Selected advanced problems on mechanics

(some require numerical methods)

- A small body begins to slide without friction from the top of a sphere or cylinder of radius R standing on a plane. Find the horizontal distance between the starting point and landing point on the plane.

- A body of mass m begins to slide without friction from the top of a sphere or cylinder of mass M and radius R that can roll on a plane without slipping. Find the horizontal distance between the starting point and landing point of the body m on the plane and the displacement of the CM of the sphere.

- A small body begins to slide down the inner surface of a sphere with a friction coefficient μ. Measuring the position of the body by the angle φ from the vertical and starting with some φ_0, find the angle at which the body stops.

- A rod standing vertically ($\varphi = 0$) on a plane with a very large friction coefficient begins to topple. Find the angle φ at which the lower end of the rod loses contact with the plane. What happens after that? Which end of the rod hits the ground first and at which distance from the initial standing point?

- Consider a two-dimensional harmonic oscillator in the plane x, y and add a magnetic field along z, ascribing the oscillating body a charge q. Find the motion of the oscillator.

- A rapidly rotating heavy top is attached by its upper end to a massless rod of length l that is attached to a fixed point of support above. The rod can swing in a plane, as an ordinary pendulum, or in all directions. Define the motion of the system. Investigate the case of both the top and the rod aligned nearly vertically.

- A rapidly rotating heavy top is supported at a fixed point at one of its ends on the symmetry axis. At the other end another heavy rapidly rotating top is attached. Describe the motion of the system.

- A rapidly rotating heavy top is supported at its lower end on the symmetry axis on a plate of mass m that can move horizontally along x and y and is confined by springs of spring constant k along x and y. Describe the motion of the system.

- A rapidly rotating heavy top is standing with its lower end on a plane with the friction coefficient μ. Describe the motion of the top.

- Describe the motion of a sphere on a plane with friction coefficient μ under the influence of a force $\mathbf{F}$ and torque $\mathbf{K}$. Consider particular cases.

- A cone is rolling on a plane without slipping. Find the condition for the cone to turn over.