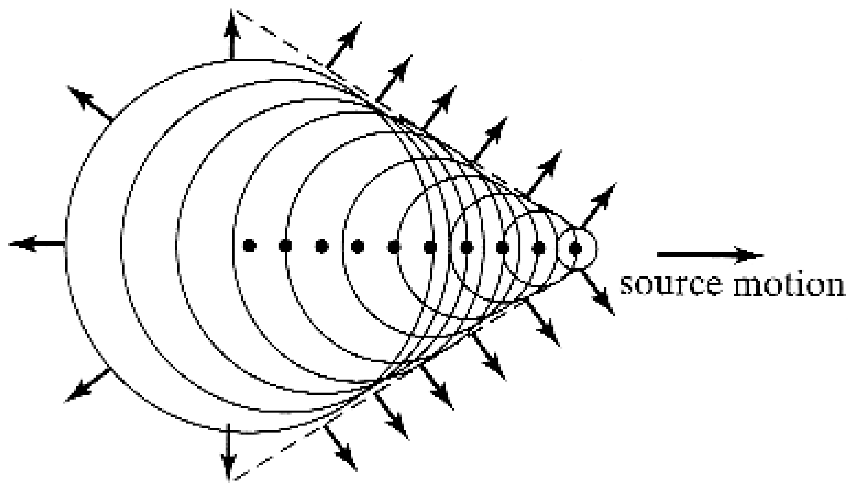


Figure 2-46 Circular waves from the sequence of points shown, in which the source velocity exceeds that of the emitted wave. The dashed lines indicate the leading edge of the combined wave front.

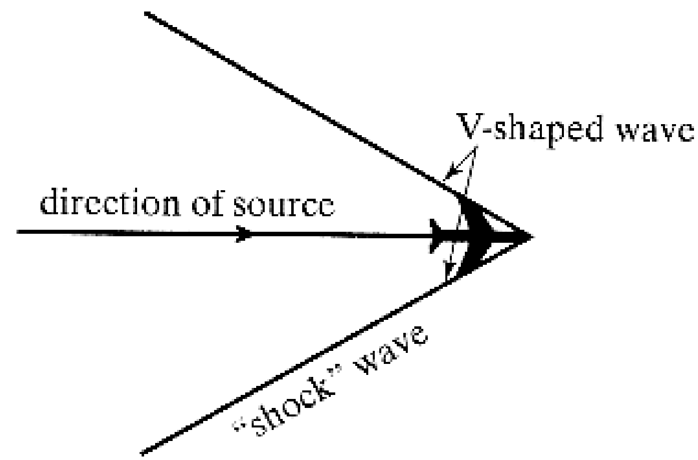


Figure 2-47 When the source velocity exceeds that of the speed of sound, the leading edges of the circular waves from the source combine to form a shock wave. The angle of the *V* decreases as the source velocity increases.

Berg and Stork, *The Physics of Sound*

See animation here (http://www.kettering.edu/~drussell/Demos/doppler/doppler.html)