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Problems set # 8 **Physics 303** October 28, 2014

1. A hypothetic velocity distribution of an ideal gas has the form $G(v) = Ae^{-kv}$. *(i)* Does $G(v)$ satisfy the molecular chaos postulate? *(ii)* Find A from the normalization condition. *(iii)* Find the most probable speed, average speed, and the rms speed. *(iv)* Find the distribution function $g(v_x)$. Check if G factorizes as $G(v) = g(v_x)g(v_y)g(v_z)$.

2. Suppose you flip 1000 unbiased coins. *(i)* What is the probability of getting 500 heads and 500 tails? (Use Stirlings approximation.) *(ii)* What is the probability of getting 450 heads and 550 tails? (Use Stirling again.) *(iii)* Compute the above two probabilities using the Gaussian approximation. How do they compare? *(iv)* Compute $\ln \Omega$ and $\ln \omega_{\rm max}$. How do they compare?

3. A drunk person is walking along the x axis. He starts at $x = 0$, and his step size is $L = 0.5$ meters. For each step, he has chance 2/3 of walking forwards (positive x) and chance 1/3 of walking backwards (negative x). What is his expected position, x, after 50 steps? What is the expected RMS variation around $\bar{x}$, $\Delta x_{\rm rms} = \sqrt{\overline{(x - \bar{x})^2}}$ after 50 steps?

4. Suppose that a system has allowed energy levels $n\varepsilon$, with $n = 0, 1, 2, 3, 4, \cdots$. There are three distinguishable particles, with total energy $U = 4\varepsilon$. *(i)* Tabulate all possible distributions of the three particles among the energy levels, satisfying $U = 4\varepsilon$. *(ii)* Evaluate ω_k for each of above distributions, and also $\Omega = \sum_k \omega_k$. *(iii)* Calculate the average occupation numbers $\bar{N}_n = \sum_k N_n^{(k)} \omega_k / \Omega$ for the three particles in the energy states. Here $\bar{N}_n$ is the average occupation number of the energy level with energy $n\varepsilon$. You should find $\bar{N}_n$ for all $n \leq 4$ (and find that $\bar{N}_n > 4 = 0$).

5. *(i)* Estimate the number of moles and molecules of water in all the Earth's oceans. Assume water covers 75% of the Earth to an average depth of 3 km. *(ii)* Estimate how many molecules of air are in each 2.0 liters breath you inhale that were also in the last breath Julius Caesar took. [Hint: Assume the atmosphere is about 10 km high and of constant density.] *(iii)* A space vehicle returning from the Moon enters the Earth's atmosphere at a speed of about $40,000$ km/h. Molecules (assume nitrogen) striking the nose of the vehicle with this speed correspond to what temperature? (Because of this high temperature, the nose of a space vehicle must be made of special materials, indeed part of it vaporize, and this is seen as a bright blaze upon reentry.)