3 – Electric current

Electric circuits consist of electric elements (resistances, condensers, coils, batteries, etc.) connected by wires.

Electric current

Electric current I through a wire is defined as charge that flows through the cross section of the wire in a unit of time:

$$I = \frac{\Delta Q}{\Delta t}$$

Unit of electric current is Ampere, 1A = 1C/s

There is an analogy between the electric current and flow of fluids. Current flowing through a cross section is nothing else than the flux of charge through it.

Current in a wire has direction and it can be positive or negative. The direction of the current I is chosen to be the direction in which positive charges would move. We know that in most of conductors (negatively charged) electrons are moving. Thus the direction of the electric current is opposite to the direction of motion of electrons

1
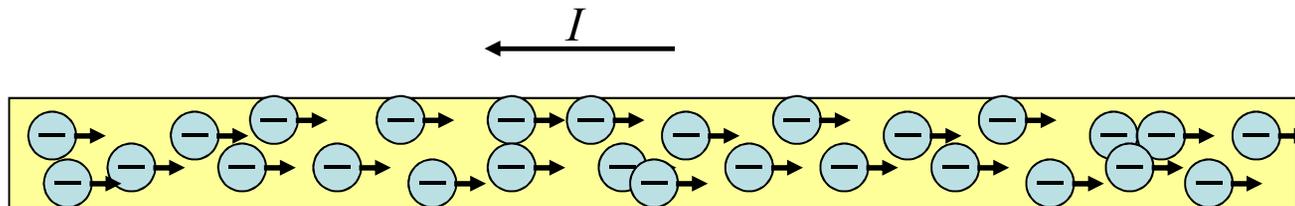
Electrons moving through a conductor under the influence of an electric field are colliding with different kinds of obstacles and thus they cannot accelerate unlimitedly. These obstacles resist the electronic motion. As a result, within a short time, electrons acquire a steady velocity. This means a steady electric current I that depends on the resistance of the material R.
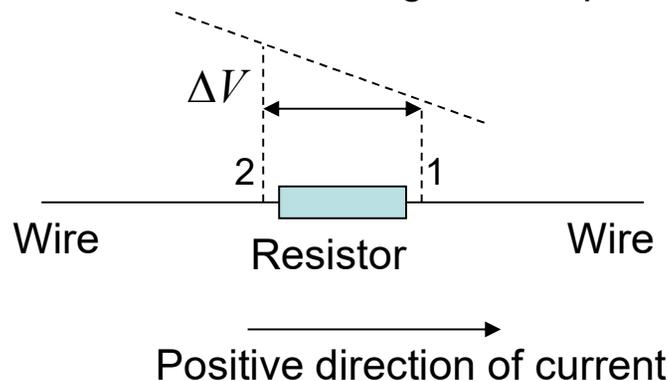
Ohm's law reads: $$I = \frac{\Delta V}{R} \quad \text{or} \quad \Delta V = RI$$ where ΔV is the potential difference (voltage) on the resistance R

Unit of resistance is 1 Ohm, denoted as 1 Ω. Evidently 1 $\Omega = 1 \ V \cdot s / C$

We will use both ΔV and simply V for the voltage, thus Ohm's law can be written as

$$\boxed{V = RI}$$

The current flows from high to low potential

Wire   Resistor   Wire

Positive direction of current

$$\Delta V = V_2 - V_1$$

If $\Delta V > 0$ then current flows in the positive direction, $I > 0$

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Resistance R introduced above depends on the properties of the material and also on its shape. The bigger the length of the wire L the bigger the resistance, and the bigger the cross-sectional area of the wire S the smaller the resistance. This is an experimental fact that can also be expected, if you have an intuition. One can introduce the so-called resistivity ρ that depends on the properties of the material only:

$$R = \rho \frac{L}{S}$$

The unit of resistivity is $\Omega \cdot m$. For metals the resistivity is in the ballpark of $10^{-8} \Omega \cdot m$.

Resistivity of metals increases with temperature
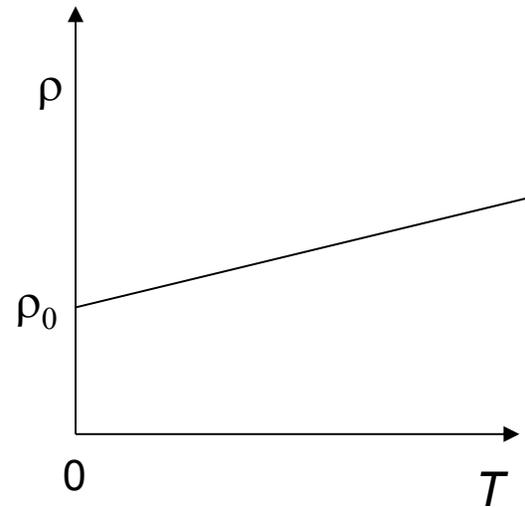
Electric current in a wire can be expressed as (see the next page)

$$I = envS,$$

where e is the charge of the electron, n is the concentration of free electrons (number of free electrons per m^3), v is the average velocity of electrons, and S is the cross-section of the wire. one can check that this formula has a correct unit C/s = A. The electrons are moving under the action of the electric field in the wire and different kinds of obstacles that in the Drude model are modeled by the drag (liquid friction) force $-\alpha v$, α being the friction constant. The Newton's second law for the electron has the form

$$ma = eE - \alpha v$$

In the stationary state the acceleration is zero, and the driving force from the electric field is balanced by the drag force. From this, one can find the electron's velocity as

$$v = eE/\alpha$$

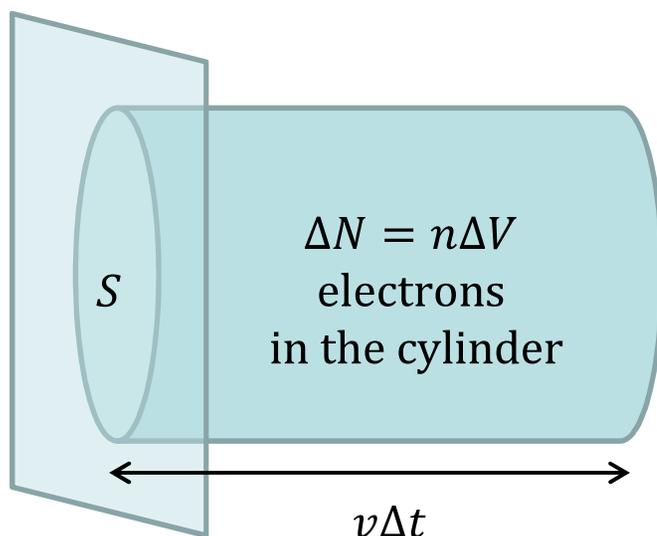Substituting it into the formula for the current and using $E = V/L$, one obtains

$$I = \frac{e^2 nS}{\alpha L} V$$

Thus we have obtained the Ohm's law

$$V = RI, \qquad R = \rho \frac{L}{S}, \qquad \rho = \frac{\alpha}{e^2 n}$$

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Derivation of the formula for the current

Cross-section of the wire

$$\Delta N = n\Delta V$$

electrons
in the cylinder

S

$$n = \frac{\Delta N}{\Delta V}$$

- concentration of electrons

$$v\Delta t$$

The charge crossing the wire's cross-section during Δt
(the charge inside the cylinder) is

$$\Delta Q = e\Delta N = en\Delta V = enSv\Delta t$$

The current:

$$I = \frac{\Delta Q}{\Delta t} = enSv$$

Power in general is defined as work per unit of time, $\quad P = \dfrac{\Delta W}{\Delta t}$

Work of the electric field on the electric charge $\Delta Q = I\Delta t$ that crosses the region with a potential difference V (say that go through a resistance with a voltage V on it, here we write V instead of ΔV) is given by

$$\Delta W = V\Delta Q$$

Then one obtains

$$P = \frac{\Delta W}{\Delta t} = \frac{V I \Delta t}{\Delta t} = IV$$

With the help of the Ohm's law it can be rewritten as

$$P = IV = I \times IR = I^2 R$$

On the other hand,

$$P = IV = \frac{V}{R}V = \frac{V^2}{R}$$

This power is being dissipated, that is, it goes over into heat.

Above we have studied the <u>direct current</u>, that is, the current that flows in one direction. Most of electric circuits use the alternating current that typically varies <u>sinusoidally</u> with time (as in the household circuits). The sinusoidal alternating current is described by

$$I(t) = I_0 \sin(\omega t + \varphi_0) \qquad \omega = 2\pi f,$$

where I_0 is the amplitude of the current, ω is the frequency, and φ_0 is an arbitrary phase. The power of this current is

$$P = I^2(t)R = I_0^2 \sin^2(\omega t + \varphi_0)R$$

As the time average of $\sin^2(\omega t)$ is ½, the average power is

$$P_{avr} = \frac{1}{2} I_0^2 R$$

That can be rewritten in terms of the effective current

$$P = I_{eff}^2 R, \qquad I_{eff} = \frac{I_0}{\sqrt{2}} \cong 0.707 I_0$$

Similarly one can introduce effective voltage

$$P = \frac{V_{eff}^2}{R}, \qquad V_{eff} = \frac{V_0}{\sqrt{2}}$$

where V_0 is the voltage amplitude. Note that the household voltage 110-120 V in North America is the effective voltage.