Mathematical Physics - PHY307 Fall 2010

Prof. D. Garanin Assignment 3

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1. Check the identities

$$\begin{aligned} \nabla \cdot (\phi \mathbf{A}) &= \phi \nabla \cdot \mathbf{A} + \nabla \phi \cdot \mathbf{A} \\ \nabla \times (\phi \mathbf{A}) &= \phi \nabla \times \mathbf{A} + \nabla \phi \times \mathbf{A} \end{aligned}$$

using Mathematica.

2. Magnetic field produced by a long wire carrying current I in the positive z direction is given by

$$\mathbf{B} = \left\{ -\frac{ya}{x^2+y^2}, \frac{xa}{x^2+y^2}, 0 \right\}, \qquad a = \frac{\mu_0 I}{2\pi}.$$

Let us regularize this expression by introducing ε^2 in the denominator,

$$\mathbf{B} = \left\{ -\frac{ya}{x^2+y^2+\varepsilon^2}, \frac{xa}{x^2+y^2+\varepsilon^2}, 0 \right\}.$$

Calculate curl of this vector. Plot $\mathbf{B}$, B, and curl $\mathbf{B}$. What happens as $\varepsilon \to 0$? What is the meaning of curl $\mathbf{B}$?

3. Electric potential of a point charge Q has the form

$$V = \frac{a}{r}, \qquad a = \frac{Q}{4\pi\varepsilon_0}$$

Let us regularize this formula by introducing $\varepsilon > 0$ in the denominator,

$$V = \frac{a}{r+\varepsilon}.$$

Calculate the electric field $\mathbf{E} = -\nabla V$ and plot its dependence on r. At which r it has a maximum? Calculate the Laplacian of V and plot its dependence on r. What is the meaning of ΔV? What happens as $\varepsilon \to 0$?