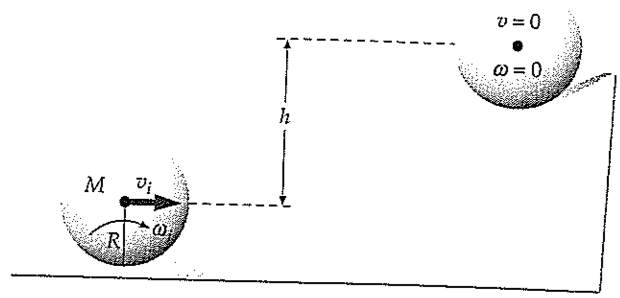
PROBLEMS 8

(1)



1. Make a labeled sketch showing the ball in both its initial and final positions (Figure ~~9-24~~).
2. No external forces act on the system, so the work done by external forces is zero, and no slipping occurs, so no energy is dissipated by kinetic friction. Thus, mechanical energy is constant:

$$W_{ext} = \Delta E_{mech} + \Delta E_{therm}$$
$$0 = \Delta E_{mech} + 0$$

3. Apply conservation of mechanical energy with $U_i = 0$ and $K_f = 0$. Write the total initial kinetic energy K_i in terms of the speed v_{cm} and the angular speed ω_i:

$$U_f + K_f = U_i + K_i$$
$$Mgh + 0 = 0 + \tfrac{1}{2}Mv_{cm\,i}^2 + \tfrac{1}{2}I_{cm}\omega_i^2$$

4. Substitute from $\omega_i = v_{cm\,i}/R$ and $I_{cm} = \frac{2}{5}MR^2$ and solve for h:

$$Mgh = \frac{1}{2}Mv_{cm\,i}^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\frac{v_{cm\,i}^2}{R^2} = \frac{7}{10}Mv_{cm\,i}^2$$

$$\text{so} \quad h = \frac{7v_{cm\,i}^2}{10g} = 0.2854\text{ m} = \boxed{29\text{ cm}}$$

(2)

Sketch a free-body diagram of the ball (Figure ~~9-36~~). We are assuming that friction between the ball and the table is negligible, so do not include this frictional force:

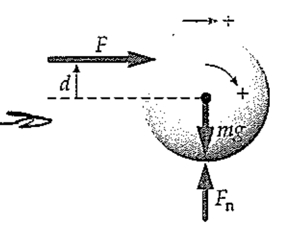


FIGURE 9-36

The torque about the horizontal axis through the center of the ball (and out of the page) equals F times d:

$$\tau = Fd$$

Apply Newton's second law for a system and Newton's second law for rotational motion about the center of the ball:

$$F = ma_{cm} \quad \text{and} \quad \tau = I_{cm}\alpha$$

The nonslip condition relates a_{cm} and α:

$$a_{cm} = R\alpha$$

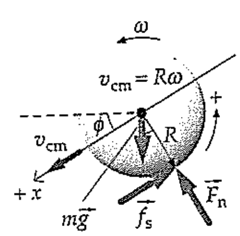
Substitute from steps 2 and 3 into step 4:

$$\frac{F}{m} = R\frac{Fd}{I_{cm}}$$

Find the moment of inertia from Table 9-1 and solve for d:

$$d = \frac{I_{cm}}{mR} = \frac{\frac{2}{5}mR^2}{mR} = \boxed{\frac{2}{5}R}$$

(3)



1. Apply Newton's second law for a system in component form for the x axis:

$$\Sigma F_x = ma_{cm\,x}$$
$$mg\sin\phi - f_s = ma_{cm}$$

2. Apply Newton's second law for rotational motion about a horizontal axis passing through the center of mass and perpendicular to $\vec{v}_{cm}$. The moment arms for the normal and gravitational forces each equal zero, so they do not exert torques on the ball:

$$\Sigma\tau_i = I_{cm}\alpha$$
$$f_sR + 0 + 0 = I_{cm}\alpha$$

3. Relate a_{cm} and α using the nonslip condition:

$$a_{cm} = R\alpha$$

4. We now have three equations and three unknowns. Solve the step-1 result for f_s and the step-3 result for α, substitute for these quantities in the step-2 result, and solve for a_{cm}:

$$(mg\sin\phi - ma_{cm})R = I_{cm}\frac{a_{cm}}{R}$$

$$\text{so} \quad a_{cm} = \frac{g\sin\phi}{1 + \dfrac{I_{cm}}{mR^2}}$$

5. Substitute the step-4 result into the step-1 result and solve for f_s:

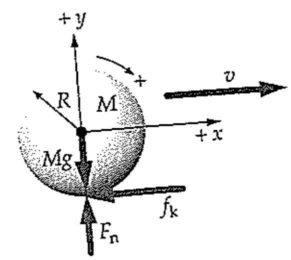
$$f_s = mg\sin\phi - ma_{cm} = mg\sin\phi - \frac{mg\sin\phi}{1 + \dfrac{I_{cm}}{mR^2}} = \frac{mg\sin\phi}{1 + \dfrac{mR^2}{I_{cm}}}$$

6. For a solid sphere, $I_{cm} = \frac{2}{5}mR^2$ (see Table 9-1). Substitute for I_{cm} in the step-4 and step-5 results:

$$a_{cm} = \frac{g\sin\phi}{1 + \frac{2}{5}} = \boxed{\frac{5}{7}g\sin\phi}$$

$$f_s = \frac{mg\sin\phi}{1 + \frac{5}{2}} = \boxed{\frac{2}{7}mg\sin\phi}$$

4



(a) 1. Sketch a free-body diagram of the ball ~~(Figure 9-11).~~

2. The net force on the ball is the force of kinetic friction f_k, which acts in the negative x direction. Apply Newton's second law:

$$\Sigma F_x = Ma_{cm\,x}$$
$$-f_k = Ma_{cm\,x}$$

3. The acceleration is in the negative x direction and $a_{cmy} = 0$. Find f_k by first finding F_n:

$$\Sigma F_y = Ma_{cmy} = 0 \Rightarrow F_n = Mg$$
$$\text{so} \quad f_k = \mu_k F_n = \mu_k Mg$$

4. Find the acceleration using the step-2 and step-3 results:

$$-\mu_k Mg = Ma_{cm\,x} \Rightarrow a_{cm\,x} = -\mu_k g$$

5. Relate the linear velocity to the constant acceleration and the time using a kinematic equation:

$$v_{cm\,x} = v_0 + a_{cm\,x}t = v_0 - \mu_k g t$$

6. Find α by applying Newton's second law for rotational motion to the ball. Compute the torques about the axis through the center of mass. Note that the free-body diagram has clockwise as positive:

$$\Sigma \tau = I_{cm}\alpha$$
$$\mu_k MgR + 0 + 0 = \tfrac{2}{5}MR^2\alpha$$
$$\text{so} \quad \alpha = \frac{5}{2}\frac{\mu_k g}{R}$$

7. Relate the angular velocity to the constant angular acceleration and the time using a kinematic equation:

$$\omega = \omega_0 + \alpha t = 0 + \alpha t = \frac{5}{2}\frac{\mu_k g}{R}t$$

8. Solve for the time t at which $v_{cm} = R\omega$:

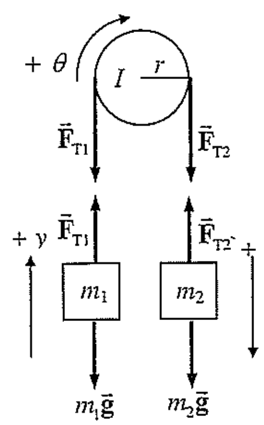
$$v_{cm} = R\omega$$
$$(v_0 - \mu_k g t) = R\left(\frac{5}{2}\frac{\mu_k g}{R}t\right)$$
$$\text{so} \quad t = \frac{2v_0}{7\mu_k g} = \frac{2(5.0\text{ m/s})}{7(0.080)(9.81\text{ m/s}^2)} = \boxed{1.8\text{ s}}$$

(b) The distance traveled while skidding is

$$\Delta x = v_0 t + \tfrac{1}{2}a_{cm}t^2 = v_0\left(\frac{2v_0}{7\mu_k g}\right) + \frac{1}{2}(-\mu_k g)\left(\frac{2v_0}{7\mu_k g}\right)^2 = \frac{12}{49}\frac{v_0^2}{\mu_k g}$$
$$= \frac{12}{49}\frac{(5.0\text{ m/s})^2}{(0.080)(9.81\text{ m/s}^2)} = \boxed{7.8\text{ m}}$$

5

We assume that $m_2 > m_1$, and so m_2 will accelerate down, m_1 will accelerate up, and the pulley will accelerate clockwise. Call the direction of acceleration the positive direction for each object. The masses will have the same acceleration since they are connected by a cord. The rim of the pulley will have that same acceleration since the cord is making it rotate, and so $\alpha_{\text{pulley}} = a/r$. From the free-body diagrams for each object, we have the following.



$$\sum F_{y1} = F_{T1} - m_1 g = m_1 a \quad \rightarrow \quad F_{T1} = m_1 g + m_1 a$$
$$\sum F_{y2} = m_2 g - F_{T2} = m_2 a \quad \rightarrow \quad F_{T2} = m_2 g - m_2 a$$

$$\sum \tau = F_{T2}r - F_{T1}r = I\alpha = I\frac{a}{r}$$

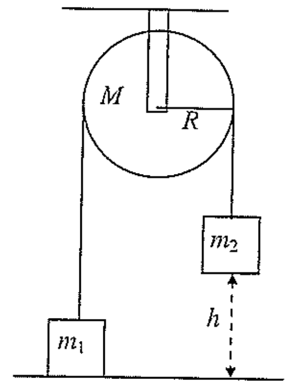
Substitute the expressions for the tensions into the torque equation, and solve for the acceleration.

$$F_{T2}r - F_{T1}r = I\frac{a}{r} \quad \rightarrow \quad (m_2 g - m_2 a)r - (m_1 g + m_1 a)r = I\frac{a}{r} \quad \rightarrow \quad \boxed{a = \frac{(m_2 - m_1)}{(m_1 + m_2 + I/r^2)}g}$$

If the moment of inertia is ignored, then from the torque equation we see that $F_{T2} = F_{T1}$, and the acceleration will be $\boxed{a_{I=0} = \frac{(m_2 - m_1)}{(m_1 + m_2)}g}$. We see that the acceleration with the moment of inertia included will be smaller than if the moment of inertia is ignored.

6

The only force doing work in this system is gravity, so mechanical energy will be conserved. The initial state of the system is the configuration with m_1 on the ground and all objects at rest. The final state of the system has m_2 just reaching the ground, and all objects in motion. Call the zero level of gravitational potential energy to be the ground level. Both masses will have the same speed since they are connected by the rope. Assuming that the rope does not slip on the pulley, the angular speed of the pulley is related to the speed of the masses by $\omega = v/R$. All objects have an initial speed of 0.



$$E_i = E_f$$

$$\tfrac{1}{2}m_1v_i^2 + \tfrac{1}{2}m_2v_i^2 + \tfrac{1}{2}I\omega_i^2 + m_1gy_{1i} + m_2gy_{2i} = \tfrac{1}{2}m_1v_f^2 + \tfrac{1}{2}m_2v_f^2 + \tfrac{1}{2}I\omega_f^2 + m_1gy_{1f} + m_2gy_{2f}$$

$$m_2gh = \tfrac{1}{2}m_1v_f^2 + \tfrac{1}{2}m_2v_f^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\left(\frac{v_f^2}{R^2}\right) + m_1gh$$

$$v_f = \sqrt{\frac{2(m_2 - m_1)gh}{(m_1 + m_2 + \frac{1}{2}M)}} = \sqrt{\frac{2(26.5\text{ kg} - 18.0\text{ kg})(9.80\text{ m/s}^2)(3.00\text{ m})}{(26.5\text{ kg} + 18.0\text{ kg} + (\frac{1}{2})7.50\text{ kg})}} = \boxed{3.22\text{ m/s}}$$

7

(a) Consider the person and platform a system for angular momentum analysis. Since the force and torque to raise and/or lower the arms is internal to the system, the raising or lowering of the arms will cause no change in the total angular momentum of the system. However, the rotational inertia increases when the arms are raised. Since angular momentum is conserved, an increase in rotational inertia must be accompanied by a decrease in angular velocity.

(b) $L_i = L_f \;\rightarrow\; I_i\omega_i = I_f\omega_f \;\rightarrow\; I_f = I_i\dfrac{\omega_i}{\omega_f} = I_i\dfrac{1.30\text{ rev/s}}{0.80\text{ rev/s}} = 1.625\,I_i \approx 1.6\,I_i$

The rotational inertia has increased by a factor of $\boxed{1.6}$.

8

The skater's angular momentum is constant, since no external torques are applied to her.

$$L_i = L_f \;\rightarrow\; I_i\omega_i = I_f\omega_f \;\rightarrow\; I_f = I_i\frac{\omega_i}{\omega_f} = \left(4.6\text{ kg}\cdot\text{m}^2\right)\frac{0.50\text{ rev/s}}{3.0\text{ rev/s}} = \boxed{0.77\text{ kg}\cdot\text{m}^2}$$

She accomplishes this by starting with her arms extended (initial angular velocity) and then $\boxed{\text{pulling her arms in to the center of her body}}$ (final angular velocity).

9

(a) For the daily rotation about its axis, treat the Earth as a uniform sphere, with an angular frequency of one revolution per day.

$$L_{\text{daily}} = I\omega_{\text{daily}} = \left(\tfrac{2}{5}MR_{\text{Earth}}^2\right)\omega_{\text{daily}}$$

$$= \tfrac{2}{5}\left(6.0\times10^{24}\text{kg}\right)\left(6.4\times10^{6}\text{m}\right)^2\left[\left(\frac{2\pi\text{ rad}}{1\text{ day}}\right)\left(\frac{1\text{ day}}{86{,}400\text{ s}}\right)\right] = \boxed{7.1\times10^{33}\text{ kg}\cdot\text{m}^2/\text{s}}$$

(b) For the yearly revolution about the Sun, treat the Earth as a particle, with an angular frequency of one revolution per year.

$$L_{\text{daily}} = I\omega_{\text{daily}} = \left(MR_{\text{Sun-Earth}}^2\right)\omega_{\text{daily}}$$

$$= \left(6.0\times10^{24}\text{kg}\right)\left(1.5\times10^{11}\text{m}\right)^2\left[\left(\frac{2\pi\text{ rad}}{365\text{ day}}\right)\left(\frac{1\text{ day}}{86{,}400\text{ s}}\right)\right] = \boxed{2.7\times10^{40}\text{ kg}\cdot\text{m}^2/\text{s}}$$

(10)

For our crude estimate, we model the hurricane as a rigid cylinder of air. Since the "cylinder" is rigid, each part of it has the same angular velocity. The mass of the air is the product of the density of air times the volume of the air cylinder.

$$M = \rho V = \rho\pi R^2 h = \left(1.3\,\text{kg/m}^3\right)\pi\left(1.00\times10^5\,\text{m}\right)^2\left(4.0\times10^3\,\text{m}\right) = 1.634\times10^{14}\,\text{kg}$$

(a) $KE = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(v_{\text{edge}}/R\right)^2 = \frac{1}{4}Mv_{\text{edge}}^2$

$$= \tfrac{1}{4}\left(1.634\times10^{14}\,\text{kg}\right)\left[\left(120\,\text{km/h}\right)\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2 = 4.539\times10^{16}\,\text{J} \approx \boxed{5\times10^{16}\,\text{J}}$$

(b) $L = I\omega = \left(\frac{1}{2}MR^2\right)\left(v_{\text{edge}}/R\right) = \frac{1}{2}MRv_{\text{edge}}$

$$= \tfrac{1}{2}\left(1.634\times10^{14}\,\text{kg}\right)\left(1.00\times10^5\,\text{m}\right)\left[\left(120\,\text{km/h}\right)\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right] = 2.723\times10^{20}\,\text{kg}\cdot\text{m}^2/\text{s}$$

$$\approx \boxed{3\times10^{20}\,\text{kg}\cdot\text{m}^2/\text{s}}$$

(11)

Angular momentum will be conserved in the Earth – asteroid system, since all forces and torques are internal to the system. The initial angular velocity of the satellite, just before collision, can be found from $\omega_{\text{asteroid}} = v_{\text{asteroid}}/R_{\text{Earth}}$. Assuming the asteroid becomes imbedded in the Earth at the surface, the Earth and the asteroid will have the same angular velocity after the collision. We model the Earth as a uniform sphere, and the asteroid as a point mass.

$$L_i = L_f \;\rightarrow\; I_{\text{Earth}}\omega_{\text{Earth}} + I_{\text{asteroid}}\omega_{\text{asteroid}} = \left(I_{\text{Earth}} + I_{\text{asteroid}}\right)\omega_f$$

The moment of inertia of the satellite can be ignored relative to that of the Earth on the right side of the above equation, and so the percent change in Earth's angular velocity is found as follows.

$$I_{\text{Earth}}\omega_{\text{Earth}} + I_{\text{asteroid}}\omega_{\text{asteroid}} = I_{\text{Earth}}\omega_f \;\rightarrow\; \frac{\left(\omega_f - \omega_{\text{Earth}}\right)}{\omega_{\text{Earth}}} = \frac{I_{\text{asteroid}}}{I_{\text{Earth}}}\frac{\omega_{\text{asteroid}}}{\omega_{\text{Earth}}}$$

(12)

Since the lost mass carries away no angular momentum, the angular momentum of the remaining mass will be the same as the initial angular momentum.

$$L_i = L_f \;\rightarrow\; I_i\omega_i = I_f\omega_f$$

$$\frac{\omega_f}{\omega_i} = \frac{I_i}{I_f} = \frac{\frac{2}{5}M_iR_i^2}{\frac{2}{5}M_fR_f^2} = \frac{\left(8.0M_{\text{Sun}}\right)\left(6.96\times10^8\,\text{m}\right)^2}{(0.25)\left(8.0M_{\text{Sun}}\right)\left(1.1\times10^4\,\text{m}\right)^2} = 1.601\times10^{10}$$

$$\omega_f = 1.601\times10^{10}\,\omega_i = 1.601\times10^{10}\left(\frac{1\,\text{rev}}{12\,\text{day}}\right) = 1.334\times10^9\,\text{rev/day}$$

$$\approx \boxed{1.3\times10^9\,\text{rev/day} = 1.5\times10^4\,\text{rev/s}}$$

3)

(a) Conservation of angular momentum relates the final speed to the initial speed and the initial and final radii:

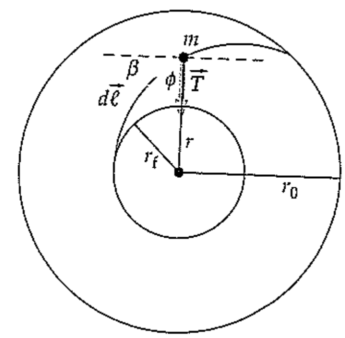
$$L_f = L_0$$
$$mv_f r_f = mv_0 r_0$$
$$\text{so} \quad v_f = \boxed{\frac{r_0}{r_f} v_0}$$

(b) 1. Apply Newton's second law to relate T to v and r. Because the particle is being pulled in slowly, the acceleration is virtually the same as if the particle were moving in a circle:

$$T \approx m\frac{v^2}{r}$$

2. Obtain a relation between L, r, and v using the definition of angular momentum. Because the particle is being pulled in slowly, $|\beta| \ll 1$ (Figure 10-32a):

$$\vec{L} = \vec{r} \times \vec{p}$$
$$L = rmv\cos\beta \approx rmv \quad (|\beta| \ll 1, \text{so } \cos\beta \approx 1)$$



(a)

3. Eliminate v by solving the Part-(b) step-2 result for v and then substituting into the Part-(b) step-1 result:

$$T = m\frac{v^2}{r} = \frac{m}{r}\left(\frac{L}{mr}\right)^2 = \boxed{\frac{L^2}{mr^3}}$$

(c) 1. Make a drawing of the particle as it moves closer to the hole (Figure 10-32b). When the particle undergoes displacement $d\vec{\ell}$, its distance r from the axis changes by dr. Because r is decreasing, dr is negative. Thus:

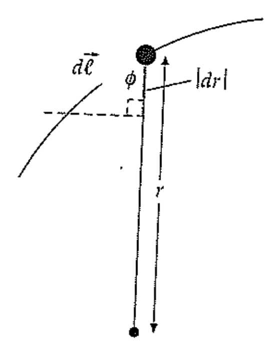
$$dr = -|dr|$$

2. Write $dW = \vec{T} \cdot d\vec{\ell}$ in terms of T and dr:

$$dW = \vec{T} \cdot d\vec{\ell} = T\,d\ell\cos\phi$$

Because $|dr| = d\ell\cos\phi$,

$$dW = T|dr| = -T\,dr$$



(b)

3. Integrate from r_0 to r_f after substituting for T from the Part-(b) step-3 result:

$$W = -\int_{r_0}^{r_f} T\,dr = -\int_{r_0}^{r_f} \frac{L^2}{mr^3}\,dr$$
$$= -\frac{L^2}{m}\int_{r_0}^{r_f} r^{-3}\,dr = -\frac{L^2}{m}\frac{r^{-2}}{-2}\bigg|_{r_0}^{r_f}$$
$$= \boxed{\frac{L^2}{2m}\left(\frac{1}{r_f^2} - \frac{1}{r_0^2}\right)}$$

(14)

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} \cdot \vec{B} = 3\hat{j} \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$= 3B_x \hat{j} \cdot \hat{i} + 3B_y \hat{j} \cdot \hat{j} + 3B_z \hat{j} \cdot \hat{k}$$

$$= 3B_y = 12 \Rightarrow B_y = 4$$

$$\vec{A} \times \vec{B} = 3\hat{j} \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$= 3B_x \hat{j} \times \hat{i} + 3B_y \hat{j} \times \hat{j} + 3B_z \hat{j} \times \hat{k}$$

$$= 3B_z \hat{i} - 3B_x \hat{k}$$

$$= 9\hat{i}$$

so $B_z = 3$ and $B_x = 0$

$$\therefore \vec{B} = 0\hat{i} + 4\hat{j} + 3\hat{k}$$