PROBLEMS 6

(1)

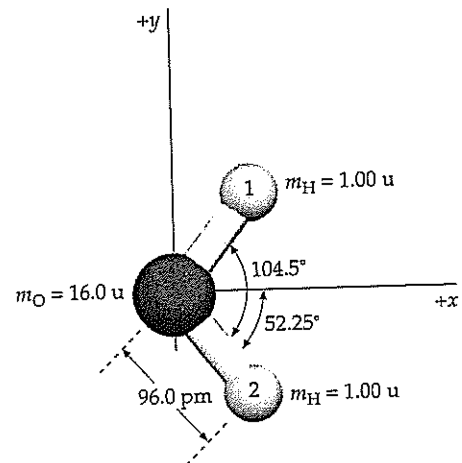


FIGURE 5-42

1. The location of the center of mass is given by its coordinates, x_{cm} and y_{cm} (Equations 5-15 and 5-16):

$$x_{cm} = \frac{\sum m_i x_i}{M}, \quad y_{cm} = \frac{\sum m_i y_i}{M}$$

2. Writing these out explicitly gives:

$$x_{cm} = \frac{m_{H1}x_{H1} + m_{H2}x_{H2} + m_O x_O}{m_{H1} + m_{H2} + m_O}$$

$$y_{cm} = \frac{m_{H1}y_{H1} + m_{H2}y_{H2} + m_O y_O}{m_{H1} + m_{H2} + m_O}$$

3. We have chosen the origin to be the location of the oxygen atom, so both the x and y coordinates of the oxygen atom are zero. The x and y coordinates of the hydrogen atoms are calculated from the 52.25° angle each hydrogen makes with the x axis:

$$x_O = y_O = 0$$
$$x_{H1} = 96.0 \text{ pm} \cos 52.25° = 58.8 \text{ pm}$$
$$x_{H2} = 96.0 \text{ pm} \cos(-52.52°) = 58.8 \text{ pm}$$
$$y_{H1} = 96.0 \text{ pm} \sin 52.25° = 75.9 \text{ pm}$$
$$y_{H2} = 96.0 \text{ pm} \sin(-52.25°) = -75.9 \text{ pm}$$

4. Substituting the coordinate and mass values into step 2 gives x_{cm}:

$$x_{cm} = \frac{(1.00 \text{ u})(58.8 \text{ pm}) + (1.00 \text{ u})(58.8 \text{ pm}) + (16.0 \text{ u})(0)}{1.00 \text{ u} + 1.00 \text{ u} + 16.0 \text{ u}} = 6.53 \text{ pm}$$

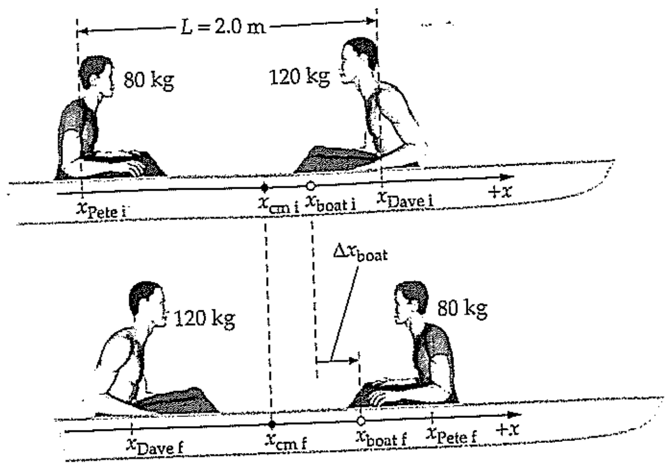
$$x_{cm} = \frac{(1.00 \text{ u})(75.9 \text{ pm}) + (1.00 \text{ u})(-75.9 \text{ pm}) + (16.0 \text{ u})(0)}{1.00 \text{ u} + 1.00 \text{ u} + 16.0 \text{ u}} = 0.00 \text{ pm}$$

5. The center of mass is on the x axis:

$$\vec{r}_{cm} = x_{cm}\hat{i} + y_{cm}\hat{j} = \boxed{6.53 \text{ pm } \hat{i} + 0.00\hat{j}}$$

(2) See slides

3



[redacted] Pete and Dave changing places viewed from the reference frame of the water. The blue dot is the center of mass of the boat and the black dot is the center of mass of the Pete–Dave–boat system.

1. Make a sketch of the system in its initial and final configurations (Figure 5-52). Let $L = 2.0$ m and let $d = \Delta x_{\text{boat}}$, the distance the boat moves forward when Pete and Dave switch places:

2. Flesh out $Mx_{\text{cm}} = \sum m_i x_i$ both before and after Pete and Dave change places. The coordinate axis measures positions in the reference frame of the water:

$$Mx_{\text{cm i}} = m_{\text{Pete}}x_{\text{Pete i}} + m_{\text{Dave}}x_{\text{Dave i}} + m_{\text{boat}}x_{\text{boat i}}$$

and

$$Mx_{\text{cm f}} = m_{\text{Pete}}x_{\text{Pete f}} + m_{\text{Dave}}x_{\text{Dave f}} + m_{\text{boat}}x_{\text{boat f}}$$

3. Subtract the third step-2 equation from the second step-2 equation. Then substitute 0 for Δx_{cm}, $d + L$ for Δx_{Pete}, $d - L$ for Δx_{Dave} and d for Δx_{boat}:

$$M\Delta x_{\text{cm}} = m_{\text{Pete}}\,\Delta x_{\text{Pete}} + m_{\text{Dave}}\,\Delta x_{\text{Dave}} + m_{\text{boat}}\,\Delta x_{\text{boat}}$$

$$0 = m_{\text{Pete}}(d + L) + m_{\text{Dave}}(d + L) + m_{\text{boat}}d$$

4. Solve for d:

$$d = \frac{(m_{\text{Dave}} - m_{\text{Pete}})}{m_{\text{Dave}} + m_{\text{Pete}} + m_{\text{boat}}}L = \frac{(120\text{ kg} - 80\text{ kg})}{120\text{ kg} + 80\text{ kg} + 60\text{ kg}}(2.0\text{ m}) = \boxed{0.31\text{ m}}$$

(4)

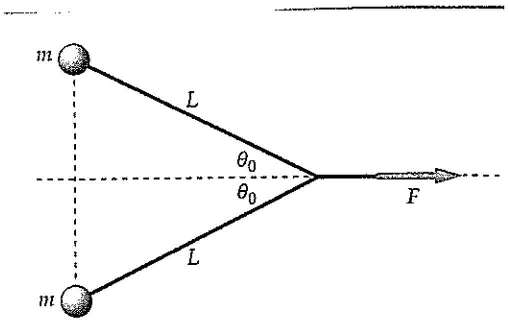


FIGURE 6-25

1. Make a drawing showing the system initially, and after it has moved distance d (Figure 6-26):

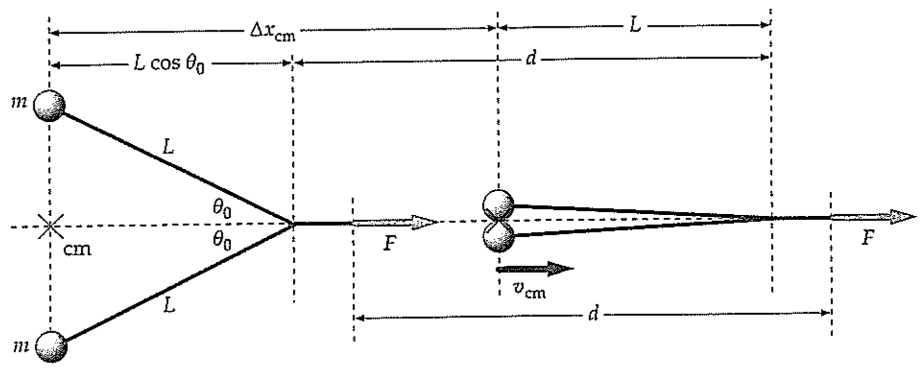


FIGURE 6-26 As the center of mass moves the distance Δx_{cm}, the point of application of the force $\vec{F}$ moves the distance d.

2. Apply the center-of-mass-work–translational-kinetic-energy relation to the system. The net force on the system is $\vec{F} = F\hat{i}$:

$$\int_i^f \vec{F}_{\text{net ext}} \cdot d\vec{\ell}_{cm} = \Delta K_{trans}$$

$$\int_i^f F\hat{i} \cdot dx_{cm}\hat{i} = K_{trans\,f} - K_{trans\,i}$$

$$F\int_i^f dx_{cm} = K_{trans\,f} - 0$$

$$F\Delta x_{cm} = \tfrac{1}{2}(2m)v_{cm}^2 = mv_{cm}^2$$

3. Find Δx_{cm} in terms of d and L. Figure 6-26 makes the calculation of Δx_{cm} fairly straightforward:

$$\Delta x_{cm} + L = L\cos\theta_0 + d$$

$$\text{so}\quad \Delta x_{cm} = d - L(1 - \cos\theta_0)$$

4. Substitute the step-3 result into the step-2 result and solve for v_{cm}:

$$F\Delta x_{cm} = mv_{cm}^2$$

$$F[d - L(1 - \cos\theta_0)] = mv_{cm}^2$$

$$\text{so}\quad v_{cm} = \boxed{\sqrt{\frac{F[d - L(1 - \cos\theta_0)]}{m}}}$$

(5)

(a) Assume that there are no non-conservative forces on the rock, and so its mechanical energy is conserved. Subscript 1 represents the rock as it leaves the volcano, and subscript 2 represents the rock at its highest point. The location as the rock leaves the volcano isthe zero location for PE $(y = 0)$. We have $y_1 = 0$, $y_2 = 500\text{ m}$, and $v_2 = 0$. Solve for v_1.

$$E_1 = E_2 \rightarrow \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \rightarrow \tfrac{1}{2}mv_1^2 = mgy_2 \rightarrow$$

$$v_1 = \sqrt{2gy_2} = \sqrt{2(9.80\text{ m/s}^2)(500\text{ m})} = 98.99\text{ m/s} \approx \boxed{1\times10^2\text{ m/s}}$$

(b) The power output is the energy transferred to the launched rocks perunit time. The launching energy of a single rock is $\tfrac{1}{2}mv_1^2$, and so the energy of 1000 rocks is $1000\left(\tfrac{1}{2}mv_1^2\right)$. Divide this energy by the time it takes to launch 1000 rocks to find the power output needed to launch the rocks.

$$P = \frac{1000\left(\tfrac{1}{2}mv_1^2\right)}{t} = \frac{500(500\text{ kg})(98.99\text{ m/s})^2}{60\text{ sec}} = \boxed{4\times10^7\text{ W}}$$

6

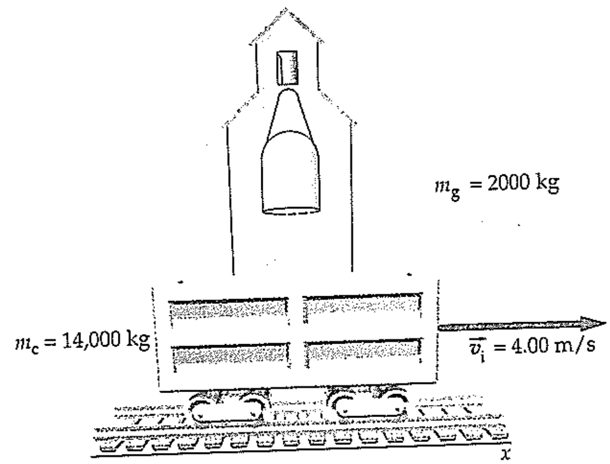


FIGURE 8-2

1. The time for the car to travel from the elevator to the yard is related to the distance to the yard d and the car's speed v_{fx} following the grain dump. We are looking for this time:

$$d = v_{fx}\,\Delta t$$

2. Sketch a free-body diagram (FBD) of the system consisting of the car, the grain already in the car, and the grain that is falling into the car (Figure 8-3). Include coordinate axes:

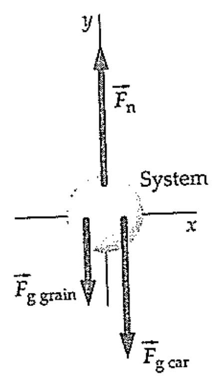


FIGURE 8-3 Three forces act on the system: the gravitational forces on the grain and the car, and the normal force of the track on the car.

3. The sum of the external forces acting on the grain–car system equals the rate of change of the momentum of the system (Equation 8-4):

$$\sum \vec{F}_{i\,\mathrm{ext}} = \vec{F}_{g\,\mathrm{grain}} + \vec{F}_{g\,\mathrm{car}} + \vec{F}_n = \frac{d\vec{P}_{\mathrm{sys}}}{dt}$$

4. Each of the external forces is vertical, so the x component of each is zero. Take the x component of each term in the step-3 result. The x component of the net external force is zero, so $P_{\mathrm{sys}\,x}$ is constant:

$$F_{g\,\mathrm{grain}\,x} + F_{g\,\mathrm{car}\,x} + F_{nx} = \frac{dP_{\mathrm{sys}\,x}}{dt}$$

$$0 + 0 + 0 = \frac{dP_{\mathrm{sys}\,x}}{dt}$$

$$\therefore\ P_{\mathrm{sys\,f}x} = P_{\mathrm{sys\,i}x}$$

5. Make a sketch of the system before the collision and again after the collision (Figure 8-4):

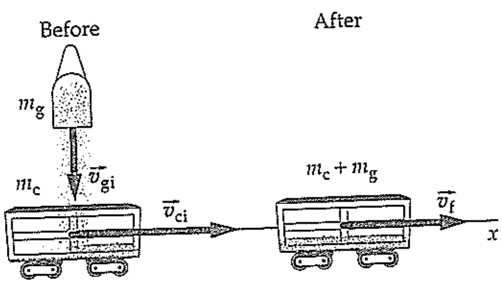


FIGURE 8-4

6. Apply conservation of momentum to relate the final velocity v_{fx} to the initial velocity v_{ix}. The x component of the system's momentum is conserved:

$$P_{\mathrm{sys\,f}x} = P_{\mathrm{sys\,i}}$$

$$(m_c + m_g)v_{fx} = m_c v_{ix} + m_g(0)$$

7. Solve for v_{fx}:

$$v_{fx} = \frac{m_c}{m_c + m_s} v_{ix}$$

8. Substitute the result for v_{fx} into step 1 and solve for the time:

$$\Delta t = \frac{d}{v_{fx}} = \frac{(m_c + m_s)d}{m_c v_{ix}}$$

$$= \frac{(14\,000\ \mathrm{kg} + 2000\ \mathrm{kg})(500\ \mathrm{m})}{(14\,000\ \mathrm{kg})(4.00\ \mathrm{m/s})}$$

$$= \boxed{1.43 \times 10^2\ \mathrm{s}}$$

7

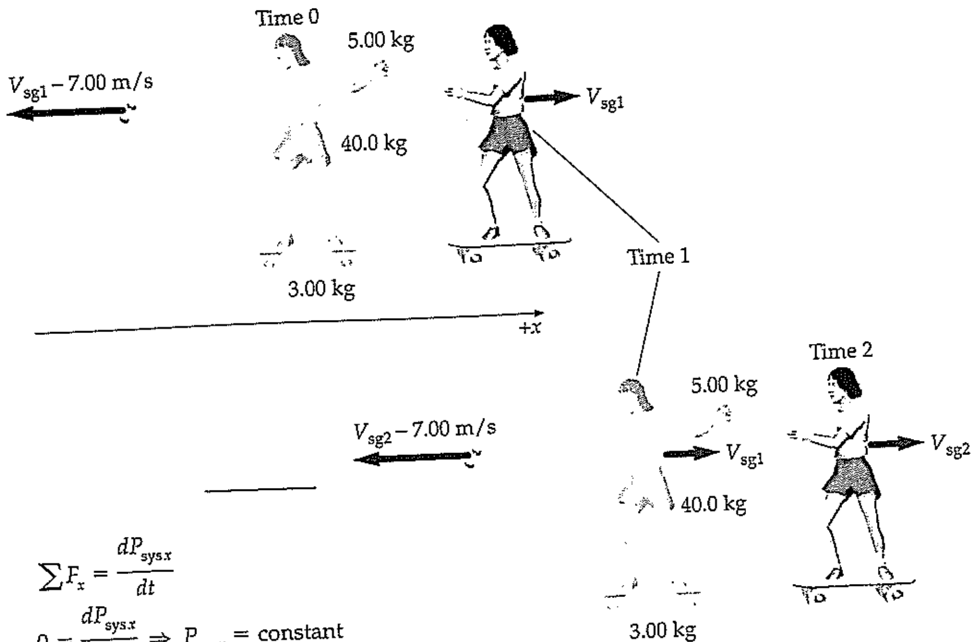


FIGURE 8-5 The numbers in the subscripts stand for times. Time 0 occurs just before the first throw, time 1 occurs between the two throws, and time 2 occurs following the second throw.

(*a*) 1. Let V_{sg1x} and v_{ws1x} be the x components of the velocities of the skateboarder and the thrown weight relative to the ground, respectively. Apply conservation of momentum for the first throw:

$$\sum F_x = \frac{dP_{sysx}}{dt}$$

$$0 = \frac{dP_{sysx}}{dt} \Rightarrow P_{sysx} = \text{constant}$$

so $P_{sys1x} = P_{sys0x}$

$$(M + m)V_{sg1x} + mv_{wg1x} = 0$$

2. The velocity of the thrown weight relative to the ground equals the velocity of the weight relative to the skateboarder plus the velocity of the skateboarder relative to the ground:

$$v_{wg1x} = v_{ws1x} + V_{sg1x}$$

3. Substitute for v_{wg1x} into the step-1 result and solve for V_{sg1x}:

$$(M + m)V_{sg1x} + m(v_{ws1x} + V_{sg1x}) = 0$$

so $$V_{sg1x} = -\frac{m}{M + 2m}v_{ws1x}$$

$$= -\frac{5.00\text{ kg}}{43.0\text{ kg} + 10.0\text{ kg}}(-7.00\text{ m/s}) = \boxed{0.660\text{ m/s}}$$

(*b*) 1. Repeat step 1 of Part (*a*) for the second throw. Let V_{sg2x} and $v_{w'g2x}$ be the x components of the respective velocities of the skateboarder and the second thrown weight relative to the ground:

$$P_{sys2x} = P_{sys1x}$$

$$MV_{sg2x} + mv_{w'g2x} = (M + m)V_{sg1x}$$

2. Repeat step 2 of Part (*a*) for the second throw.

$$v_{w'g2x} = v_{w's2x} + V_{sg2x}$$

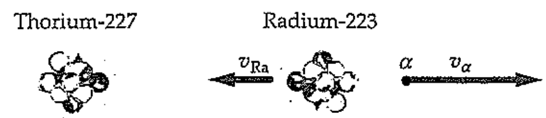
3. Substitute for $v_{w'g2x}$ in the Part-(*b*) step-1 result and solve for V_{sg2x}:

$$MV_{sg2x} + m(v_{w's2x} + V_{sg2x}) = (M + m)V_{sg1x}$$

so $$V_{sg2x} = \frac{(M + m)V_{sg1x} - mv_{w's2x}}{M + m} = V_{sg1x} - \frac{m}{M + m}v_{w's2x}$$

$$= 0.660\text{ m/s} - \frac{5.00\text{ kg}}{48.0\text{ kg}}(-7.00\text{ m/s}) = \boxed{1.39\text{ m/s}}$$

8



SOLVE

1. Write the kinetic energy of the radium nucleus K_{ra} in terms of its mass m_{ra} and speed v_{ra}.
   $K_{ra} = \frac{1}{2}m_{ra}v_{ra}^2$
2. Write the kinetic energy of the alpha particle K_α in terms of its mass m_α and speed v_α.
   $K_\alpha = \frac{1}{2}m_\alpha v_\alpha^2$
3. Use conservation of momentum to relate v_{ra} to v_α. The thorium nucleus was at rest, so the momentum of the system is zero:
   $m_\alpha v_\alpha = m_{ra}v_{ra}$
4. Solve the step-1 and step 2-results for the speeds v_{ra} and v_α, and substitute these expressions into the step-3 result.
   $K_{ra} = \frac{1}{2}m_{ra}v_{ra}^2 \qquad K_\alpha = \frac{1}{2}m_\alpha v_\alpha^2$
   $v_{ra} = \left(\frac{2K_{ra}}{m_{ra}}\right)^{1/2} \qquad v_\alpha = \left(\frac{2K_\alpha}{m_\alpha}\right)^{1/2}$
   so $m_\alpha\left(\frac{2K_\alpha}{m_\alpha}\right)^{1/2} = m_{ra}\left(\frac{2K_{ra}}{m_{ra}}\right)^{1/2}$
5. Solve the step-4 result for K_{ra}.
   $K_{ra} = \frac{m_\alpha}{m_{ra}}K_\alpha = \frac{4.00\text{ u}}{223\text{ u}}(6.00\text{ MeV}) = \boxed{0.107\text{ MeV}}$

9

(*a*) For a perfectly elastic collision, [redacted] $v_A - v_B = -(v_A' - v_B')$. Substitute that into the coefficient of restitution definition.

$$e = \frac{v_A' - v_B'}{v_B - v_A} = -\frac{(v_A - v_B)}{v_B - v_A} = 1.$$

For a completely inelastic collision, $v_A' = v_B'$. Substitute that into the coefficient of restitution definition.

$$e = \frac{v_A' - v_B'}{v_B - v_A} = 0.$$

(*b*) Let A represent the falling object, and B represent the heavy steel plate. The speeds of the steel plate are $v_B = 0$ and $v_B' = 0$. Thus $e = -v_A'/v_A$. Consider energy conservation during the falling or rising path. The potential energy of body A at height h is transformed into kinetic energy just before it collides with the plate. Choose down to be the positive direction.

$$mgh = \tfrac{1}{2}mv_A^2 \quad \rightarrow \quad v_A = \sqrt{2gh}$$

The kinetic energy of body A immediately after the collision is transformed into potential energy as it rises. Also, since it is moving upwards, it has a negative velocity.

$$mgh' = \tfrac{1}{2}mv_A'^2 \quad \rightarrow \quad v_A' = -\sqrt{2gh'}$$

Substitute the expressions for the velocities into the definition of the coefficient of restitution.

$$e = -v_A'/v_A = -\frac{-\sqrt{2gh'}}{\sqrt{2gh}} \quad \rightarrow \quad \boxed{e = \sqrt{h'/h}}$$

10

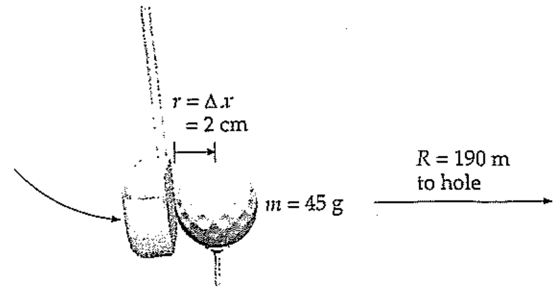


FIGURE 8-12

(a) 1. Set the impulse equal to the change in momentum of the ball:

$$I_x = F_{avx}\,\Delta t = \Delta p_x$$

2. Make a sketch showing the ball in both the pre- and postcollision positions (Figure 8-13):

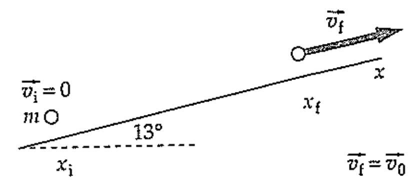


FIGURE 8-13

3. The speed v_f immediately after the collision is related to the range R, which is given by $R = (v_0^2/g)\sin 2\theta_0$ (Equation 2-23) with v_0 equal to the post-collision speed v_f:

$$R = \frac{v_f^2}{g}\sin 2\theta_0$$

4. Take $\theta_0 = 13°$ and calculate the initial speed for the projectile motion:

$$v_0 = \sqrt{\frac{Rg}{\sin 2\theta_0}} = \sqrt{\frac{(190\text{ m})(9.81\text{ m/s}^2)}{\sin 26°}} = 65.2\text{ m/s}$$

5. Use this value of v_0 to calculate the impulse:

$$I_x = \Delta p_x = m(v_{0x} - 0) = (0.045\text{ kg})(65.2\text{ m/s})$$
$$= 2.93\text{ kg}\cdot\text{m/s} = \boxed{2.90\text{ N}\cdot\text{s}}$$

(b) Calculate the collision time Δt using $\Delta x = 2.0$ cm and $v_{avx} = \frac{1}{2}(v_{ix} + v_{fx})$:

$$\Delta t = \frac{\Delta x}{v_{avx}} = \frac{\Delta x}{\frac{1}{2}(0 + v_0)} = \frac{0.020\text{ m}}{\frac{1}{2}(65.2\text{ m/s})}$$
$$= 6.13 \times 10^{-4}\text{ s} = \boxed{6.1 \times 10^{-4}\text{ s}}$$

(c) Use the calculated values of I_x and Δt to find the magnitude of the average force:

$$F_{av} = F_{avx} = \frac{I_x}{\Delta t} = \frac{2.93\text{ N}\cdot\text{s}}{6.13 \times 10^{-4}\text{ s}} = 4.78\text{ kN} = \boxed{4.8\text{ kN}}$$

11

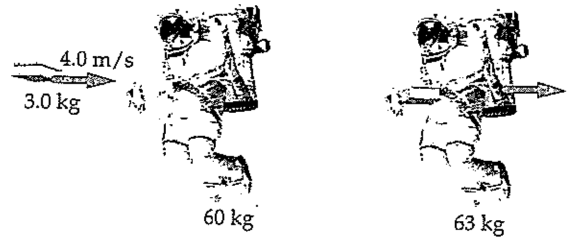


FIGURE 8-16

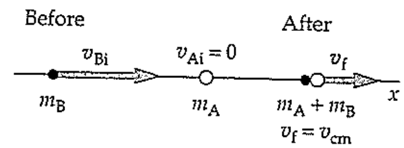


FIGURE 8-17

(a) 1. Make a drawing (Figure 8-17) showing the objects just before and just after the catch. Let the direction you throw the book be the $+x$ direction:

2. Use conservation of momentum to relate the final velocity of the system v_f to the initial velocities:

$$m_B v_{Bi} + m_A v_{Ai} = (m_A + m_B) v_f$$

3. Solve for v_f:

$$v_f = \frac{m_B v_B + m_A v_A}{m_B + m_A} = \frac{(3.0\,\text{kg})(4.0\,\text{m/s}) + (60\,\text{kg})(0\,\text{m/s})}{3.0\,\text{kg} + 60\,\text{kg}}$$
$$= 0.190\,\text{m/s} = \boxed{0.19\,\text{m/s}}$$

(b) 1. Because the astronaut is initially at rest, the initial kinetic energy of the book–astronaut system is the initial kinetic energy of the book:

$$K_{\text{sys i}} = K_{Bi} = \tfrac{1}{2} m_B v_{Bi}^2 = \tfrac{1}{2}(3.0\,\text{kg})(4.0\,\text{m/s})^2 = \boxed{24\,\text{J}}$$

2. The final kinetic energy is the kinetic energy of the book and astronaut moving together at v_f:

$$K_{\text{sys f}} = \tfrac{1}{2}(m_B + m_A) v_f^2 = 2(63\,\text{kg})(0.190\,\text{m/s})^2 = 1.14\,\text{J} = \boxed{1.1\,\text{J}}$$

(c) Set the impulse exerted on the astronaut equal to the change in momentum of the astronaut:

$$I_{\text{by B on A}} = \Delta p_A = m_A \Delta v_A = (60\,\text{kg})(0.190\,\text{m/s} - 0)$$
$$= 11.4\,\text{kg}\cdot\text{m/s} = \boxed{11\,\text{N}\cdot\text{s}}$$

12

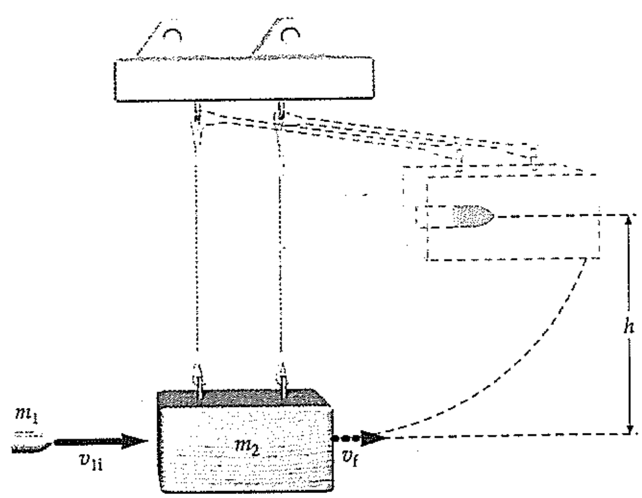


FIGURE 8-18

1. Using conservation of mechanical energy *after* the collision, we relate the postcollision speed v_f to the maximum height h:

$$\tfrac{1}{2}(m_1 + m_2) v_f^2 = (m_1 + m_2) g h$$
$$v_f = \sqrt{2gh}$$

2. Using conservation of momentum *during* the collision we relate velocities v_{1i} and v_f:

$$m_1 v_{1i} = (m_1 + m_2) v_f$$
$$v_{1i} = \frac{m_1 + m_2}{m_1} v_f$$

3. Substituting for v_f in the step-2 result, we can solve for v_{1i}:

$$v_{1i} = \frac{m_1 + m_2}{m_1} v_f = \boxed{\frac{m_1 + m_2}{m_1}\sqrt{2gh}}$$

(13)

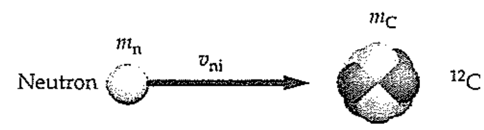


FIGURE 8-24

1. Use conservation of momentum to obtain one relation for the final velocities: $m_n v_{ni} = m_n v_{nf} + m_C v_{Cf}$

2. Use Equation 8-23 to equate the speeds of recession and approach:
$$v_{Cf} - v_{nf} = v_{ni} - v_{Ci} = v_{ni} - 0$$
so $v_{Cf} = v_{ni} + v_{nf}$

3. To eliminate v_{Cf}, substitute the expression for v_{Cf} from step 2 into the step-1 result: $m_n v_{ni} = m_n v_{nf} + m_C(v_{ni} + v_{nf})$

4. Solve for v_{nf}:
$$v_{nf} = \boxed{-\frac{m_C - m_n}{m_n + m_C} v_{ni}}$$

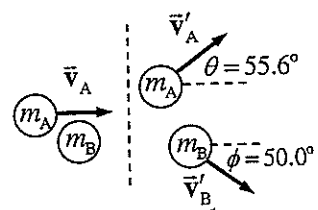
5. Substitute the step-4 result into the step-2 result and solve for v_{Cf}:
$$v_{Cf} = v_{ni} - \frac{m_C - m_n}{m_n + m_C} v_{ni} = \boxed{\frac{2m_n}{m_n + m_C} v_{ni}}$$

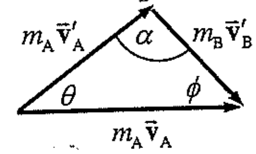
1. The collision is elastic, so the kinetic energy lost by the neutron is the final kinetic energy of the carbon nucleus:
$$f = \frac{-\Delta K_n}{K_{ni}} = \frac{K_{Cf}}{K_{ni}} = \frac{\frac{1}{2}m_C v_{Cf}^2}{\frac{1}{2}m_n v_{ni}^2} = \frac{m_C}{m_n}\left(\frac{v_{Cf}}{v_{ni}}\right)^2$$

2. Solve the Part-(*a*) step-5 result for the ratio of the velocities; substitute into the Part-(*b*) step-1 result, and solve for the fractional energy loss of the neutron:
$$f = \frac{m_C}{m_n}\left(\frac{2m_n}{m_n + m_C}\right)^2 = \boxed{\frac{4m_n m_C}{(m_n + m_C)^2}}$$

(14)

Let A represent the incoming neon atom, and B represent the target atom. A momentum diagram of the collision looks like the first figure. The figure can be re-drawn as a triangle, the second figure, since $m_A \vec{v}_A = m_A \vec{v}'_A + m_B \vec{v}'_B$. Write the law of sines for this triangle, relating each final momentum magnitude to the initial momentum magnitude.





$$\frac{m_A v'_A}{m_A v_A} = \frac{\sin\phi}{\sin\alpha} \rightarrow v'_A = v_A \frac{\sin\phi}{\sin\alpha}$$

$$\frac{m_B v'_B}{m_A v_A} = \frac{\sin\theta}{\sin\alpha} \rightarrow v'_B = v_A \frac{m_A}{m_B}\frac{\sin\theta}{\sin\alpha}$$

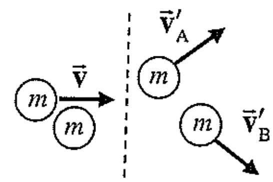
The collision is elastic, so write the KE conservation equation, and substitute the results from above. Also note that $\alpha = 180.0 - 55.6° - 50.0° = 74.4°$

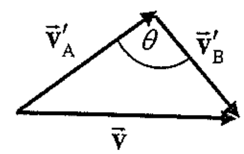
$$\tfrac{1}{2}m_A v_A^2 = \tfrac{1}{2}m_A v_A'^2 + \tfrac{1}{2}m_B v_B'^2 \rightarrow m_A v_A^2 = m_A\left(v_A \frac{\sin\phi}{\sin\alpha}\right)^2 + m_B\left(v_A \frac{m_A}{m_B}\frac{\sin\theta}{\sin\alpha}\right)^2 \rightarrow$$

$$m_B = \frac{m_A \sin^2\theta}{\sin^2\alpha - \sin^2\phi} = \frac{(20.0\text{ u})\sin^2 55.6°}{\sin^2 74.4 - \sin^2 50.0°} = \boxed{39.9\text{ u}}$$

(15)

In an elastic collision between two objects of equal mass, with the target object initially stationary, the angle between the final velocities of the objects is 90°. Here is a proof of that fact. Momentum conservation as a vector relationship says $m\vec{v} = m\vec{v}'_A + m\vec{v}'_B \rightarrow \vec{v} = \vec{v}'_A + \vec{v}'_B$. Kinetic energy conservation says $\frac{1}{2}mv^2 = \frac{1}{2}mv'^2_A + \frac{1}{2}mv'^2_B \rightarrow v^2 = v'^2_A + v'^2_B$. The vector equation resulting from momentum conservation can be illustrated by the second diagram. Apply the law of cosines to that triangle of vectors, and then equate the two expressions for v^2.



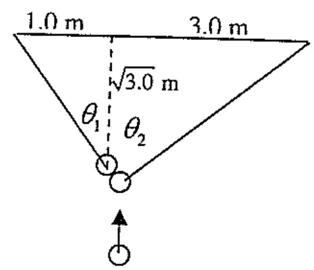


$$v^2 = v'^2_A + v'^2_B - 2v'_A v'_B \cos\theta$$

Equating the two expressions for v^2 gives

$$v'^2_A + v'^2_B - 2v'_A v'_B \cos\theta = v'^2_A + v'^2_B \rightarrow \cos\theta = 0 \rightarrow \theta = 90°$$

For this specific circumstance, see the third diagram. We assume that the target ball is hit "correctly" so that it goes in the pocket. Find θ_1 from the geometry of the "left' triangle: $\theta_1 = \tan^{-1}\frac{1.0}{\sqrt{3.0}} = 30°$. Find θ_2 from the geometry of the "right" triangle: $\theta_2 = \tan^{-1}\frac{3.0}{\sqrt{3.0}} = 60°$. Since the balls will separate at a 90° angle, if the target ball goes in the pocket, this does appear to be a good possibility of a scratch shot.



(16)

Write momentum conservation in the x and y directions, and KE conservation. Note that both masses are the same. We allow $\vec{v}_A$ to have both x and y components.

$$p_x:\quad mv_B = mv'_{Ax} \rightarrow v_B = v'_{Ax}$$

$$p_y:\quad mv_A = mv'_{Ay} + mv'_B \rightarrow v_A = v'_{Ay} + v'_B$$

$$KE:\quad \tfrac{1}{2}mv_A^2 + \tfrac{1}{2}mv_B^2 = \tfrac{1}{2}mv'^2_A + \tfrac{1}{2}mv'^2_B \rightarrow v_A^2 + v_B^2 = v'^2_A + v'^2_B$$

Substitute the results from the momentum equations into the KE equation.

$$(v'_{Ay} + v'_B)^2 + (v'_{Ax})^2 = v'^2_A + v'^2_B \rightarrow v'^2_{Ay} + 2v'^2_{Ay}v'_B + v'^2_B + v'^2_{Ay} = v'^2_A + v'^2_B \rightarrow$$

$$v'^2_A + 2v'^2_{Ay}v'_B + v'^2_B = v'^2_A + v'^2_B \rightarrow 2v'^2_{Ay}v'_B = 0 \rightarrow v'_{Ay} = 0 \text{ or } v'_B = 0$$

Since we are given that $v'_B \neq 0$, we must have $v'_{Ay} = 0$. This means that the final direction of A is the x direction. Put this result into the momentum equations to find the final speeds.

$$v'_A = v'_{Ax} = v_B = \boxed{3.7\,\text{m/s}} \qquad v'_B = v_A = \boxed{2.0\,\text{m/s}}$$