PHY 167 Recitation 3, Spring 2020

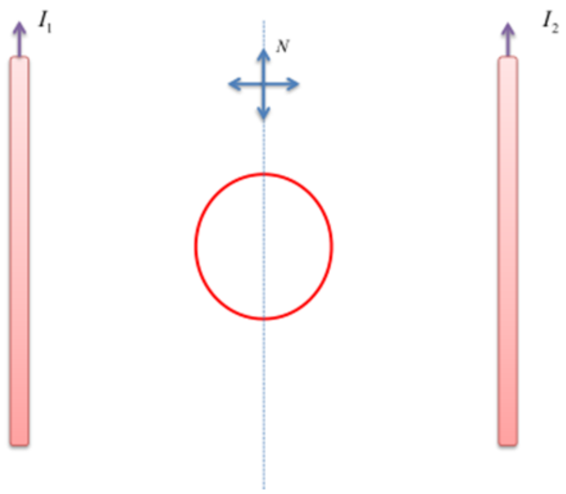


Fig. 1 Picture of the setup in problem 1.

1. A circular loop is placed between two long parallel wires of carrying currents I 1 and I 2, as shown in Fig.1. For each case below, what will be the direction of the induced current, if any, on the wire loop:

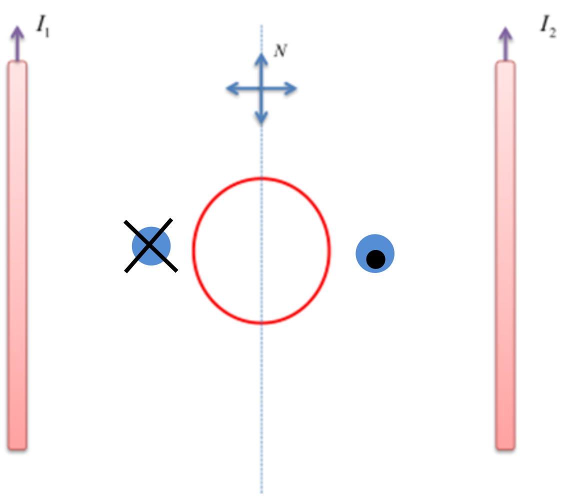
(a) The wire loop is at rest directly in the center of the wires, as initially shown.

(b) The wire loop is moved to the left, towards I_1.

(c) The wire loop is moved to the right, towards I_2.

(d) The wire loop is moved upwards, north, along the dashed line.

Solution:



The magnetic field created by currents obeys the screw rule and is directed as shown in the figure (into the plane closer to I_1 and out of the plane closer to I_2). In the middle between the two wires, shown by the dotted line, their magnetic fields cancel each other. The EMF in the loop is given by the Faraday-Lenz law

$$\mathcal{E} = -\frac{\Delta\Phi}{\Delta t},$$

where Φ is the magnetic flux in the loop. The minus sign, introduced by Lenz, tells us that the direction of the EMF is such that the magnetic flux created by the current induced by the EMF is partially compensating the change of the external magnetic flux in the loop.

(a) If the loop is not moving, the magnetic flux through it is not changing in time, thus $\mathcal{E} = 0$. In this case, by symmetry, $\Phi = 0$.
(b) If the loop moves to the left, the external magnetc flux (created by the two long wires) becomes negative (if we choose the direction out of the plane towards us as positive). Thus the current in the loop should create a positive magnetic flux to compensate the change of Φ. This current should flow counterclockwise that gives the direction of the EMF.
(c) If the loop moves to the right, everything is vice versa and the EMF is clockwise.
(d) If the loop moves up or down, the external magnetic flux does not change, thus $\mathcal{E} = 0$.

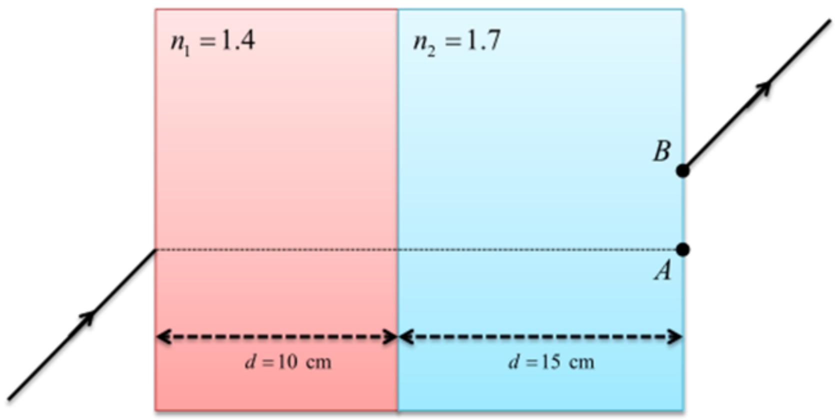


Figure 2: Picture of the setup in problem 2.

2. Two parallel-sided blocks of glass of refraction index $n_1 = 1.4$ and $n_2 = 1.7$ lie next to each other as shown above. A light ray strikes the first one at an incident angle of 40°.

(a) Calculate the angle of refraction θ as the ray emerges from the second block to air.

(b) Calculate the distance AB, where B is the point of exit.

Solution:

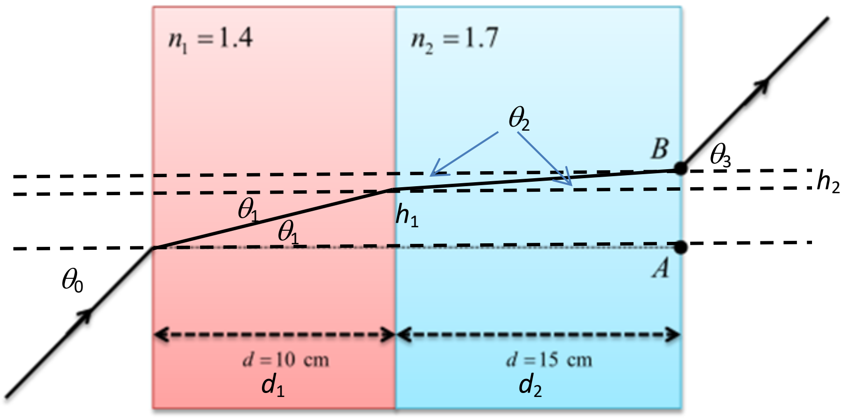


Figure 2: Picture of the setup in problem 2.

First, we enhance the original drawing adding the light ray inside the system and necessary notations.

(a) As all surfaces are parallel, the light ray will exit the system in the same direction as it entered the system, the only effect being the lateral displacement of the ray. This means $\theta_3 = \theta_0$. This has to be explicitly demonstrated in (a). To do this, we apply the Snell's law for all three intercaces:

$$\sin\theta_0 = n_1 \sin\theta_1, \quad n_1 \sin\theta_1 = n_2 \sin\theta_2, \quad n_2 \sin\theta_2 = \sin\theta_3.$$

These three equations can be compacted into one:

$$\sin\theta_0 = \sin\theta_3$$

that implies $\theta_3 = \theta_0$.

(b) The distance AB is the sum of the vertical distances in each of the two layers:

$$AB = h_1 + h_2.$$

From the triangles one obtains

$$h_1 = d_1 \tan\theta_1, \quad h_2 = d_2 \tan\theta_2,$$

where the angles can be found from the Snell's law above:

$$\theta_1 = \arcsin\frac{\sin\theta_0}{n_1}, \quad \theta_2 = \arcsin\frac{\sin\theta_0}{n_2}.$$

The final result is

$$AB = d_1 \tan\arcsin\frac{\sin\theta_0}{n_1} + d_2 \tan\arcsin\frac{\sin\theta_0}{n_2}.$$

One can produce a smarter solution expressing tan via sin with the help of the formula

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sin\theta}{\sqrt{1-(\sin\theta)^2}} = \frac{1}{\sqrt{\frac{1}{(\sin\theta)^2} - 1}}.$$

Finding $\sin\theta_1$ and $\sin\theta_2$ from the Snell's law, one obtains

$$AB = \frac{d_1}{\sqrt{\left(\frac{n_1}{\sin\theta_0}\right)^2 - 1}} + \frac{d_2}{\sqrt{\left(\frac{n_2}{\sin\theta_0}\right)^2 - 1}}.$$

or

$$AB = \frac{d_1 \sin\theta_0}{\sqrt{n_1^2 - (\sin\theta_0)^2}} + \frac{d_2 \sin\theta_0}{\sqrt{n_2^2 - (\sin\theta_0)^2}}.$$

All three analytical expressions for AB are acceptable.
Substituting numbers, one obtains $\sin\theta = \sin 40° = 0.6428$ and

$$AB = \frac{10}{\sqrt{\left(\frac{1.4}{0.6428}\right)^2 - 1}} + \frac{15}{\sqrt{\left(\frac{1.7}{0.6428}\right)^2 - 1}} = 11.3\ cm.$$

3. An object 10 mm tall is placed 12 cm in front of a convex spherical mirror whose radius of curvature is 20 cm.

(a.) Determine the position, size and orientation of the image.
(b.) Draw the corresponding ray diagram.
(c.) Repeat parts (a.) and (b.) if the convex mirror is replaced by a concave one.

Solution: Formalize the problem. Convex mirror, d_o = 12 cm, h_o = 10 mm, R = -20 cm, $f = R/2$ = -10 cm.

(a) Use

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

and solve for d_i:

$$d_i = \frac{1}{\frac{1}{f} - \frac{1}{d_o}} = \frac{d_o f}{d_o - f} = -\frac{d_o|f|}{d_o + |f|} = -\frac{12 \times 10}{12 + 10} = -5.45\ cm. \qquad (4)$$

The image is virtual. Use

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

to solve for h_i:

$$h_i = -h_o \frac{d_i}{d_o} = -\frac{h_o f}{d_o - f} = \frac{h_o f}{f - d_o} = \frac{h_o|f|}{|f| + d_o} = \frac{10\ mm\ \times 10\ cm}{12\ cm + 10\ cm} = 4.52\ mm. \qquad (5)$$

(b) Use the applet on ophysics: https://ophysics.com/l10.html

(c) Concave mirror, R = 20 cm, everything else the same. Here

$$d_i = \frac{d_o f}{d_o - f} = \frac{12 \times 10}{12 - 10} = 60\ cm \qquad (4)$$

and

$$h_i = -\frac{h_o f}{d_o - f} = -\frac{10\ mm\ \times 10\ cm}{12\ cm - 10\ cm} = -50\ mm. \qquad (5)$$

The image is virtual, upside-down, and magnified.

4. This problem with the lens is very similar to the preceding problems with the mirror. I suggest the students to solve it themselves and exchange the results in the BB Discussion board to find out the correct numerical results.