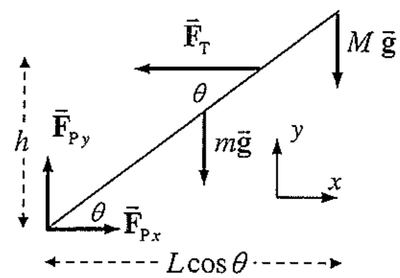
PROBLEMS 9

(1) See slides

(2) See slides

(3)

(*a*) The pole is in equilibrium, and so the net torque on it must be zero. From the free-body diagram, calculate the net torque about the lower end of the pole, with counterclockwise torques as positive. Use that calculation to find the tension in the cable. The length of the pole is L.



$$\sum \tau = F_T h - mg\left(L/2\right)\cos\theta - MgL\cos\theta = 0$$

$$F_T = \frac{\left(m/2 + M\right) gL\cos\theta}{h}$$

$$= \frac{\left(6.0\text{ kg} + 21.5\text{ kg}\right)\left(9.80\text{ m/s}^2\right)\left(7.50\text{ m}\right)\cos 37^\circ}{3.80\text{ m}} = 424.8\text{ N} \approx \boxed{4.25\times10^2\text{ N}}$$

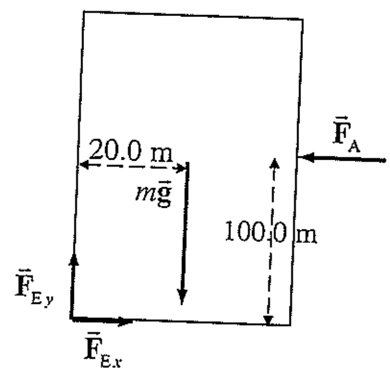
(*b*) The net force on the pole is also zero since it is in equilibrium. Write Newton's 2nd law in both the x and y directions to solve for the forces at the pivot.

$$\sum F_x = F_{Px} - F_T = 0 \rightarrow F_{Px} = F_T = \boxed{4.25\times10^2\text{ N}}$$

$$\sum F_y = F_{Py} - mg - Mg = 0 \rightarrow F_{Py} = \left(m+M\right)g = \left(33.5\text{ kg}\right)\left(9.80\text{ m/s}^2\right) = \boxed{3.28\times10^2\text{ N}}$$

(4)

Assume that the building has just begun to tip, so that it is essentially vertical, but that all of the force on the building due to contact with the Earth is at the lower left corner, as shown in the figure. Take torques about that corner, with counterclockwise torques as positive.

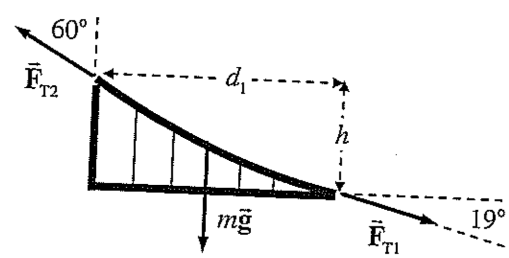


$$\sum\tau = F_A(100.0\text{ m}) - mg(20.0\text{ m})$$
$$= (950\text{ N/m}^2)(200.0\text{ m})(70.0\text{ m})(100.0\text{ m}) - (1.8\times10^7\text{ kg})(9.80\text{ m/s}^2)(20.0\text{ m})$$
$$= -2.2\times10^9\text{ m}\cdot\text{N}$$

Since this is a negative torque, the building will tend to rotate clockwise, which means it will rotate back down to the ground. Thus **the building will not topple**.

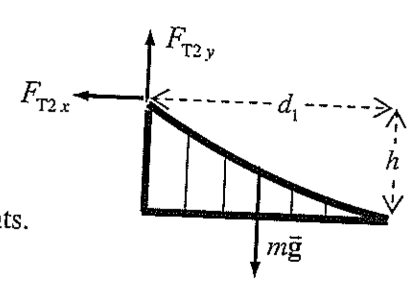
Draw a force diagram for the cable that is supporting the right-hand section. The forces will be the tension at the left end, $\vec{F}_{T2}$, the tension at the right end, $\vec{F}_{T1}$, and the weight of the section, $m\vec{g}$. The weight acts at the midpoint of the horizontal span of the cable. The system is in equilibrium. Write Newton's 2nd law in both the x and y directions to find the tensions.

(5)



$$\sum F_x = F_{T1}\cos 19° - F_{T2}\sin 60° = 0 \rightarrow$$
$$F_{T2} = F_{T1}\frac{\cos 19°}{\sin 60°}$$
$$\sum F_y = F_{T2}\cos 60° - F_{T1}\sin 19° - mg = 0 \rightarrow$$
$$F_{T1} = \frac{F_{T2}\cos 60° - mg}{\sin 19°} = \frac{F_{T1}\frac{\cos 19°}{\sin 60°}\cos 60° - mg}{\sin 19°} \rightarrow$$
$$F_{T1} = mg\frac{\sin 60°}{(\cos 19°\cos 60° - \sin 19°\sin 60°)} = 4.539\,mg \approx \boxed{4.5\,mg}$$
$$F_{T2} = F_{T1}\frac{\cos 19°}{\sin 60°} = 4.539\frac{\cos 19°}{\sin 60°}mg = 4.956\,mg \approx \boxed{5.0\,mg}$$

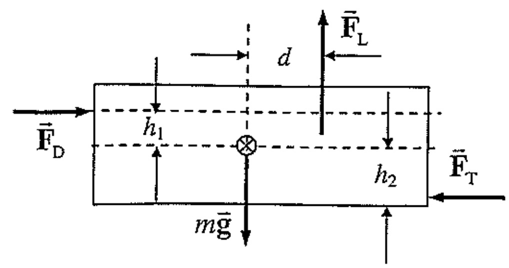
To find the height of the tower, take torques about the point where the roadway meets the ground, at the right side of the roadway. Note that then $\vec{F}_{T1}$ will exert no torque. Take counterclockwise torques as positive. For purposes of calculating the torque due to $\vec{F}_{T2}$, split it into x and y components.



$$\sum\tau = mg\left(\tfrac{1}{2}d_1\right) + F_{T2x}h - F_{T2y}d_1 = 0 \rightarrow$$
$$h = \frac{\left(F_{T2y} - \frac{1}{2}mg\right)}{F_{T2x}}d_1 = \frac{\left(F_{T2}\cos 60° - \frac{1}{2}mg\right)}{F_{T2}\sin 60°}d_1 = \frac{\left(4.956\,mg\cos 60° - 0.50\,mg\right)}{4.956\,mg\sin 60°}(343\text{ m})$$
$$= \boxed{158\text{ m}}$$

(6)

The airplane is in equilibrium, and so the net force in each direction and the net torque are all equal to zero. First write Newton's 2nd law for both the horizontal and vertical directions, to find the values of the forces.



$$\sum F_x = F_D - F_T = 0 \rightarrow F_D = F_T = \boxed{5.0\times10^5\,\mathrm{N}}$$

$$\sum F_y = F_L - mg = 0$$

$$F_L = mg = \left(6.7\times10^4\,\mathrm{kg}\right)\left(9.8\,\mathrm{m/s^2}\right) = \boxed{6.6\times10^5\,\mathrm{N}}$$

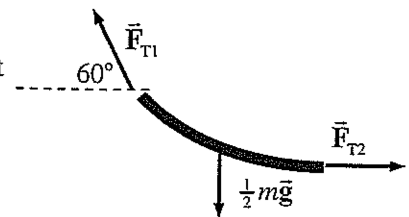
Calculate the torques about the CM, calling counterclockwise torques positive.

$$\sum \tau = F_L d - F_D h_1 - F_T h_2 = 0$$

$$h_1 = \frac{F_L d - F_T h_2}{F_D} = \frac{\left(6.6\times10^5\,\mathrm{N}\right)(3.2\,\mathrm{m}) - \left(5.0\times10^5\,\mathrm{N}\right)(1.6\,\mathrm{m})}{\left(5.0\times10^5\,\mathrm{N}\right)} = \boxed{2.6\,\mathrm{m}}$$

(7)

Draw a free-body diagram for half of the cable. Write Newton's 2nd law for both the vertical and horizontal directions, with the net force equal to 0 in each direction.



$$\sum F_y = F_{T1}\sin 60^\circ - \tfrac{1}{2}mg = 0 \rightarrow F_{T1} = \tfrac{1}{2}\frac{mg}{\sin 60^\circ} = 0.58mg$$

$$\sum F_x = F_{T2} - F_{T1}\cos 60^\circ = 0 \rightarrow F_{T2} = 0.58mg\left(\cos 60^\circ\right) = 0.29mg$$

So the results are:

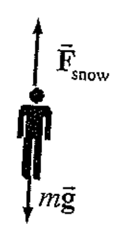
(a) $F_{T2} = \boxed{0.29mg}$

(b) $F_{T1} = \boxed{0.58mg}$

(c) The direction of the tension force is tangent to the cable at all points on the cable. Thus the direction of the tension force is [horizontal at the lowest point], and is [60° above the horizontal at the attachment point].

(8)

Assume a constant acceleration as the person is brought to rest, with up as the positive direction. Use Eq. 2-11c to find the acceleration. From the acceleration, find the average force of the snow on the person, and compare the force per area to the strength of body tissue.



$$v^2 = v_0^2 - 2a\left(x - x_0\right) \rightarrow a = \frac{v^2 - v_0^2}{2\left(x - x_0\right)} = \frac{0 - \left(60\,\mathrm{m/s}\right)^2}{2\left(-1.0\,\mathrm{m}\right)} = 1800\,\mathrm{m/s^2}$$

$$\frac{F}{A} = \frac{ma}{A} = \frac{\left(75\,\mathrm{kg}\right)\left(1800\,\mathrm{m/s^2}\right)}{0.30\,\mathrm{m^2}} = 4.5\times10^5\,\mathrm{N/m^2} < \text{Tissue strength} = 5\times10^5\,\mathrm{N/m^2}$$

Since the average force on the person is less than the strength of body tissue, the person may escape serious injury. Certain parts of the body, such as the legs if landing feet first, may get more than the average force, though, and so still sustain injury.

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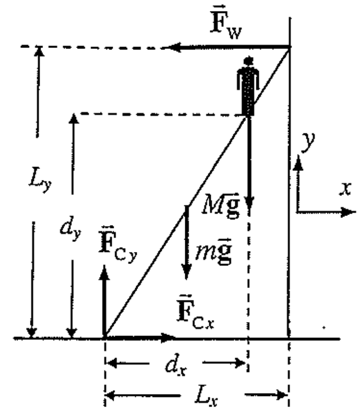

The ladder is in equilibrium, so the net torque and net force must be zero. By stating that the ladder is on the verge of slipping, the static frictional force at the ground, F_{Cx} is at its maximum value and so $F_{Cx} = \mu_s F_{Cy}$. Since the person is standing 70% of the way up the ladder, the height of the ladder is $L_y = d_y/0.7 = 2.8\text{ m}/0.7 = 4.0\text{ m}$. The width of the ladder is $L_x = d_x/0.7 = 2.1\text{ m}/0.7 = 3.0\text{ m}$. Torques are taken about the point of contact of the ladder with the ground, and counterclockwise torques are taken as positive. The three conditions of equilibrium are as follows.

$$\sum F_x = F_{Cx} - F_W = 0 \rightarrow F_{Cx} = F_W$$

$$\sum F_y = F_{Gy} - Mg - mg = 0 \rightarrow$$

$$F_{Gy} = (M + m)g = (67.0\text{ kg})(9.80\,\text{m/s}^2) = 656.6\,\text{N}$$

$$\sum \tau = F_W L_y - mg\left(\tfrac{1}{2}L_x\right) - Mgd_x = 0$$

Solving the torque equation gives

$$F_W = \frac{\frac{1}{2}mL_x + Md_x}{L_y}g = \frac{\frac{1}{2}(12.0\text{ kg})(3.0\text{ m}) + (55.0\text{ kg})(2.1\text{ m})}{4.0\text{ m}}(9.80\,\text{m/s}^2) = 327.1\,\text{N}.$$

The coefficient of friction then is found to be

$$\mu_s = \frac{F_{Gx}}{F_{Gy}} = \frac{327.1\text{ N}}{656.6\text{ N}} = \boxed{0.50}.$$

10

If the lamp is just at the point of tipping, then the normal force will be acting at the edge of the base, 12 cm from the lamp stand pole. We assume the lamp is in equilibrium and just on the verge of tipping, and is being pushed sideways at a constant speed. Take torques about the center of the base, with counterclockwise torques positive. Also write Newton's 2nd law for both the vertical and horizontal directions.

$$\sum F_y = F_N - mg = 0 \rightarrow F_N = mg \qquad \sum F_x = F_P - F_{fr} = 0 \rightarrow F_P = F_{fr} = \mu F_N = \mu mg$$

$$\sum \tau = F_N(0.12\text{ m}) - F_P x = 0 \rightarrow x = \frac{F_N}{F_P}(0.12\text{ m}) = \frac{mg}{\mu mg}(0.12\text{ m}) = \frac{0.12\text{ m}}{0.20} = \boxed{0.60\text{ m}}$$