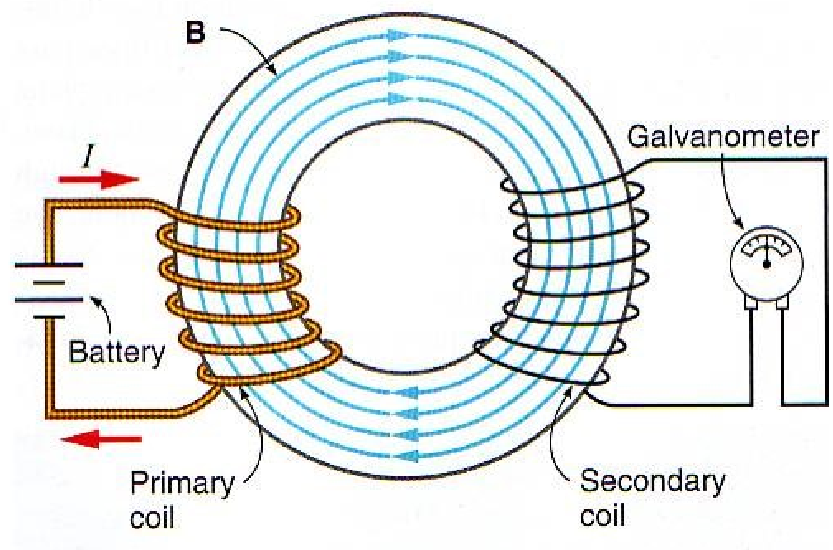
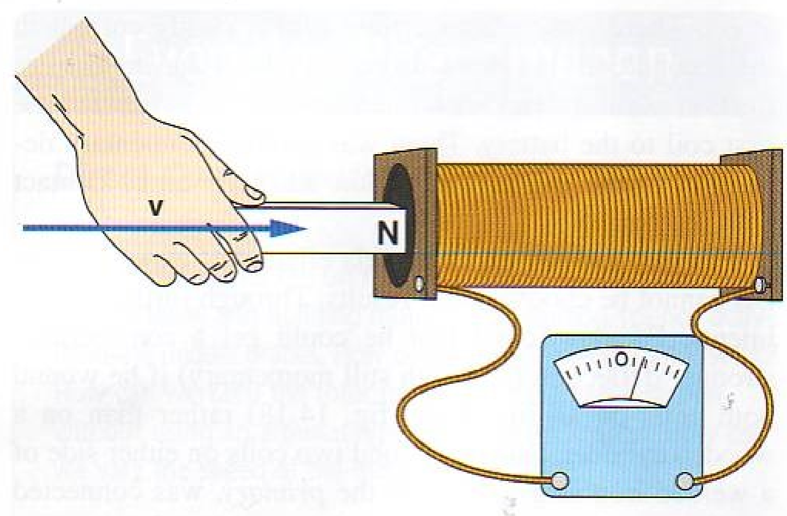
6 – Electromagnetic induction

Michael Faraday´s experiment: Changing current in the primary coil results in surges of current in the secondary coil. The effect is stronger if both coils are winded around an iron ring. The primary coil creates a magnetic field that goes throgh the secondary coil, too. If the magnetic field changes with time, an electromotive force (EMF) develops in both coils. Faraday has seen this effect on the secondary coil, as shown in the figure.

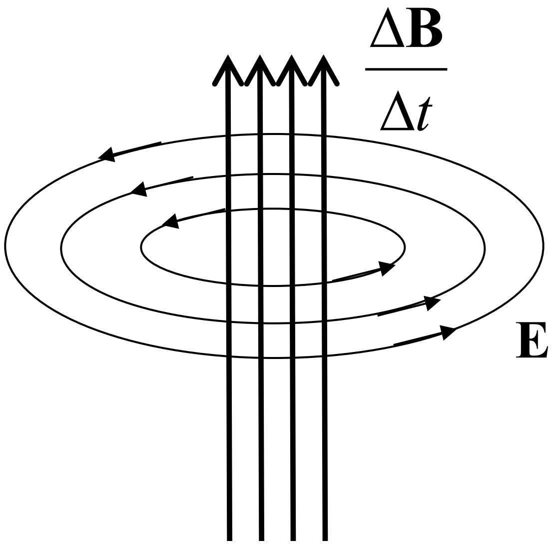


One year later Joseph Henry detected the EMF in the primary coil, the so-called self-induction (in fact he used only one coild in his experiments).



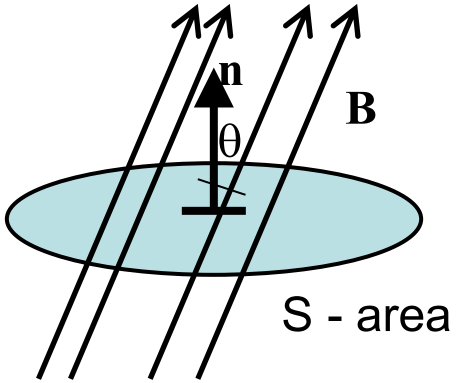
Electromotive force in a coil can also be produced by changing the magnetic field as a result of the motion of a magnet. Moreover, moving a wire in a constant magnetic field produces an EMF in it. The latter is used in generators of electric current, where a coils rotate in the magnetic field created by permanent magnets.

Physical nature of the EMF in Faraday´s experiments: Changing magnetic field in time creates an electric field with closed lines. This electric field is different from the static electric field that originates and terminates on electric charges. The structure of this field is similar to that of magnetic field that also has closed lines.



If a wire is moving in a constant magnetic field, then the physical nature of the EMF is different. It is due to the magnetic forces on moving electric charges, see below.

Summarizing the results of all these experiments leads to the Faraday´s law that can be formulated in terms of the magnetic flux Φ. that for a flat loop of area S in a uniform magnetic field $\mathbf{B}$ is given by



$$\Phi = \mathbf{B} \bullet \mathbf{n} S = BS\cos\theta$$

Note that Φ can be positive or negative, depending on the direction of $\mathbf{B}$ with respect to the unit vector $\mathbf{n}$ that is perpendicular to the loop. If $\mathbf{B}$ is perpendicular to the loop, the absolute value of Φ is maximal. If $\mathbf{B}$ is parallel to the loop, then Φ = 0.

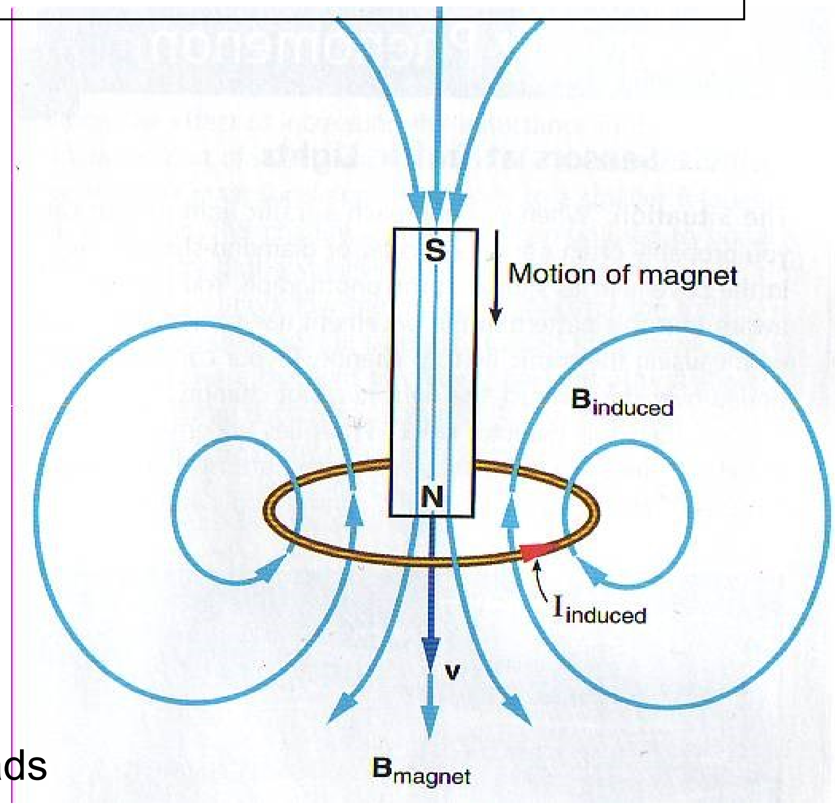
Faraday´s law for the electromagnetic induction

Faraday´s law: The EMF in a loop is equal to the rate of change of the magnetic flux through this loop

$$\mathcal{E} = -\frac{\Delta\Phi}{\Delta t}$$

The minus here reminds us of the Lenz´s law that determines the direction of the electric current induced by this EMF:

Lenz´s law: A current produced by an induced EMF flows in a direction so that its magnetic field tends to compensate the original change of the magnetic flux.

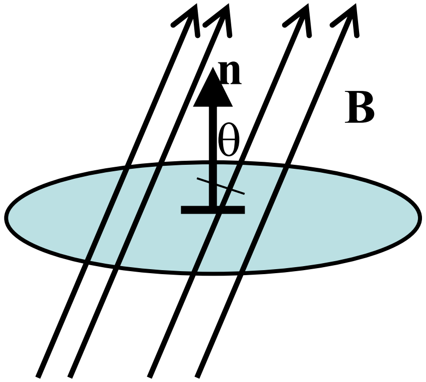


For a loop of wire with N turns the Faraday´s law reads

$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t}$$

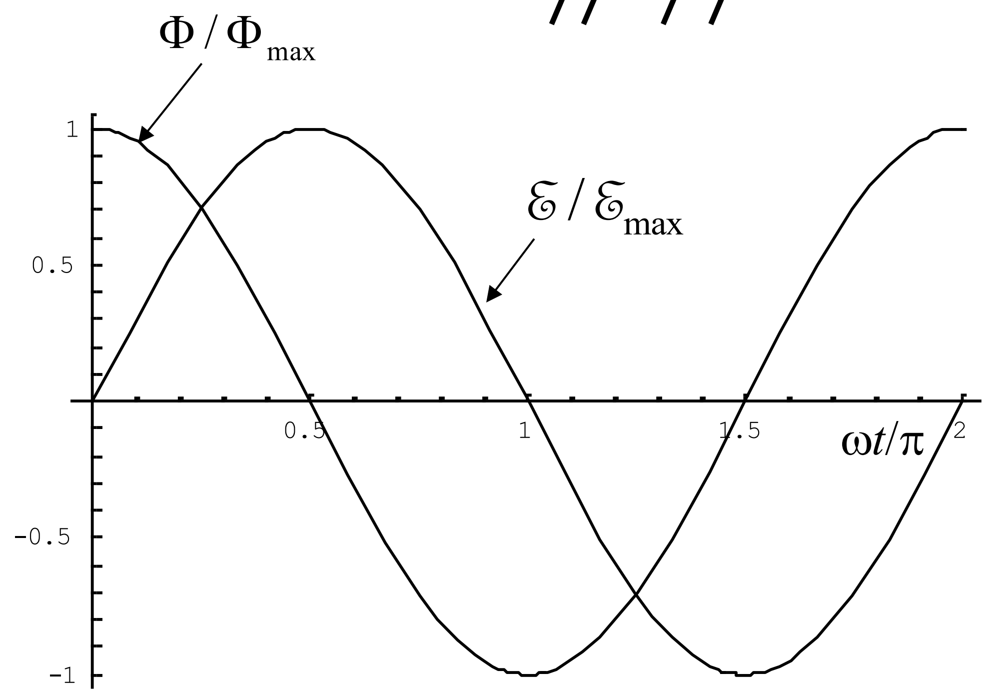
The beauty of the Faraday´s law is in its universality: The result is the same for changing strength of the magnetic field, changing direction of the magnetic field. changing the orientation of the loop, or changing the area of the loop.

Generators



In electric generators, loops of wire (coils) rotate in a static magnetic field created by permanent magnets. The angle describing the orientation of the wire changes linearly with time: $\theta = \omega t$, where ω is the angular frequency. Then the magnetic flux through the coil thdepends on time as

$$\Phi = \Phi_{max} \cos(\omega t + \varphi_0)$$



The EMF following from the Faraday´s law has the form

$$\mathcal{E} = -\frac{\Delta\Phi}{\Delta t}$$

has the form

$$\mathcal{E} = \omega\Phi_{max} \sin(\omega t + \varphi_0) = \mathcal{E}_{max} \sin(\omega t + \varphi_0)$$

This electromotive force generates <u>alternating current</u>.

Transformers

Transformers are used to change the EMF (in the everyday language voltage) for alternating currents. Transformer consists of two coils with different numner of wire turns that have the common core made low-loss iron alloy (permalloy). Because of the core, magnetic field created by the primary coil all enters the secondary coil. Thus the magnetic flux is practically the same in both coils, that means that the EMF per wire turn is the same in both coils:

$$\mathcal{E}_1 = -N_1 \frac{\Delta\Phi}{\Delta t}, \quad \mathcal{E}_2 = -N_2 \frac{\Delta\Phi}{\Delta t}$$

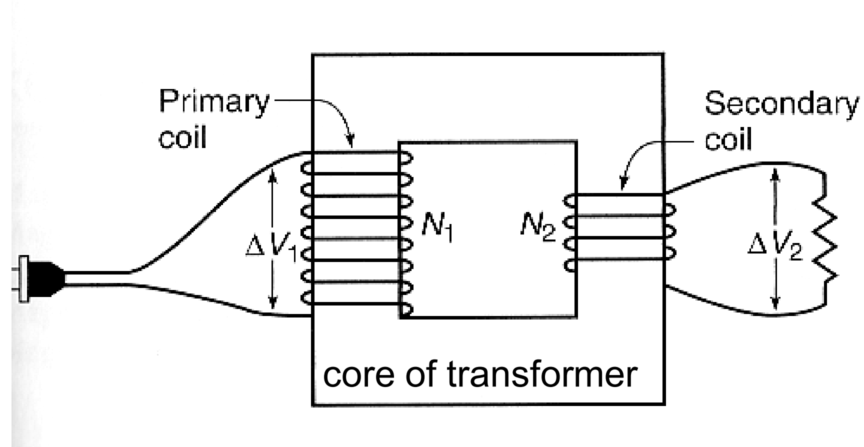
One obtains

$$-\frac{\Delta\Phi}{\Delta t} = \boxed{\frac{\mathcal{E}_1}{N_1} = \frac{\mathcal{E}_2}{N_2}}$$

that is the relation between the emf´s on the coils of a transformer

For instance, to step-up the voltage, one should use a transformer with $N_2 > N_1$

$$\boxed{\mathcal{E}_2 = \frac{N_2}{N_1}\mathcal{E}_1} \quad \text{if} \;\; N_2 > N_1, \quad \text{then} \quad \mathcal{E}_2 > \mathcal{E}_1$$



The condition on electric currents in the coils can be obtained from the energy conservation (in the absence of losses) in the form input power = output power,

$$P_1 = \mathcal{E}_1 I_1 = P_2 = \mathcal{E}_2 I_2$$

that is

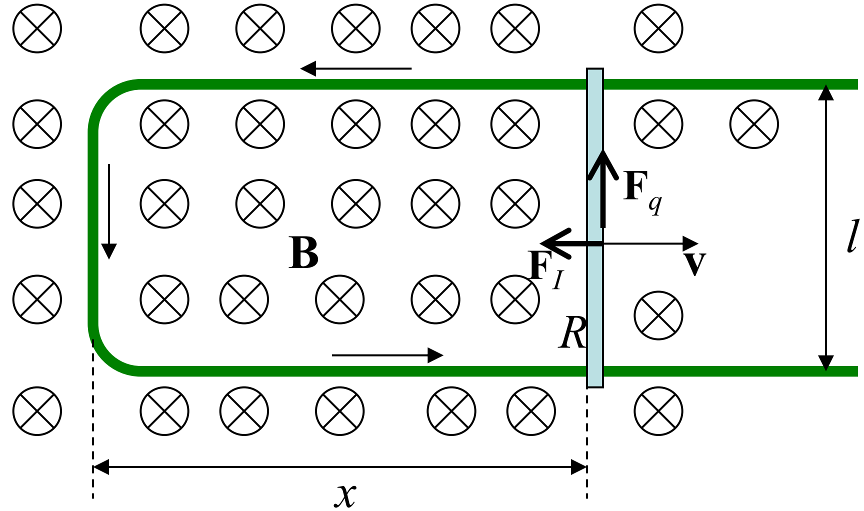
$$\boxed{\mathcal{E}_1 I_1 = \mathcal{E}_2 I_2}$$

Combining this with the relation for the emf leads to the relation

$$\boxed{N_1 I_1 = N_2 I_2}$$

Problem

U-shaped conductor and a movable rod



Resistance of the U-shaped conductor is zero, resistance of the rod is R. Find the emf, current in the circuit and the pulling force on the rod.

Solution: The EMF can be found from the Lenz' law as

$$\mathcal{E} = -\frac{\Delta\Phi}{\Delta t} = -\frac{\Delta(BS)}{\Delta t} = -B\frac{\Delta S}{\Delta t} = -B\frac{\Delta(lx)}{\Delta t} = -Bl\frac{\Delta x}{\Delta t} = -Blv$$

The same result can be obtained if we calculate work of the Lorenz force acting on the moving charges and divide this work by the charge q:

$$\mathcal{E} = \frac{W}{q} = \frac{F_q l}{q} = \frac{qvBl}{q} = Blv$$

The current is given by

$$I = \frac{\mathcal{E}}{R} = \frac{Blv}{R}$$

Magnetic field exerts the force on the current I

$$F_I = BIl = B\frac{Blv}{R}l = \frac{B^2l^2v}{R}$$

Power dissipated:

$$P = I^2R = \frac{B^2l^2v^2}{R}$$

Also

$$P = F_I v = \frac{B^2l^2v^2}{R}$$

Self-inductance

If the current I in a coil changes, the magnetic flux Φ in the coil changes, too, proportionally to the current. This results in the EMF in the coil that is given by the Lenz law

$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t}$$

This EMF can be represented in the form

$$\boxed{\mathcal{E} = -L\frac{\Delta I}{\Delta t},}$$

where L is the self-inductance or simply the inductance of the coil. For a solenoid the magnetic flux is given by

$$\Phi = BS = \frac{\mu_0 IN}{l}S,$$

thus

$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t} = -N\frac{\Delta\left(\frac{\mu_0 IN}{l}S\right)}{\Delta t} = -\frac{\mu_0 N^2 S}{l}\frac{\Delta I}{\Delta t} = -L\frac{\Delta I}{\Delta t}$$

and, finally,

$$L = \frac{\mu_0 N^2 S}{l}$$

The unit of inductance in Henry or H.

Energy stored in a coil

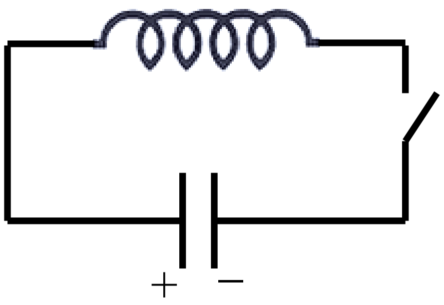
If the current in a coil changes, the emf generated in a coil tries to oppose the change of the magnetic flux and thus of the current. As a result, to make a current flow through a coil, one has to do work that is then stored in the coil as energy. Calculation leads to the result for the energy of the coil

$$\boxed{E_L = \frac{1}{2} L I^2}$$

This energy is related to the flowing current and it is a kind of kinetic energy. It can be compared with the energy of a charged capacitor

$$E_C = \frac{1}{2}\frac{Q^2}{C}$$

that is a kind of potential energy. A circuit containing a coil and a capacitor is similar to the mechanical system mass on a spring. In the illustration, the energy at first is stored as the potential energy on the capacitor.



Switching the circuit on leads to the discharge of the capacitor through the coil. When the capacitor is fully discharged (Q = 0), the current reaches its maximum. At this moment, the whole energy is stored in the coil as a kinetic energy. Then the capacitor begins to charge in the opposite direction as the current cannot stop immediately. The system performs oscillations similar to those of a mass on a spring. The period of these oscillations is

$$T = 2\pi\sqrt{LC}$$