Analytical Dynamics - Graduate Center CUNY - Fall 2008

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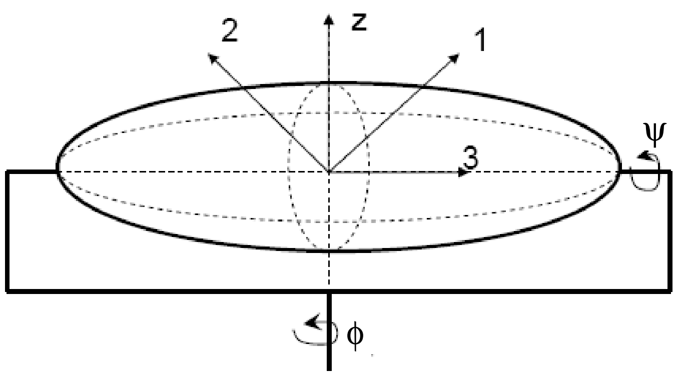
Problem set 02 Due: We, Nov 26

1 Euler equation for a heavy symmetric top

(10 points) Write down the Euler equations of motion for a heavy symmetric top. What integrals of motion can you find immediately? Try to express L_z via L_1, L_2, and L_3 and check its conservation with the help of the Euler equations.

2 Asymmetric top with the $\theta = 0$ holder

(10 points) Consider an asymmetric top with moments of inertia $I_1 < I_2$ supported by a holder that allows the top to freely rotate changing its Euler angles ϕ and ψ while preserving $\theta = \pi/2$; see figure. The axes of the holder cross at the center of mass of the top.



a) Set up the Lagrange equations for this top, find integrals of motion;

b) Eliminate ϕ to obtain an effective energy for ψ. What kinds of motion for ψ are possible? Analyze the behavior of ψ near the minimum of the effective potential energy.

c) If you have access to mathematical software, you can try to produce numerical solutions with particular initial conditions.

3 Self-rotation

(10 points) How can a cat manage always to land on her feet? How can a system with zero angular momentum set itself into rotation? Consider a person standing on a rotating platform without friction, so that its angular momentum is conserved and is zero, $L_z = 0$. The person having together with the platform a moment of inertia $I_{zz} \equiv I$ moves a point mass m by a (massless) hand around a closed contour in the x, y plane, defined in the frame of the platform. By which angle the person on the platform rotates as the mass m makes a full turn?

a) Write down the condition $L_z = 0$ in terms of the projections of the point-mass position and velocity on the axes of the body (platform) frame.

b) Change to the polar coordinates (r, φ) for the point mass and obtain a relation between $d\varphi$ and the in¯nitesimal rotation of the platform $d\theta$. Let $\Delta\theta$ be the angle of rotation of the platform corresponding to one full turn of the point mass. What do you expect for $\Delta\theta$ in the limits $I \to \infty$ and $I \to 0$?

c) Consider a particular case of rotation of the point mass around a circle with radius R and the center at the distance $l > R$ from the center of the platform and calculate $\Delta\theta$. What is the condition for $\Delta\theta$ to be maximal?