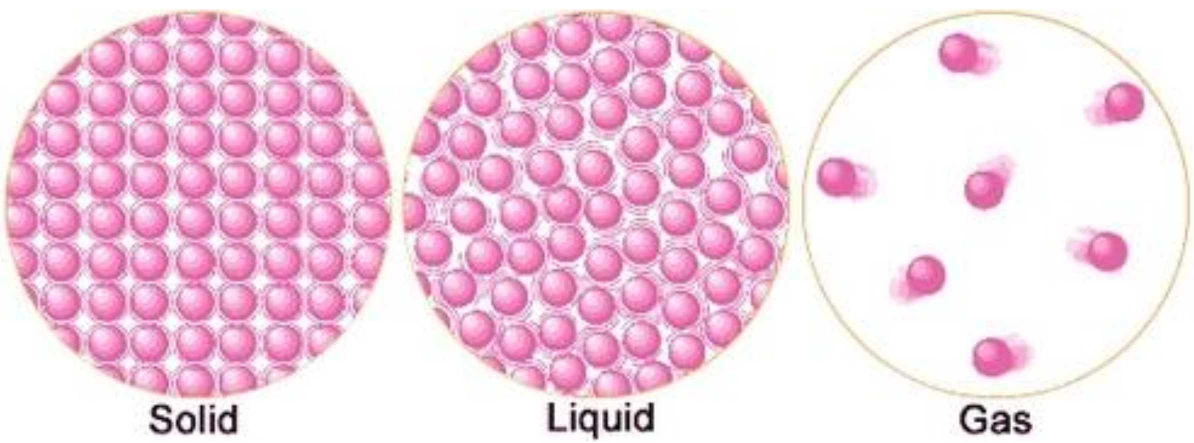
9 – Fluids

Three *ordinary* states (phases) of the matter:

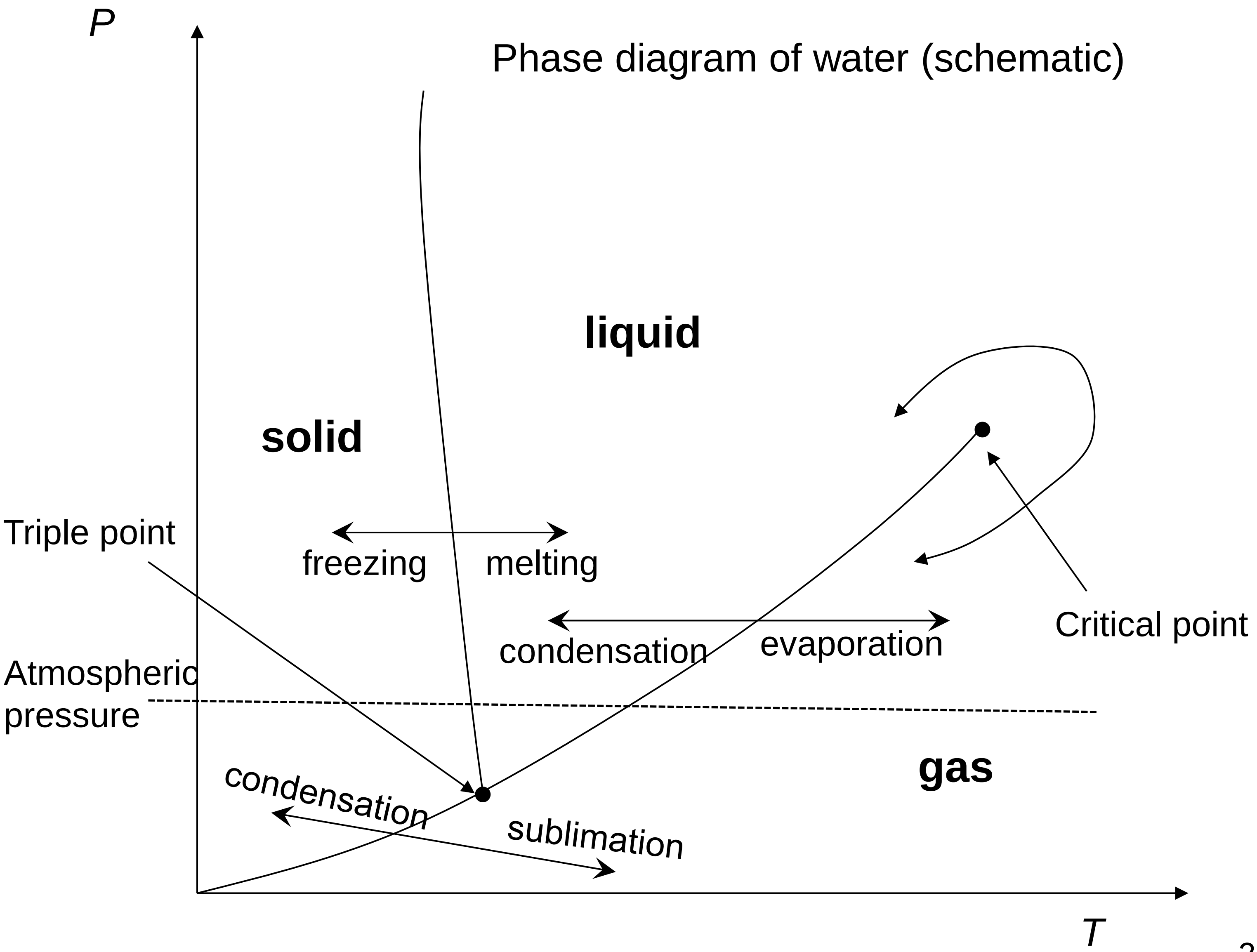
- Solids – maintain volume and shape
- Liquids – maintain volume but do not mantain shape
- Gases – do not maintain volume, spread over the whole region

Further states of the matter: Glass, butter, liquid crystals, colloids, plasma, etc.

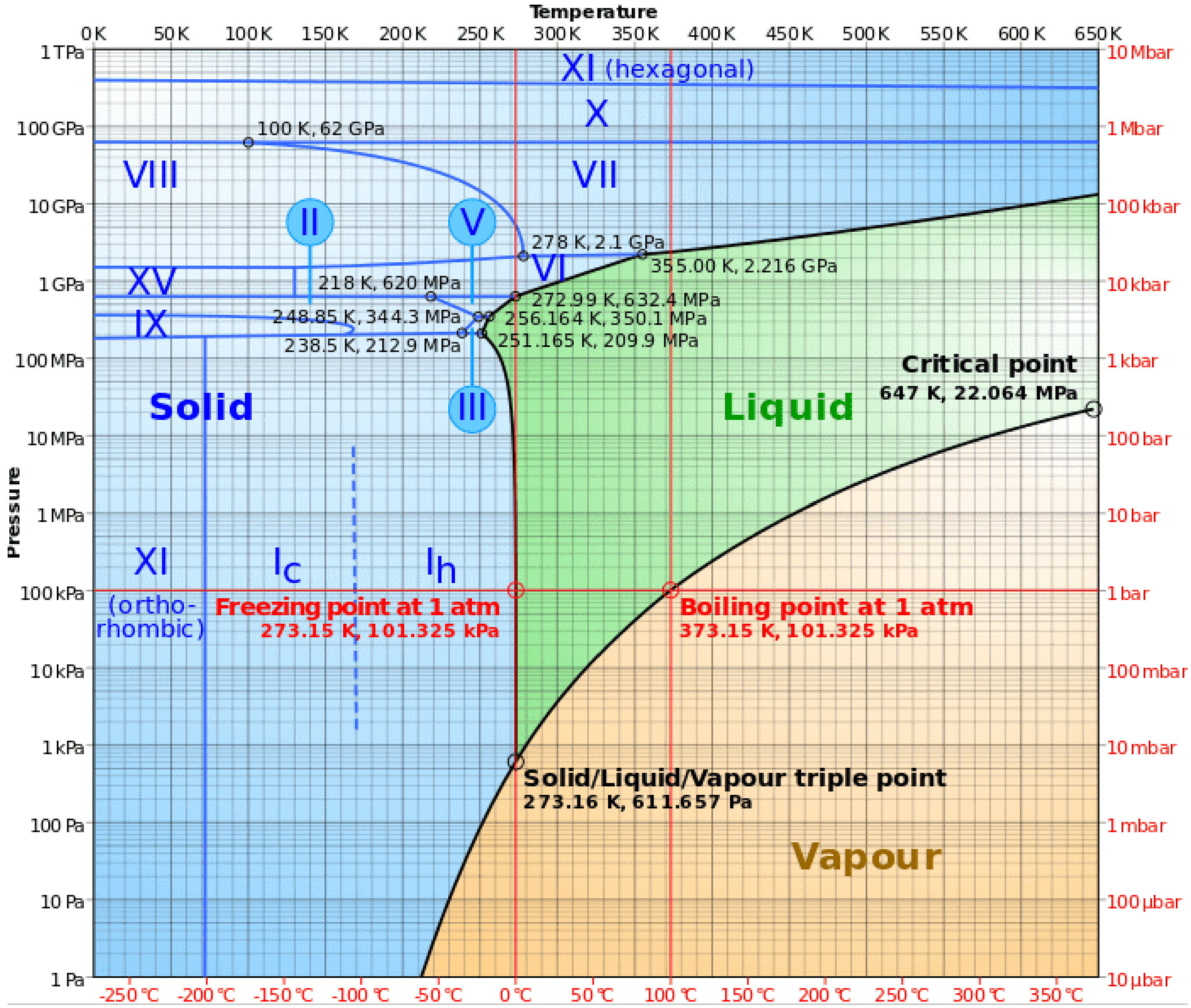
Liquids and gases are called <u>fluids</u>



There is a boundary between solids and fluids since solids have crystal lattice and fluids not. On the contrary, liquids can be made gases without crossing any boundary.



Phase diagram of water



Density and pressure

In contrast to the mechanics of solid bodies that retain their shape and thus can be considered as the whole, performing translational and rotational motions, fluids usually cannot be considered as the whole (think of water flowing through a pipe). For this reason fluids are usually described by *local* quantities such as density (instead of mass) and pressure (instead of force).

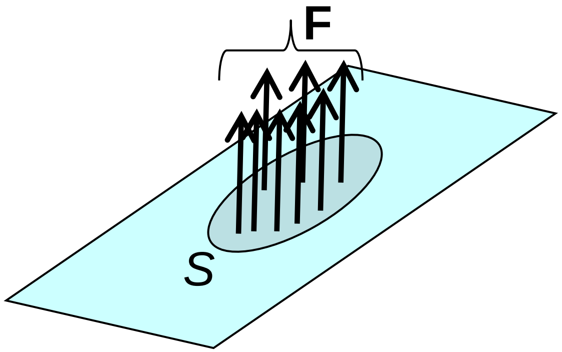
Density: $$\rho = \frac{m}{V}, \quad V \to 0$$

For liquids density is practically the same everywhere in the liquid. For gases, such as the Earths' athmosphere, the density can vary in space.

Density of the air at normal conditions: 1.204 kg/m^3

Substance	**Density, g/cm^3**
Water	1.00 (or 1000 kg/m^3)
Mercury	13.6
Gasoline	0.68
Alcohole	0.79
Ice	0.917
Concrete	2.3
Aluminium	2.7
Iron	7.8
Lead	11.3
Gold	19.3
Uranium	19.1
Wood	0.3-0.9

Pressure



$$P = \frac{F_\perp}{S}, \quad F_\perp \text{ - total force applied } \perp \text{ to a surface of area } S$$

For fluids at rest there are no forces parallel to the surface since fluids do not retain their shape and thus they cannot resist such forces. As soon as a parallel force is applied, fluid begins to move.

Unit of the pressure: 1 N/m^2 = 1 Pa [Blaise Pascal (1623-1662)]

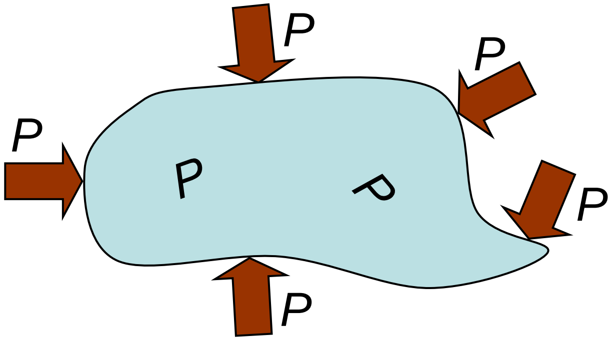
Other units:

- 1 atm = 1.013×10^5 Pa
- 1 bar = 10^5 Pa $\approx$ 1 atm
- 1 mm mercury (Hg) = 133 Pa (1 atm = 760 mm Hg)

Contrary to the force, pressure is not a vector but scalar. According to Newton´s third law, pressure from the wall of a vessel onto the fluid is the same as pressure from the fluid onto the wall. Similarly, one can consider an <u>imaginary membrane</u> inside the fluid and define pressure exerted by the fluid on one side of the membrane upon the fluid on its other side. So one can define pressure everywhere in the fluid. Pressure in fluids cannot be negative (we cannot pull a fluid)

Pascal´s law

In the absence of gravity, pressure in the fluid at rest is uniformly transfomed in all directions and it is the same everywhere.



We will see that in the presence of gravity pressure changes from one point to the other but still, it does not depend on the direction of the imaginary membrane we use to define the pressure inside the fluid.

Pascal´s law follows from the work principle (remember consideration of simple mechanical machines)

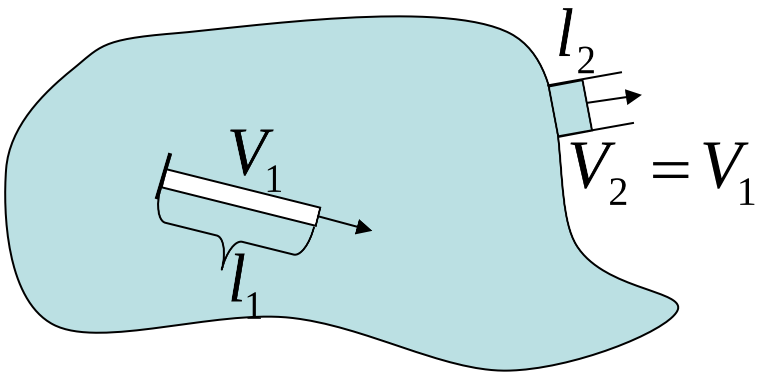
$$W_{\text{in}} = W_{\text{out}} \quad \text{or} \quad W_1 = W_2$$

A membrane expands from a zero volume to V_1 inside the liquid, pushing the liquid away in the direction shown. Conserving its volume, the liquid pushes the wall away in the region with surface area S_2, where the wall is allowed to move away. The work of the membrane on the liquid W_1 goes over into the work of the liquid on the wall W_2.

$$W_1 = F_1 l_1 = P_1 S_1 l_1 = P_1 V_1 = W_2 = P_2 V_2$$

From $V_1 = V_2$ (incompressibility) follows

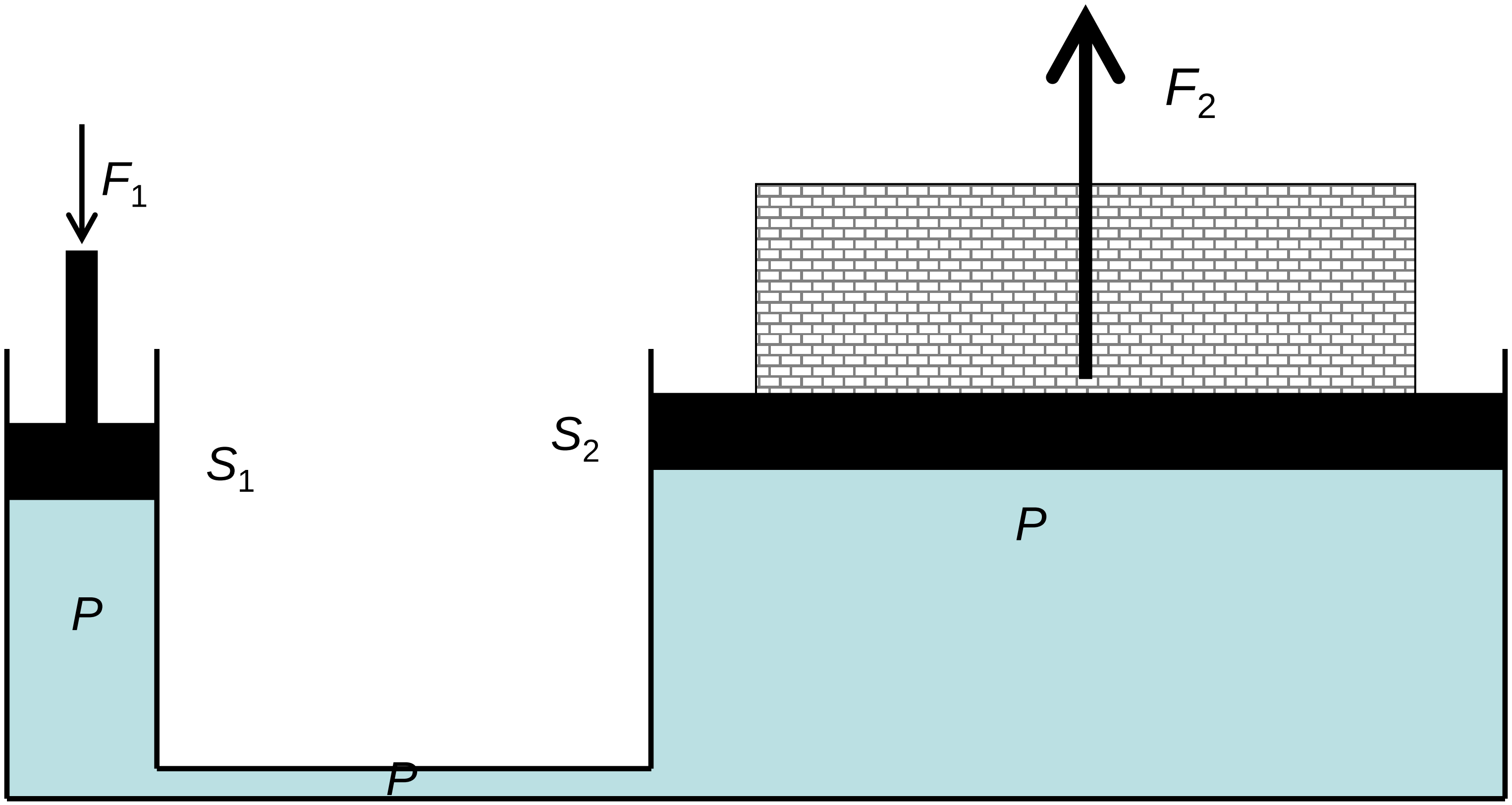
$$P_1 = P_2 = P$$



Application of Pascal´s law

Hydraulic jack

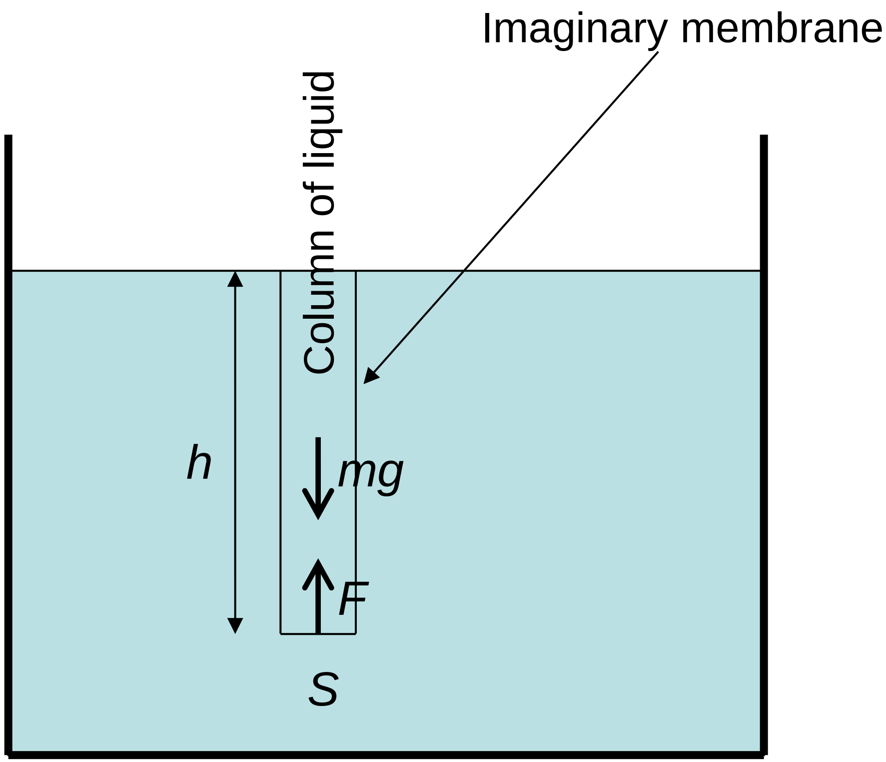
$$F_2 = PS_2 = \frac{F_1}{S_1} S_2 = \frac{S_2}{S_1} F_1 >> F_1, \quad \text{if} \quad S_2 >> S_1$$



Pressure in liquids with gravity

Weight of the column of liquid is balanced by the pressure on its bottom.

$$mg = \rho Vg = \rho hSg = F = PS$$



Thus pressure at the depth h is

$$\boxed{P = \rho gh}$$

gauge pressure (without the atmospheric pressure)

With the athmospheric pressure P_0 :

$$\boxed{P = \rho gh + P_0}$$

- absolute pressure

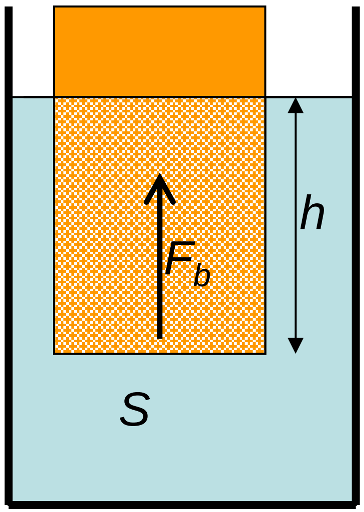
Note that forces acting on the sides of the column of liquid are horizontal and thus they do not play a role.

In the presence of gravity, Pascal's law is valid in its limited form: pressure everywhere at the same height (or depth) is the same.

Buoyancy and Archimede´s law

The buoyant force acting on an object immersed in a fluid is equal to the weight of the liquid displaced by the object

Proof 1
(object of columnar shape)

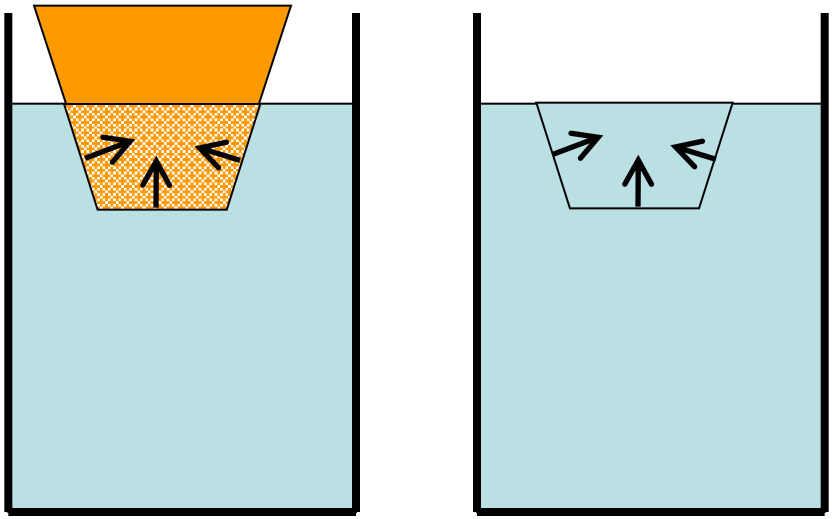


Buoyant force acts on the bottom only:

$$F_b = PS = \rho g h S = \rho g V = mg$$

(Weight of the displaced fluid)

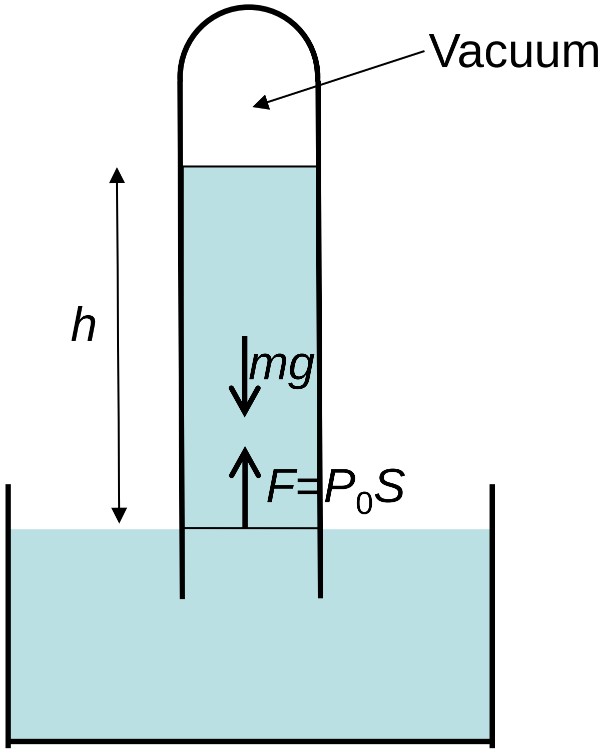
Proof 2
(general)



Replace immersed part of the object (left) by the fluid separated from the rest of the fluid by an imaginary membrane (right). The pressure forces applied are the same, thus one can use the picture on the right for the calculation of the bouyant force. Since the immersed piece of the fluid is in mechanical equilibrium,

$$F_b = mg$$

Athmospheric pressure and barometer



Torricelli´s experiment: Athmospheric pressure balances the weight of the column of liquid

$$F = mg \quad \Rightarrow \quad P_0 S = \rho h S g$$

$$\Rightarrow \quad P_0 = \rho g h$$

One can thus measure the athmospheric pressure in the units of height

For Hg (mercury)

$$h = P_0/(\rho g) = \frac{1.013\times10^5\ \mathrm{N/m^2}}{13.6\times10^3\ \mathrm{kg/m^3}\ 9.8\,\mathrm{m/s^2}} = 0.760\,\mathrm{m} = 760\,\mathrm{mm}$$

For $H_2$0 (water)

$$h = P_0/(\rho g) = \frac{1.013\times10^5\ \mathrm{N/m^2}}{1\times10^3\ \mathrm{kg/m^3}\ 9.8\,\mathrm{m/s^2}} = 10.34\,\mathrm{m}$$

Problem: Is the crown gold?

Formulation: When a crown of mass 14.7 kg is submerged in water, a scale reads only 13.4 kg. Is the crown made of gold?

Given: m=14.7 kg (mass of the crown), m´=13.4 kg (apparent mass of the crown submerged)
ρ_W=1 × 10^3 kg/m^3 (density of the water), ρ_G=19.3 × 10^3 kg/m^3 (density of gold)

To find: ρ (crown)

Solution: $$m = \rho V \qquad (1)$$

$$m' g = mg - F_b = \rho V g - \rho_W V g = (\rho - \rho_W) V g$$

$$\Rightarrow \quad m' = (\rho - \rho_W) V \qquad (2)$$

To eliminate unknown volume of the crown *V*, we divide (2)/(1):

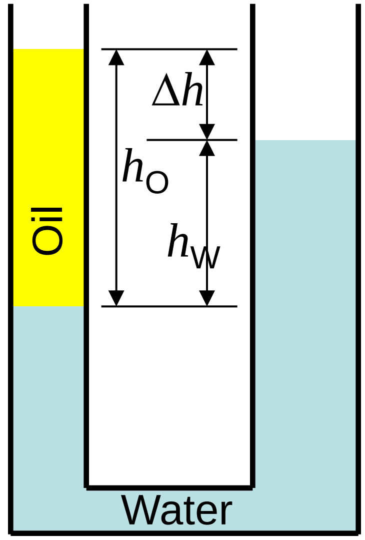
$$\frac{m'}{m} = \frac{\rho - \rho_W}{\rho} = 1 - \frac{\rho_W}{\rho} \quad \Rightarrow \quad \frac{\rho_W}{\rho} = 1 - \frac{m'}{m} \quad \Rightarrow \quad \frac{\rho}{\rho_W} = \frac{1}{1 - m'/m} = \frac{m}{m - m'}$$

Analytical result: $$\rho = \rho_W \frac{m}{m - m'}$$

Plug numbers: $$\rho = 1\,\text{g/cm}^3 \frac{14.7}{14.7 - 13.4} = 11.3\,\text{g/cm}^3$$

Verdict: This is not gold but lead!

Problem: Water and then oil (which don‘t mix) are poured into a U-shaped tube, open at both ends. They come to equilibrium as shown in Fig. with h_O = 27.2 cm and Δh = 9.41 cm. What is the density of the oil ρ_O?



Solution: The pressure at the boundary between the oil and water can be considered as produced by the column of oil and it is

$$P = \rho_O g h_O$$

On the other hand, the same pressure is produced by the column of water of the height $h_W = h_O - \Delta h$ that, according to the Pascal principle, is transmitted from the right to the left vertical pipe. That is,

$$P = \rho_O g h_O = \rho_W g (h_O - \Delta h)$$

From this equation one finds

$$\rho_O = \rho_W \frac{h_O - \Delta h}{h_O} = \rho_W \left(1 - \frac{\Delta h}{h_O}\right)$$

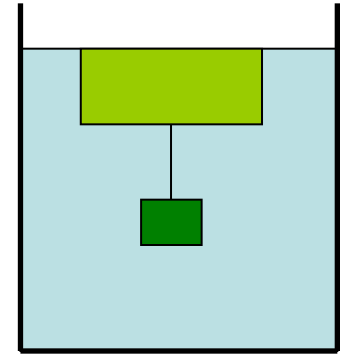
- Final analytical result

Plug numbers:

$$\rho_O = 1\,\mathrm{g/cm^3}\left(1 - \frac{9.41}{27.2}\right) = 0.654\,\mathrm{g/cm^3}$$

Problem: A 5.25-kg piece of wood (specific gravity: SG=0.50) floats on water. What minimum mass of lead, hung from the wood by a string, will cause it to sink?

Solution: When the system is about to sink, ist total weight is balanced by the buoyant force as follows



$$\left(m_{Wood} + m_{Lead}\right)g = \rho_{Water}\left(V_{Wood} + V_{Lead}\right)g$$

or
$$m_{Wood} + m_{Lead} = \rho_{Water}\left(V_{Wood} + V_{Lead}\right)$$

Here we find the volumes of wood and lead as: $V_{Wood} = \dfrac{m_{Wood}}{\rho_{Wood}}$, $V_{Lead} = \dfrac{m_{Lead}}{\rho_{Lead}}$ Then we obtain

$$m_{Wood} + m_{Lead} = \rho_{Water}\left(\frac{m_{Wood}}{\rho_{Wood}} + \frac{m_{Lead}}{\rho_{Lead}}\right) \Rightarrow m_{Lead}\left(1 - \frac{\rho_{Water}}{\rho_{Lead}}\right) = m_{Wood}\left(\frac{\rho_{Water}}{\rho_{Wood}} - 1\right)$$

Thus the final result is

$$\boxed{m_{Lead} = m_{Wood}\frac{\dfrac{1}{SG_{Wood}} - 1}{1 - \dfrac{1}{SG_{Lead}}}}$$

Definition:

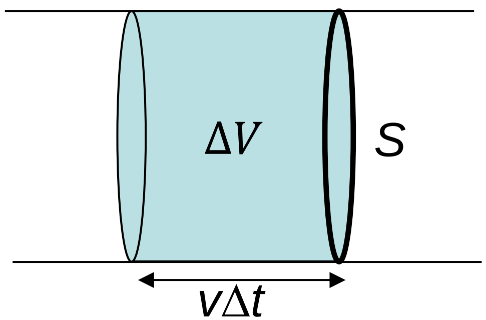
$$SG = \frac{\rho}{\rho_{\text{Water}}}$$

Plug numbers:
$$m_{Lead} = 5.25\frac{\dfrac{1}{0.5} - 1}{1 - \dfrac{1}{11.3}} = 5.25 \times 1.097 = 5.76\ \text{kg}$$

Hydrodynamics

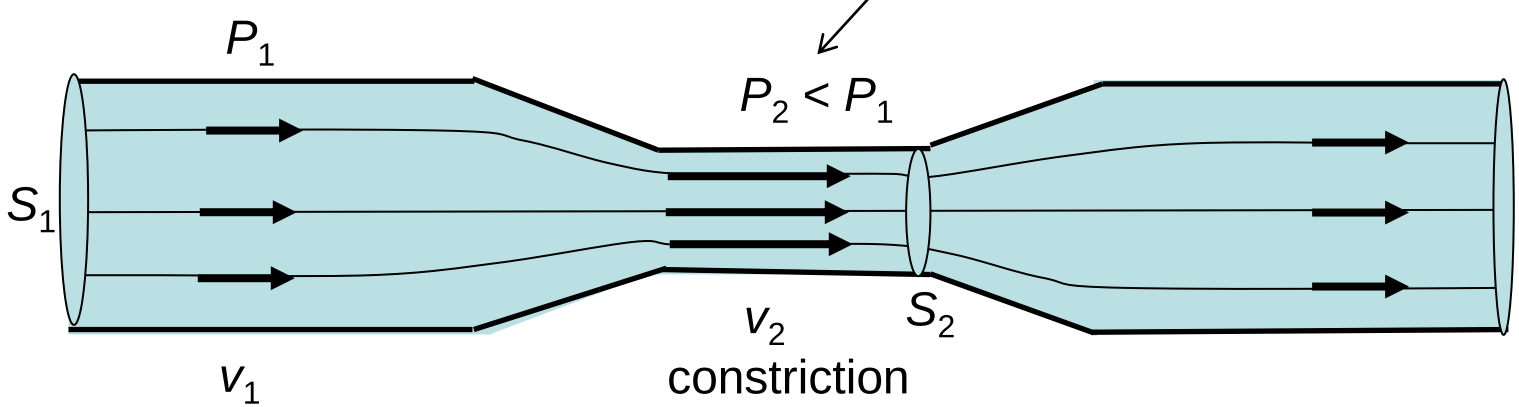
<u>Continuity condition</u> for the uncompressible fluid (liquid): The volume of liquid ΔV flowing through each cross-section of the pipe during the time Δt is the same for each cross-section. Reason: because of the uncompressibility, the liquid cannot accumulate anywhere. What enters any area from one side must exit from the other side.

The rate of flow: $\frac{\Delta V}{\Delta t} = \frac{Sv\Delta t}{\Delta t} = Sv = const$ everywhere in the pipe



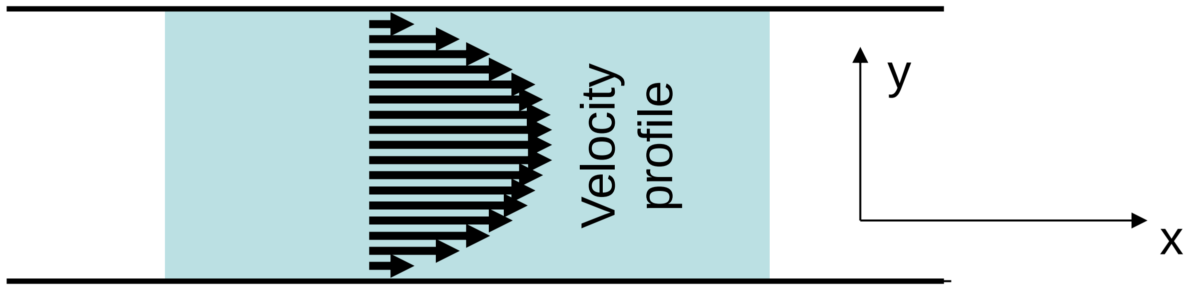
$$\Delta V = Sv\Delta t$$

Follows from the Bernoulli's law!

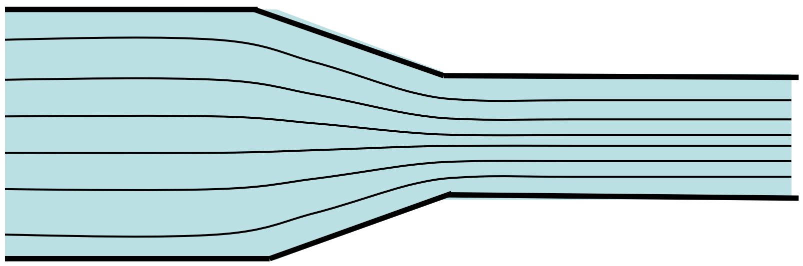


Viscosity

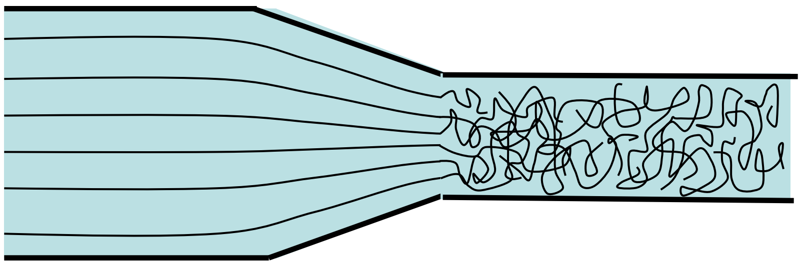
Layers of fluid having different speeds exert friction onto each other that is described in terms of viscosity η



Laminar and turbulent flows



Laminar – stable in time and space



Turbulent – unstable time and space

The following factors favor turbulence:
High speeds, large spacial dimensions, low viscosity

Bernoulli´s law

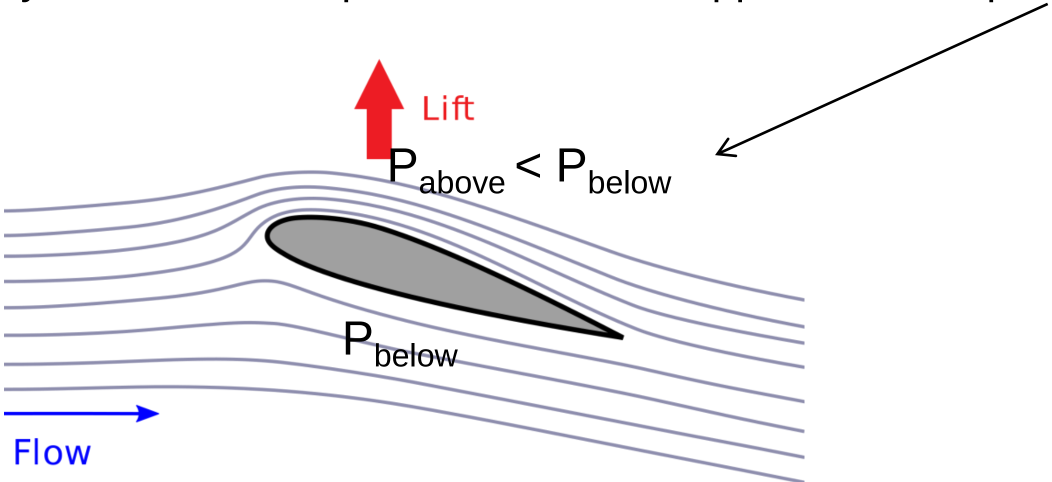
For a laminar flow of an incompressible fluid of zero viscosity

$$P + \rho g h + \frac{\rho v^2}{2} = \text{const} \qquad (h - \text{height, not depth})$$

along the flow lines

Bernoulli´s law can be derived from Newton´s second law: Entering a constriction, the element of the fluid accelerates, thus the pressure behind it is greater than the pressure before it – thus the pressure in the constriction is lower.

Its meaning: When an element of the fluid is moving from the area with a higher pressure into the area with a lower pressure, applied forces do positive work on it and its kinetic energy increases. An important consequence: In constrictions where the speed of the fluid is higher because of the continuity of the flow, the pressure is lower. Applications: Airplane wing, curveball.



Problem: Estimate the air pressure inside a cathegory 5 hurricane, where the wind speed is 300 km/h

Given: v = 300 km/h, ρ_{air} = 1.29 kg/m^3.

To find: P (inside the hurricane)

Solution: Assuming that the hurricane is stationary and laminar (that is not perfectly true!) and that far from the hurricane the air speed is zero and the pressure is equal to the athmospheris pressure P_0, use the Bernoulli equation

$$P + \frac{\rho v^2}{2} = P_0$$

And obtain

$$P = P_0 - \frac{\rho v^2}{2} = 10^5 \text{ N/m}^2 - \frac{1.29 \text{ kg/m}^3 \times (300 \text{ km/h})^2}{2}$$

$$= 1.013 \times 10^5 \text{ N/m}^2 - \frac{1.29 \text{ kg/m}^3 \times (300 \times 1000 / 3600 \text{ m/s})^2}{2}$$

$$= (1.013 \times 10^5 - 0.45 \times 10^4) \text{ N/m}^2 = 0.968 \times 10^5 \text{ N/m}^2$$

$$= 0.968 / 1.013 \text{ atm} = 0.956 \text{ atm} = 726 \text{ mm Hg}$$