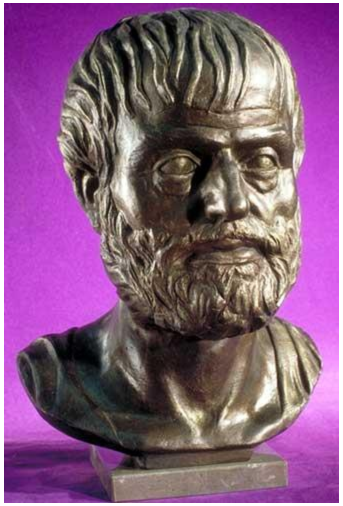
4 – Newton´s laws

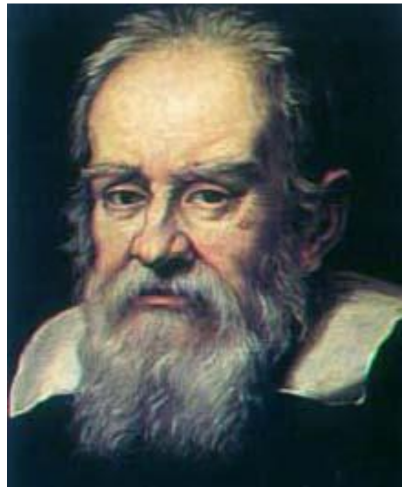


Aristotle (384-322 b.c.)

Dominating views for centuries

„Forces cause objects to move;
No force – no velocity“

Aristotle disregarded friction forces



Galilei (1564-1642
+2000 years!)

Questioned Aristotles

„Forces cause objects to accelerate;
No force – velocity constant“

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

Newton (1642-1727)
1687 – The mathematical Principles of Natural Philosophy (Principia)

Newton´s laws: Foundation of modern Physics

Newton mechanics →

- Relativistic mechanics: $v \sim c = 3 \cdot 10^8$ m/s
- Quantum Mechanics: $m \sim m_e \sim 10^{-30}$ kg

♦ **Newton´s first law**:

Objects that are not subject to action of forces are moving with zero or constant velocity

(follows from the second law; has historical significance – disproves Aristotle)

♦ **Newton´s second law**: $\mathbf{F} = m\mathbf{a}$

F – force; *m* – mass; **a** – acceleration

Unit of force:
kg m/s^2 = N(ewton)

Allows to compute acceleration from force etc.

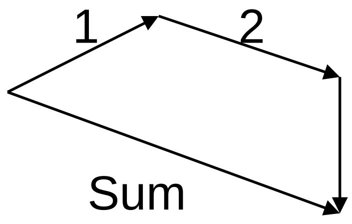
Consequences: (i) First law for **F** = 0;
(ii) Statics for **a** = 0, i.e., **F** = 0 (important applications in engineering)

Example: gravitational force: $\mathbf{F}_G = m\mathbf{g}$, where $\mathbf{g}$ is directed down and $g = 9.8\ m/s^2$

Force **F** is the sum of all forces acting on the object:

$$\mathbf{F} = \sum_i \mathbf{F}_i = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \ldots$$

Addition of vectors

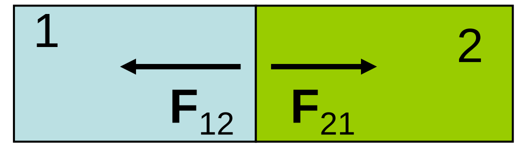


Mass *m* is a positive scalar, the measure of inertia of the object

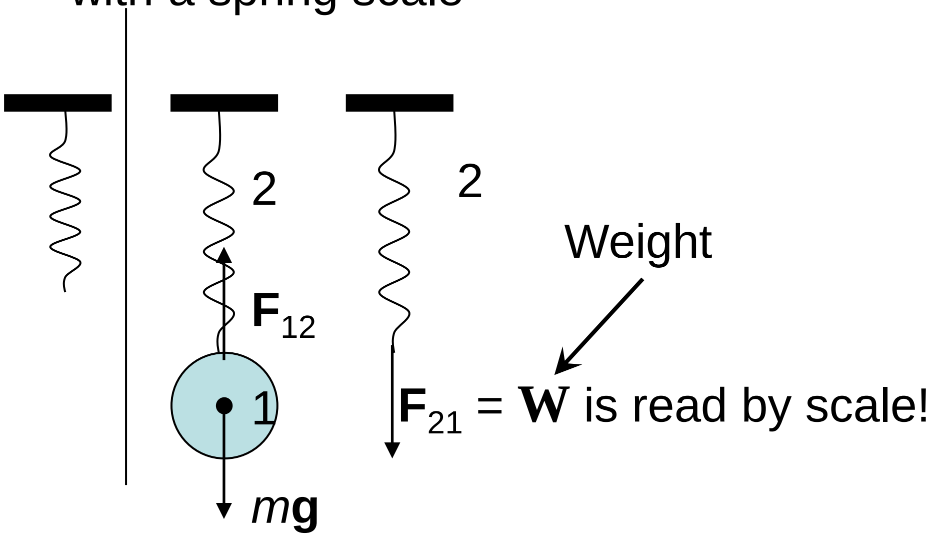
♦ **Newton´s third law**: $\mathbf{F}_{12} = -\mathbf{F}_{21}$

Helps to identify forces in systems of interacting bodies

Two objects pressed against each other



Example: Measuring weight with a spring scale



Using the 2-nd law:

$m\mathbf{g} + \mathbf{F}_{12} = 0$

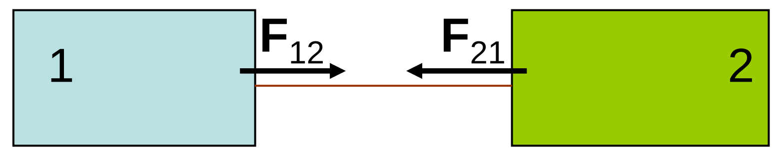
$\Rightarrow \mathbf{F}_{12} = -m\mathbf{g}$

Using the 3-rd law:

$\mathbf{F}_{21} = \mathbf{W} = -\mathbf{F}_{12} = m\mathbf{g}$

In this way the weight **W** = *m***g** is measured

Two objects connected by a cord and pulled apart



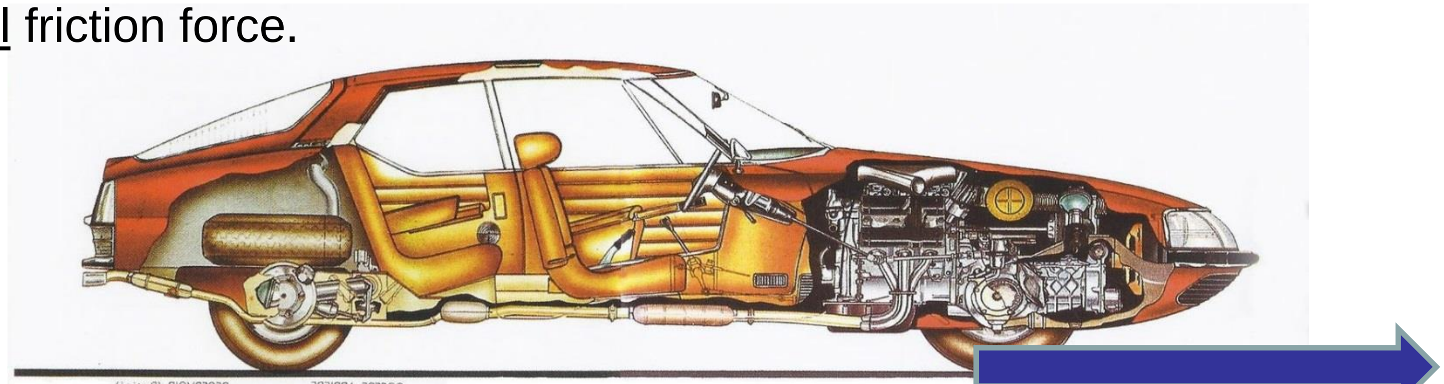
In *isolated* systems
the sum of all (*internal*) forces is zero;

Only *external* forces
can accelerate the system as the whole

Examples:
- rocket
- car,
- smart mule
- Baron Münchhausen

Examples of the workings of Newton's third law

Accelerating car: internal forces, including those in the motor and transmission cancel each other according to Newton's third law. The only force that makes the car accelerate is the <u>external</u> friction force.



$\mathbf{F}_{friction}$

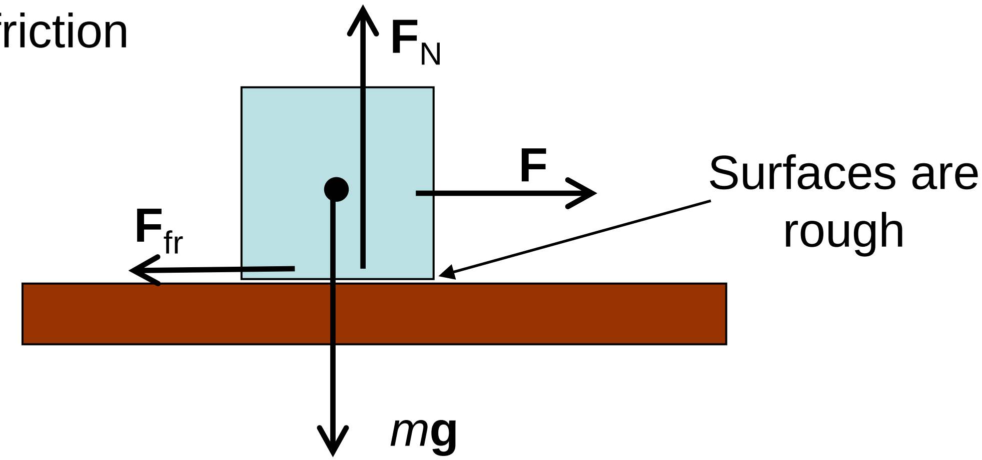
Horse and a cart

Internal forces cancel

$\mathbf{F}_{friction}$

Friction

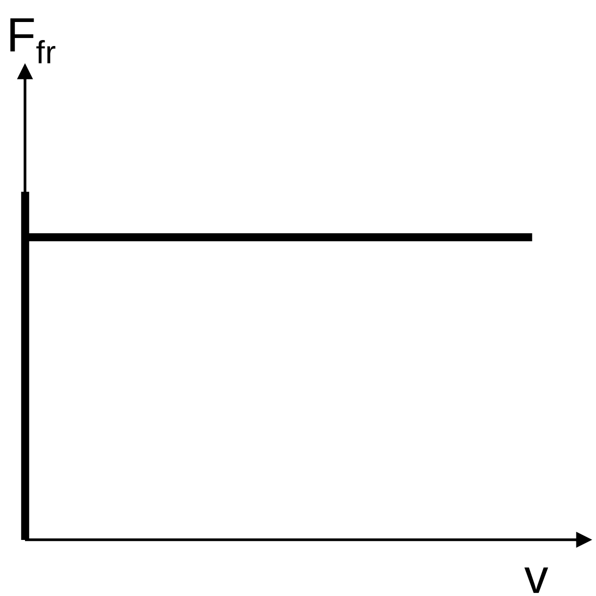
- Dry friction
- Rolling friction
- Viscous friction

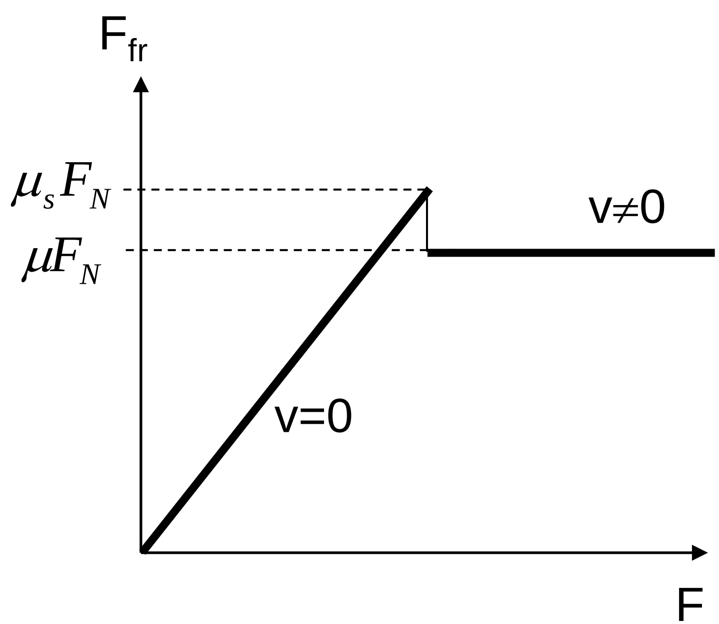


Dry friction

$$F_{fr} = \mu F_N, \qquad v \neq 0$$
$$F_{fr} = F \leq \mu_s F_N, \qquad v = 0$$

μ_s is the static friction coefficient, $\mu_s > \mu$ for all pairs of materials





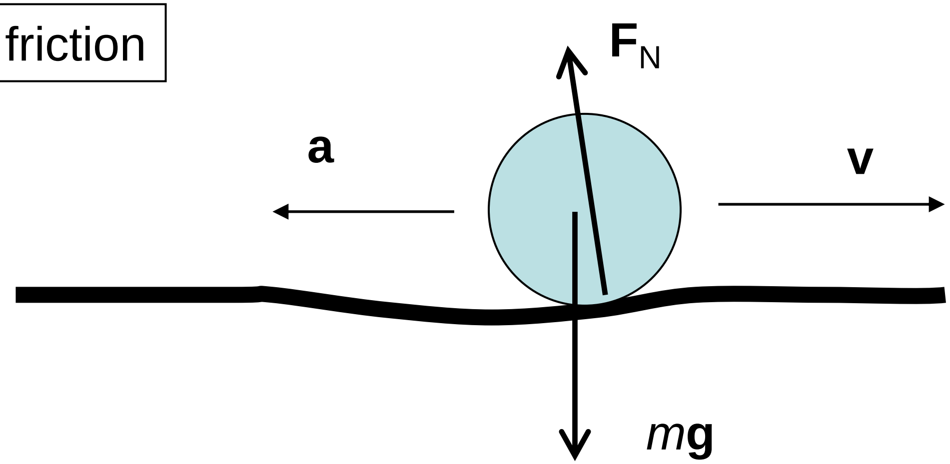
Here $\mu_k = \mu$

TABLE 4–2 Coefficients of Friction†

Surfaces	Coefficient of Static Friction, μ_s	Coefficient of Kinetic Friction, μ_k
Wood on wood	0.4	0.2
Ice on ice	0.1	0.03
Metal on metal (lubricated)	0.15	0.07
Steel on steel (unlubricated)	0.7	0.6
Rubber on dry concrete	1.0	0.8
Rubber on wet concrete	0.7	0.5
Rubber on other solid surfaces	1–4	1
Teflon® on Teflon in air	0.04	0.04
Teflon on steel in air	0.04	0.04
Lubricated ball bearings	<0.01	<0.01
Synovial joints (in human limbs)	0.01	0.01

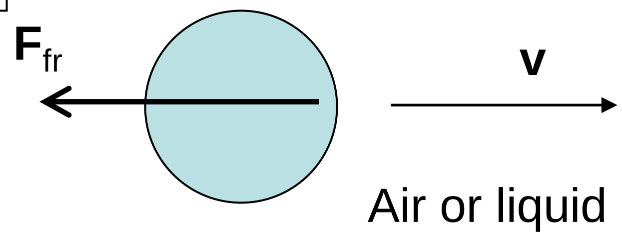
† Values are approximate and intended only as a guide.

Rolling friction



If the surface is elastically deformable, the rolling object pushes it down and creates a dynamic pit that is moving with the rolling body. The latter goes uphill and the total force $m\mathbf{g} + \mathbf{F}_M$ is nonzero and directed back, being the rolling friction force. The same effect arises if the body is deformable. If both the surface and the rolling body are absolutely rigid, there is no rolling friction.

Viscous friction

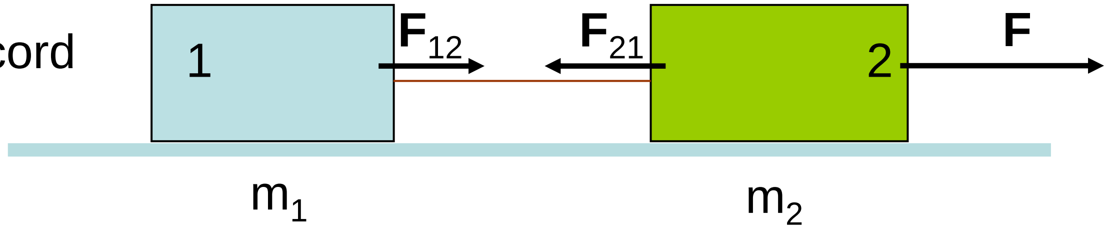


$$\mathbf{F}_{viscous\ friction} = -\alpha \mathbf{v}$$

The coefficient of viscous friction α increases with the size of the body and is proportional to the viscosity of the fluid.

Problem: Two connected blocks

- Two blocks joined by a cord



a - ?

F$_{12}$ - ? (tension of the cord)

Solution 1: Use Newton's second law for the individual masses + Newton's third law. As all vectors are directed along the same line, one can discard vector notations.

$$m_1 a = F_{12}, \qquad m_2 a = F + F_{21}$$

Add these equations and use Newton's third law

$$(m_1 + m_2)a = F + \underbrace{F_{12} + F_{21}}_{0} = F \longrightarrow \boxed{a = \frac{F}{m_1 + m_2}}$$

$$F_{12} = m_1 a = \frac{m_1}{m_1 + m_2} F$$

Solution 2: Use Newton's second law for the whole system, taking into account only external forces

$$(m_1 + m_2)a = F \longrightarrow \boxed{a = \frac{F}{m_1 + m_2}}$$

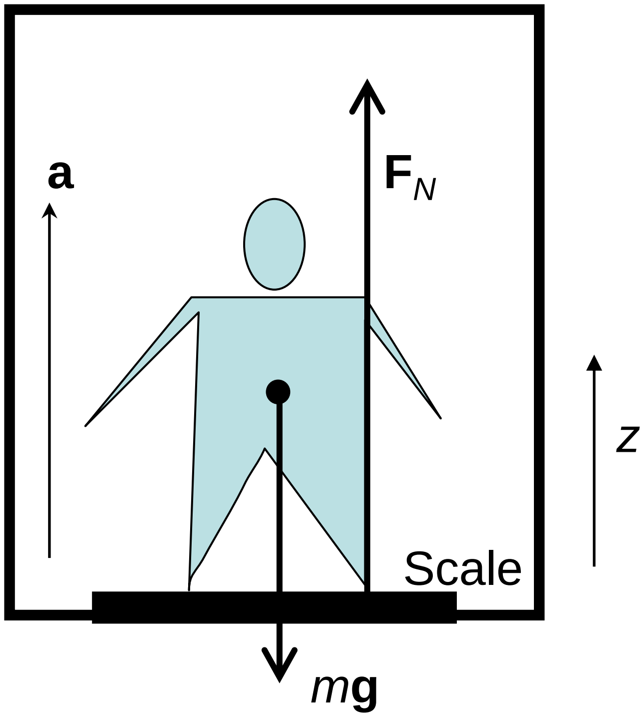
Problem: Apparent weight of a person in the elevator

Find the apparent weight of a person in an elevator moving with the acceleration a

Solution. The apparent weight is defined as the normal force $\mathbf{F}_N$ acting on the person from the floor. This is what the person feels as his/her weight

Newton's second law:

$$F_N - mg = ma \longrightarrow F_N = m(g + a)$$



For $a > 0$ (acceleration up) the apparent weight is greater than the actual weight, $F_N > mg$.

The elevator is an example of a non-inertial frame. The person inside does not know that it is moving with acceleration and can think that there is a gravity force $F_G = m(g + a)$

♦ Simple mechanical devices: pulley

Mass M is the load

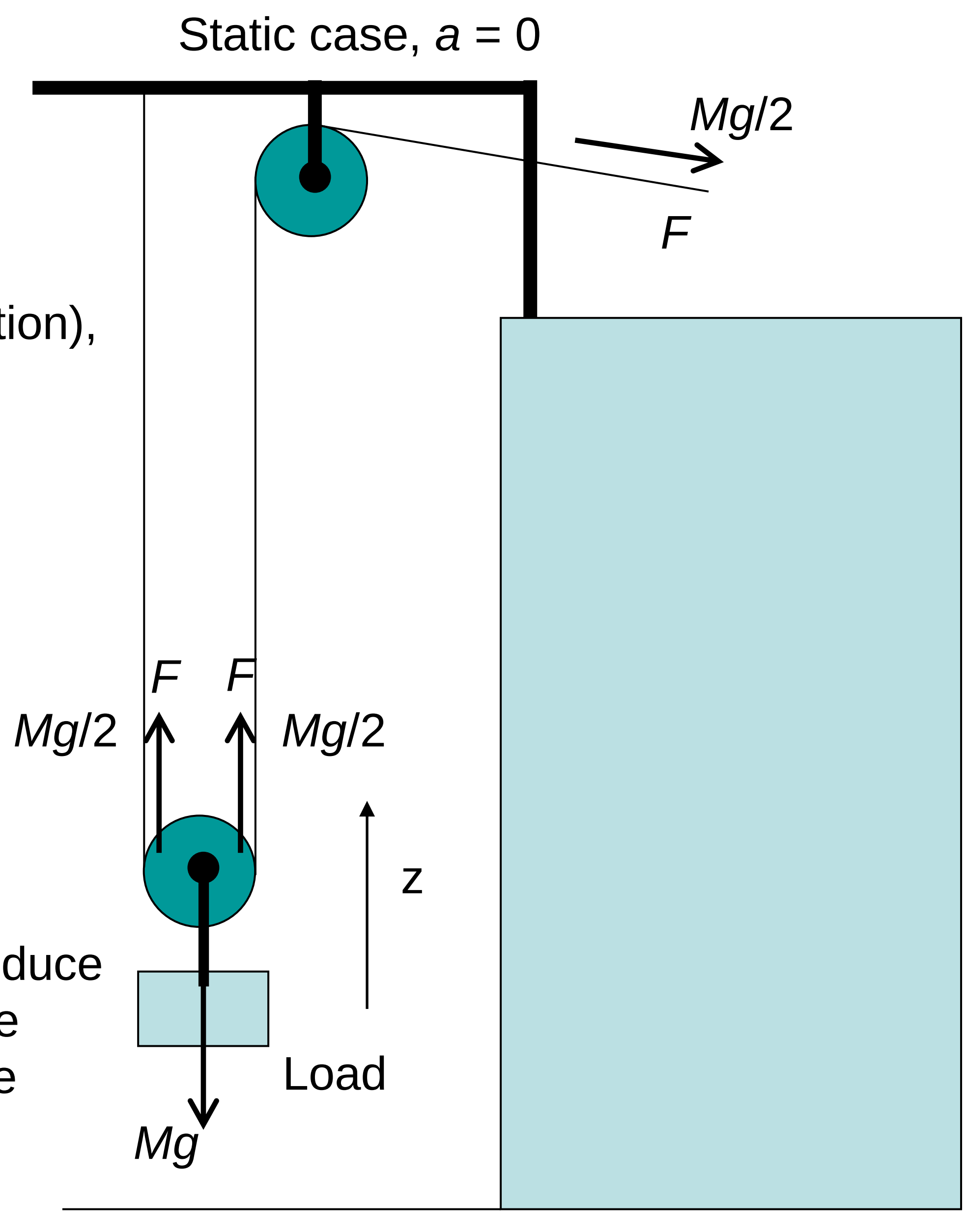
To lift the load slowly (without acceleration), One has to pull with the force F so that

$$2F - Mg = Ma = 0,$$

that is, only

$$F = Mg/2$$

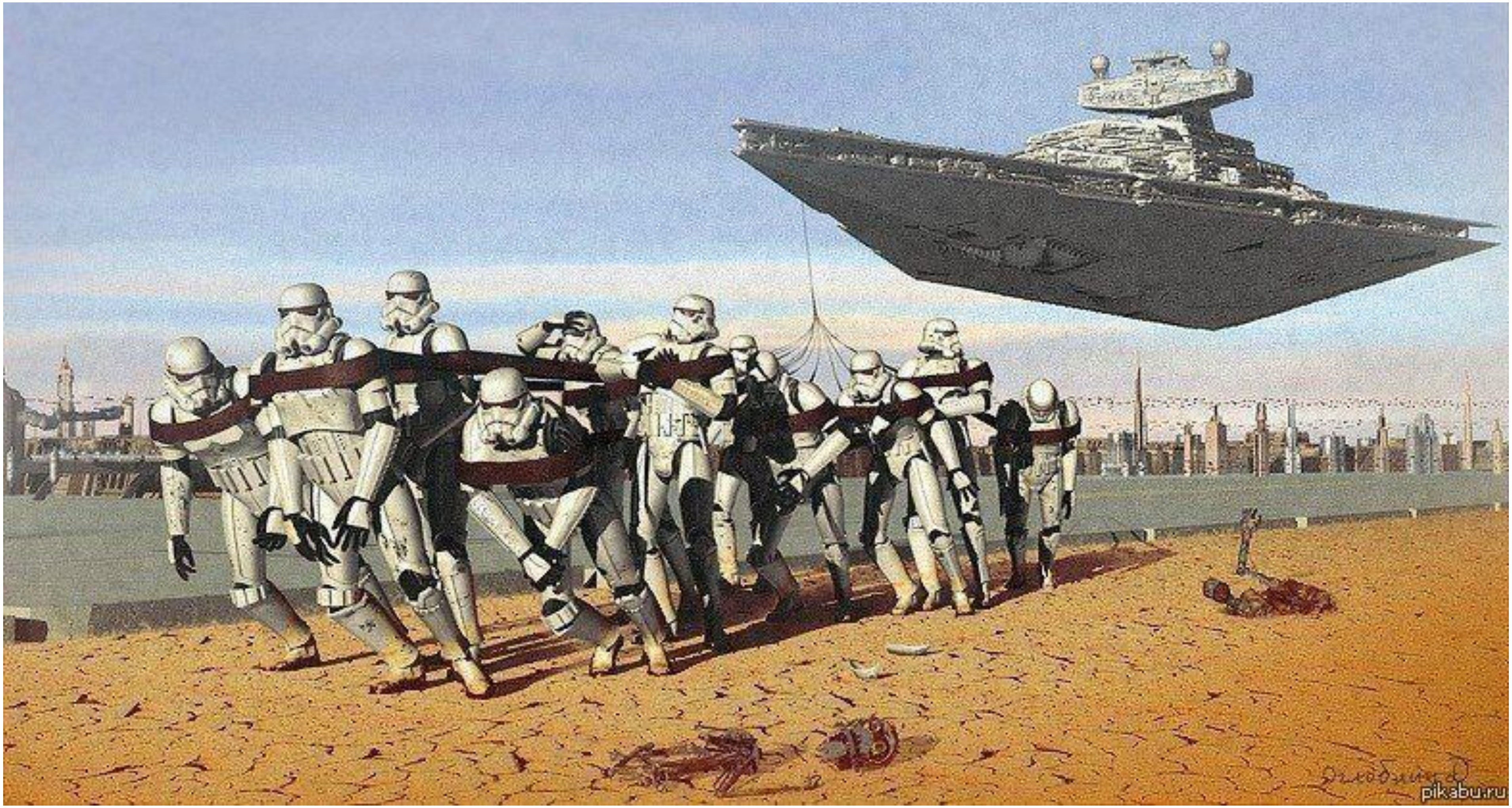
Using more blocks, one can further reduce the pulling force. However, in this case the weight of many blocks reduces the efficacy of the machine.



Pulling a non-motorized boat

Ilya Repin: “Burlaki on Volga” (1873, a cult painting!)

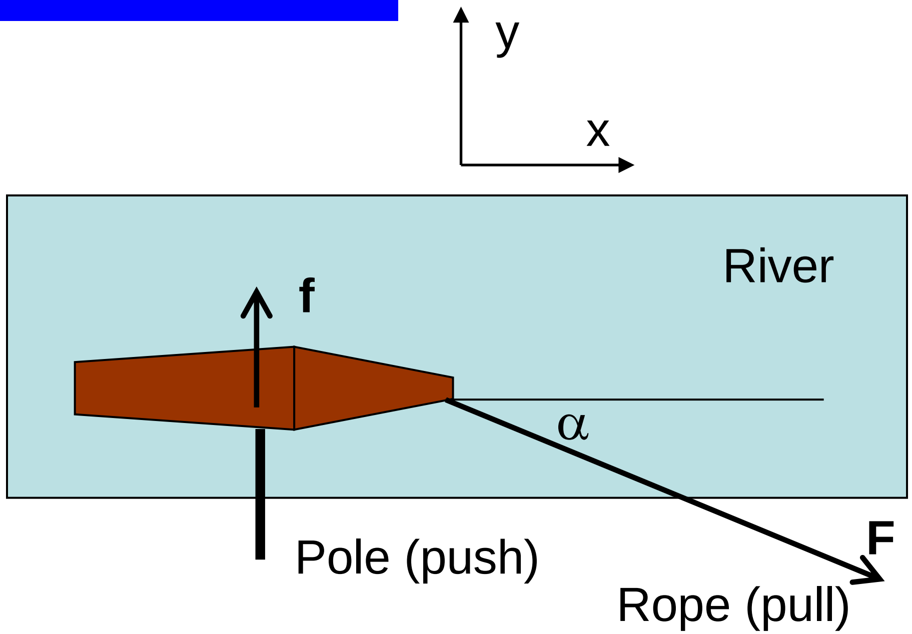
Live reconstruction of Repin’s painting



Alien burlaki

A Chinese caricature of the Soviet-Russian leadership

Problem: Pulling a non-motorized boat



Known: F, α

Find: pushing force f, net force

Solution. To ensure that the boat moves horizontally, one should apply a pushing force that cancels the vertical component of the pulling force.
Projecting forces onto the *y* axis, one obtains:

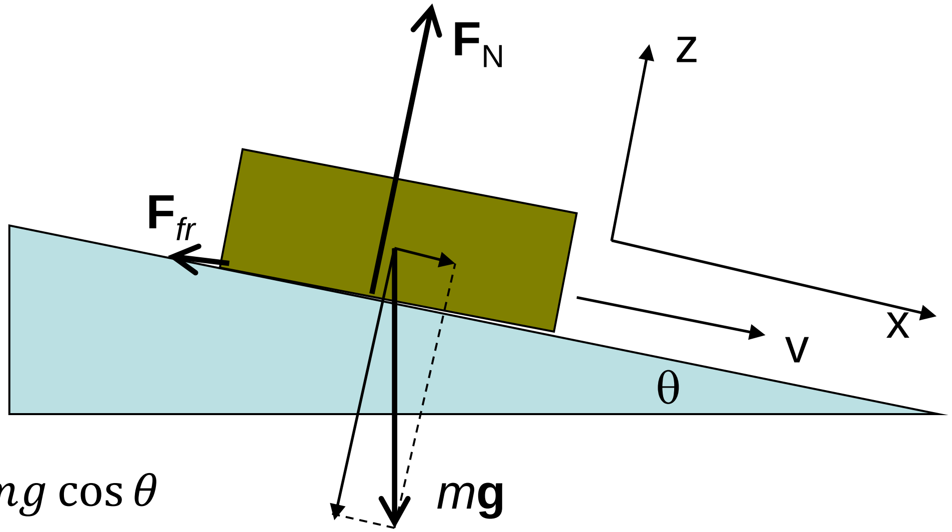
$$f - F\sin\alpha = 0 \longrightarrow \boxed{f = F\sin\alpha}$$

The net force is then directed along the direction of the river:

$$\boxed{F_{net} = F_x = F\cos\alpha}$$

There is also a resistance force acting on the boat from the water and directed to the left so that it compensates for the net force and ensures zero acceleration.

Incline



Newton's second law

$$\mathbf{mg} + \mathbf{F}_N + \mathbf{F}_{fr} = m\mathbf{a}$$

In components

"z": $F_N - mg\cos\theta = 0$ → $F_N = mg\cos\theta$

"x": $-F_{fr} + mg\sin\theta = ma$

If the block is *sliding*, $F_{fr} = \mu F_N$

$$a = g(\sin\theta - \mu\cos\theta) > 0$$

Condition for sliding:

$$\tan\theta > \mu$$

If the result for the acceleration is negative, it means that the block is not moving and the friction force just cancels the gravity force's x-component:

$$F_{fr} = mg\sin\theta \le \mu_s F_N = \mu_s mg\cos\theta$$

→ $\tan\theta < \mu_s$ - condition for resting

Example: A person in shoes with *rubber* soles is on a *concrete* slope. For what slope angles θ the person will be resting? Sliding?

Condition for sliding: $\tan\theta > \mu$ → $\theta > \arctan\mu = \arctan 0.8 = 38.7°$

Condition for resting: $\tan\theta < \mu_s$ → $\theta < \arctan\mu_s = \arctan 1 = 45°$

In the interval $38.7° \le \theta \le 45°$ both resting and sliding are possible