PROBLEMS 4

(1) We are to calculate the force on Earth, so we need the distance of each planet from Earth.

$$r_{\substack{\text{Earth}\\\text{Venus}}} = (150-108)\times10^6\ \text{km} = 4.2\times10^{10}\,\text{m} \qquad r_{\substack{\text{Earth}\\\text{Jupiter}}} = (778-150)\times10^6\ \text{km} = 6.28\times10^{11}\,\text{m}$$

$$r_{\substack{\text{Earth}\\\text{Saturn}}} = (1430-150)\times10^6\ \text{km} = 1.28\times10^{12}\,\text{m}$$

Jupiter and Saturn will exert a rightward force, while Venus will exert a leftward force. Take the right direction as positive.

$$F_{\substack{\text{Earth-}\\\text{planets}}} = G\frac{M_{\text{Earth}}M_{\text{Jupiter}}}{r^2_{\substack{\text{Earth}\\\text{Jupiter}}}} + G\frac{M_{\text{Earth}}M_{\text{Saturn}}}{r^2_{\substack{\text{Earth}\\\text{Saturn}}}} - G\frac{M_{\text{Earth}}M_{\text{Venus}}}{r^2_{\substack{\text{Earth}\\\text{Venus}}}}$$

$$= GM^2_{\text{Earth}}\left(\frac{318}{(6.28\times10^{11}\,\text{m})^2} + \frac{95.1}{(1.28\times10^{12}\,\text{m})^2} - \frac{0.815}{(4.2\times10^{10}\,\text{m})^2}\right)$$

$$= (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\,\text{kg})^2(4.02\times10^{-22}\,\text{m}^{-2}) = \boxed{9.56\times10^{17}\,\text{N}}$$

The force of the Sun on the Earth is as follows.

$$F_{\substack{\text{Earth-}\\\text{Sun}}} = G\frac{M_{\text{Earth}}M_{\text{Sun}}}{r^2_{\substack{\text{Earth}\\\text{Sun}}}} = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(5.97\times10^{24}\,\text{kg})(1.99\times10^{30}\,\text{kg})}{(1.50\times10^{11}\,\text{m})^2} = 3.52\times10^{22}\,\text{N}$$

And so the ratio is $F_{\substack{\text{Earth-}\\\text{planets}}}\Big/F_{\substack{\text{Earth-}\\\text{Sun}}} = 9.56\times10^{17}\,\text{N}/3.52\times10^{22}\,\text{N} = \boxed{2.71\times10^{-5}}$, which is 27 millionths.

(2) The expression for the acceleration due to gravity at the surface of a body is $g_{\text{body}} = G\dfrac{M_{\text{body}}}{R^2_{\text{body}}}$, where R_{body} is the radius of the body. For Mars, $g_{\text{Mars}} = 0.38\,g_{\text{Earth}}$. Thus

$$G\frac{M_{\text{Mars}}}{R^2_{\text{Mars}}} = 0.38\,G\frac{M_{\text{Earth}}}{R^2_{\text{Earth}}} \quad\rightarrow$$

$$M_{\text{Mars}} = 0.38M_{\text{Earth}}\left(\frac{R_{\text{Mars}}}{R_{\text{Earth}}}\right)^2 = 0.38(5.97\times10^{24}\,\text{kg})\left(\frac{3400\ \text{km}}{6380\ \text{km}}\right)^2 = \boxed{6.4\times10^{23}\,\text{kg}}$$

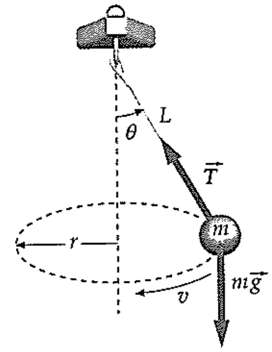
(3) Since mass m is dangling, the tension in the cord must be equal to the weight of mass m, and so $F_T = mg$. That same tension is in the other end of the cord, maintaining the circular motion of mass M, and so $F_T = F_R = Ma_R = M\,v^2/r$. Equate the two expressions for the tension and solve forthe velocity.

$$M\,v^2/r = mg \quad\rightarrow\quad v = \boxed{\sqrt{mgR/M}}.$$

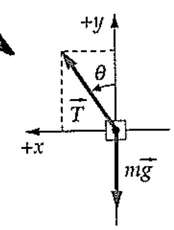
4

Cover the column to the right and try these on your own before looking at the answers.

Steps	Answers
(*a*) 1. The ball is moving in a horizontal circle at constant speed. The acceleration is in the centripetal direction.	The acceleration is horizontal and directed from the ball toward the center of the circle it is moving in.
(*b*) 1. Draw a free-body diagram for the ball [redacted]. Choose as the $+x$ direction the direction of the ball's acceleration (toward the center of the circular path).	
2. Apply $\Sigma F_y = ma_y$ to the ball and solve for the tension T.	$\Sigma F_y = ma_y \Rightarrow T\cos\theta - mg = 0$ so $\boxed{T = \dfrac{mg}{\cos\theta}}$
(*c*) 1. Apply $\Sigma F_x = ma_x$ to the ball.	$\Sigma F_x = ma_x \Rightarrow T\sin\theta = m\dfrac{v^2}{r}$
2. Substitute $mg/\cos\theta$ for T and solve for v.	$\dfrac{mg}{\cos\theta}\sin\theta = m\dfrac{v^2}{r} \Rightarrow g\tan\theta = \dfrac{v^2}{r}$ so $\boxed{v = \sqrt{rg\tan\theta}}$

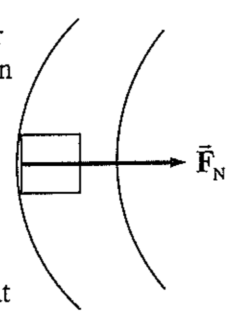


[redacted]



[redacted]

5 ans To experience a gravity-type force, objects must be on the inside of the outer wall of the tube, so that there can be a centripetal force to move the objects in a circle. See the free-body diagram for an object on the inside of the outer wall, and a portion of the tube. The normal force of contact between the object and the wall must be maintaining the circular motion. Write Newton's 2nd law for the radial direction.



$$\sum F_R = F_N = ma = mv^2/r$$

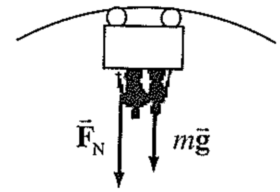
If this is to have the same effect as Earth gravity, then we must also have that $F_N = mg$. Equate the two expressions for normal force and solve for the speed.

$$F_N = mv^2/r = mg \rightarrow v = \sqrt{gr} = \sqrt{(9.8\,\mathrm{m/s^2})(550\,\mathrm{m})} = 73\,\mathrm{m/s}$$

$$(73\,\mathrm{m/s})\left(\frac{1\ \mathrm{rev}}{2\pi(550\ \mathrm{m})}\right)\left(\frac{86{,}400\ \mathrm{s}}{1\ \mathrm{d}}\right) = 1825\ \mathrm{rev/d} \approx \boxed{1.8\times10^3\ \mathrm{rev/d}}$$

(6)

At the top of a circle, a free-body diagram for the passengers would be as shown, assuming the passengers are upside down. Then the car's normal force would be pushing DOWN on the passengers, as shown in the diagram. We assume no safety devices are present. Choose the positive direction to be down, and write Newton's 2nd law for the passengers.



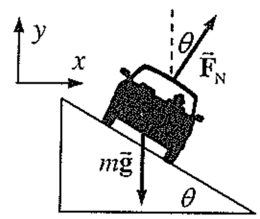
$$\sum F = F_N + mg = ma = mv^2/r \quad \rightarrow \quad F_N = m\left(v^2/r - g\right)$$

We see from this expression that for a high speed, the normal force is positive, meaning the passengers are in contact with the car. But as the speed decreases, the normal force also decreases. If the normal force becomes 0, the passengers are no longer in contact with the car– they are in free fall. The limiting condition is

$$v_{min}^2/r - g = 0 \quad \rightarrow \quad v_{min} = \sqrt{rg} = \sqrt{\left(9.8\,\mathrm{m/s^2}\right)\left(7.4\,\mathrm{m}\right)} = \boxed{8.5\,\mathrm{m/s}}$$

(17)

The car moves in a horizontal circle, and so there must be a net horizontal centripetal force. The car is not accelerating vertically. Write Newton's 2nd law for both the x and y directions.



$$\sum F_y = F_N \cos\theta - mg = 0 \quad \rightarrow \quad F_N = \frac{mg}{\cos\theta}$$

$$\sum F_x = \sum F_R = F_N \sin\theta = ma_x$$

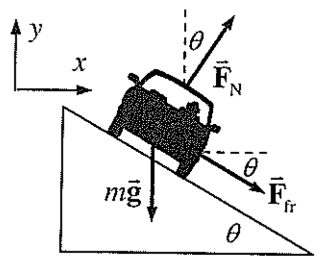
The amount of centripetal force needed for the car to round the curve is

$$F_R = mv^2/r = (1200\text{ kg})\frac{\left[(95\,\mathrm{km/h})\left(\dfrac{1.0\,\mathrm{m/s}}{3.6\,\mathrm{km/h}}\right)\right]^2}{67\text{ m}} = 1.247\times10^4\,\mathrm{N}.$$

The actual horizontal force available from the normal force is

$$F_N \sin\theta = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta = (1200\text{ kg})\left(9.80\,\mathrm{m/s^2}\right)\tan 12^\circ = 2.500\times10^3\,\mathrm{N}.$$

Thus more force is necessary for the car to round the curve than can be supplied by the normal force. That extra force will have to have a horizontal component to the right in order to provide the extra centripetal force. Accordingly, we add a frictional force pointed down the plane. That corresponds to the car not being able to make the curve without friction.



Again write Newton's 2nd law for both directions, and again the y acceleration is zero.

$$\sum F_y = F_N \cos\theta - mg - F_{fr}\sin\theta = 0 \quad \rightarrow \quad F_N = \frac{mg + F_{fr}\sin\theta}{\cos\theta}$$

$$\sum F_x = F_N \sin\theta + F_{fr}\cos\theta = mv^2/r$$

Substitute the expression for the normal force from the y equation into the x equation, and solve for the friction force.

$$\frac{mg + F_{fr}\sin\theta}{\cos\theta}\sin\theta + F_{fr}\cos\theta = mv^2/r \quad \rightarrow \quad \left(mg + F_{fr}\sin\theta\right)\sin\theta + F_{fr}\cos^2\theta = m\frac{v^2}{r}\cos\theta$$

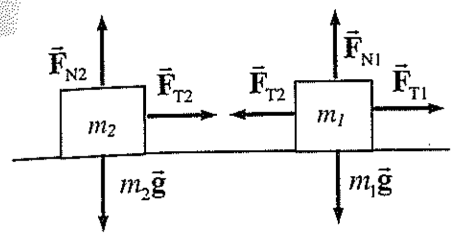
$$F_{fr} = m\frac{v^2}{r}\cos\theta - mg\sin\theta = \left(1.247\times10^4\,\mathrm{N}\right)\cos 12^\circ - (1200\text{ kg})\left(9.80\,\mathrm{m/s^2}\right)\sin 12^\circ$$

$$= 9.752\times10^3\,\mathrm{N}$$

So a frictional force of $\boxed{9.8\times10^3\,\mathrm{N}\text{ down the plane}}$ is needed to provide the necessary centripetal force to round the curve at the specified speed.

8

If the masses are in line and both have the same frequency, then they will always stay in line. Consider a free-body diagram for both masses, from a side view, at the instant that they are to the left of the post. Note that the same tension that pulls inward on mass 2 pulls outward on mass 1, by Newton's 3rd law. Also notice that since there is no vertical acceleration, the normal force on each mass is equal to its weight. Write Newton's 2nd law for the horizontal direction for both masses, noting that they are in uniform circular motion.



$$\sum F_{1R} = F_{T1} - F_{T2} = m_1 a_1 = m_1 v_1^2/r_1 \qquad \sum F_{2R} = F_{T2} = m_2 a_2 = m_2 v_2^2/r_2$$

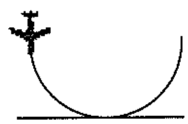
The speeds can be expressed in terms of the frequency as follows: $v = \left(f\,\frac{\text{rev}}{\text{sec}}\right)\left(\frac{2\pi r}{1\text{ rev}}\right) = 2\pi r f$.

$$F_{T2} = m_2 v_2^2/r_2 = m_2 (2\pi r_2 f)^2/r_2 = \boxed{4\pi^2 m_2 r_2 f^2}$$

$$F_{T1} = F_{T2} + m_1 v_1^2/r_1 = 4\pi m_2 r_2 f^2 + m_1 (2\pi r_1 f)^2/r_1 = \boxed{4\pi^2 f^2 (m_1 r_1 + m_2 r_2)}$$

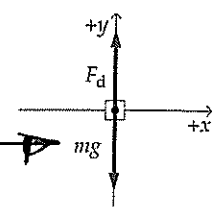
9

The fact that the pilot can withstand 9.0 g's without blacking out, along with the speed of the aircraft, will determine the radius of the circle that he must fly as he pulls out of the dive. To just avoid crashing into the sea, he must begin to form that circle (pull out of the dive) at a height equal to the radius of that circle.



$$a_R = v^2/r = 9.0g \quad \rightarrow \quad r = \frac{v^2}{9.0g} = \frac{(310\,\text{m/s})^2}{9.0(9.80\,\text{m/s}^2)} = \boxed{1.1\times10^3\,\text{m}}$$

10



(a) 1. Draw a free-body diagram ~~(Figure 5-22)~~.

2. Apply $\Sigma F_y = ma_y$. Because the skydiver is moving with constant velocity, the acceleration is zero:

$$\Sigma F_y = ma_y \Rightarrow F_d - mg = 0$$

so $F_d = mg = (64.0\text{ kg})(9.81\text{ N/kg}) = \boxed{628\text{ N}}$

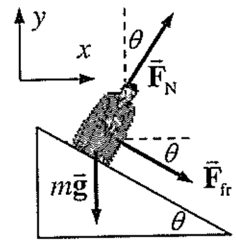
(11)

. The speed of the train is $(160\,\text{km/h})\left(\dfrac{1\,\text{m/s}}{3.6\,\text{km/h}}\right) = 44.44\,\text{m/s}$

(a) If there is no tilt, then the friction force must supply the entire centripetal force on the passenger.

$$F_R = mv^2/R = \frac{(75\text{ kg})(44.44\,\text{m/s})^2}{(620\text{ m})} = 238.9\text{ N} \approx \boxed{2.4\times10^2\text{ N}}$$

(b) For the banked case, the normal force will contribute to the radial force needed. Write Newton's 2nd law for both the x and y directions. The y acceleration is zero, and the x acceleration is radial.



$$\sum F_y = F_N\cos\theta - mg - F_{fr}\sin\theta = 0 \;\rightarrow\; F_N = \frac{mg + F_{fr}\sin\theta}{\cos\theta}$$

$$\sum F_x = F_N\sin\theta + F_{fr}\cos\theta = mv^2/r$$

Substitute the expression for the normal force from the y equation into the x equation, and solve for the friction force.

$$\frac{mg + F_{fr}\sin\theta}{\cos\theta}\sin\theta + F_{fr}\cos\theta = mv^2/r \;\rightarrow$$

$$(mg + F_{fr}\sin\theta)\sin\theta + F_{fr}\cos^2\theta = m\frac{v^2}{r}\cos\theta \;\rightarrow$$

$$F_{fr} = m\left(\frac{v^2}{r}\cos\theta - g\sin\theta\right)$$

$$= (75\text{ kg})\left[\frac{(44.44\,\text{m/s})^2}{620\text{ m}}\cos 8.0^\circ - (9.80\,\text{m/s}^2)\sin 8.0^\circ\right] = 134\text{ N} \approx \boxed{1.3\times10^2\text{ N}}$$

(12)

For an object to be apparently weightless would mean that the object would have a centripetal acceleration equal to g. This is really the same as asking what the orbital period would be for an object orbiting the Earth with an orbital radius equal to the Earth's radius. To calculate, use $g = a_C = v^2/R_{\text{Earth}}$, along with $v = 2\pi R_{\text{Earth}}/T$, and solve for T.

$$g = \frac{v^2}{R_{\text{Earth}}} = \frac{4\pi^2 R_{\text{Earth}}}{T^2} \;\rightarrow\; T = 2\pi\sqrt{\frac{R_{\text{Earth}}}{g}} = 2\pi\sqrt{\frac{6.38\times10^6\,\text{m}}{9.80\,\text{m/s}^2}} = \boxed{5.07\times10^3\,\text{s}}\ (\sim 84.5\text{ min})$$