Prof. Anchordoqui

Solutions of problems set # 2 **Physics 168**

1. See slides.

2. See slides.

3. Call the direction of the flow of the river the x-direction and the direction across the river the y-direction. From Fig. 1 we have $\tan\theta = 110$ m/260 m and so $\theta = 22.9°$. Now we know that $\vec{v}_{\rm bs} = \vec{v}_{\rm bw} + \vec{v}_{\rm ws}$, where $\vec{v}_{\rm bs}$ is the velocity of the boat with respect to the shore, $v_{\rm bw}$ is the velocity of the boat with respect to the water, and $\vec{v}_{\rm ws}$ is the velocity of the water with respect to the shore. Since $v_{{\rm ws}_y} = 0$ we can equate the vertical components of the velocity to find the speed of the boat relative to the shore $|\vec{v}_{\rm bs}| \cos theta = |v_{\rm bw}| \sin 45°$, yielding $|\vec{v}_{\rm bs}| = 1.305$ m/s. Now we can equate the horizontal components of the velocity to obtain $|\vec{v}_{\rm bs}| \sin\theta = |\vec{v}_{\rm bw}| \cos 45° - v_{{\rm ws}_x}$; this leads to $|\vec{w}_{\rm ws}| = 0.69$ ms.

4. Choose upward to be the positive direction and $y_0 = 0$ to be the height from which the cap is dropped. The equation for the vertical component of the position vector is $y = v_{0_y} t - \frac{1}{2} g t^2$. We obtain the flying time from $t(v_{0_y} - \frac{1}{2} g t) = 0$, yielding $t_1 = 0$ (initial time) and $t_2 = 2 v_{0_y}/g = 3.00$ s. Substituting the flying time in the equation for the horizontal component of the the displacement we have $x = v_{0_x} t_2 = v_0 \;\cos theta_0 \; t_2 = 58.8$ m.

5. The velocity is $\vec{v} = 12$ m/s $\hat{\imath} + 16$ m/s $- 9.8$ m/s$^2\, t\, \hat{\jmath}$ and hence the acceleration is given by $\vec{a} = -9.8$ m/s$^2\, \hat{\jmath}$.

6. See slides.

7. First we reso;ve the initial velocity into its components $v_{0_x} = v_0 \cos 37° = 16.0$ m/s and $v_{0_y} = v_0 \sin 37° = 12.0$ m/s. To obtain the maximum height we first obtain the time of maximum altitude from the velocity function in the vertical direction $v_y(t)$. This is obtained by impossing the condition of $v_y = 0$ at maximum height, yielding $t_{\rm max} = v_{0_y}/g = 1.23$ s. The maximum altitude is then $y_{\rm max} = v_{0_y} t_{\rm max} - \frac{1}{2} g t_{\rm max}^2 = 7.39$ m. To obtain the time travelled before the football hits the ground we equate to zero the vertical component of the displacement, i.e., $t(t v_{0_y} - \frac{1}{2} g t) = 0$. The solutions to this equation are $t_1 = 0$ (initial time) and $t_2 = 2 v_{0_y}/g = 2.46$ s. This could have been obtained by symmetry as the movement is symmetric with respect to the maximum altitude, i.e. $t_2 = 2 t_{\rm max}$. The maximum distance travelled is $x = v_{0_x} t_2 = 39.3$ m. At the maximum height there is no vertical component so $v = v_x = v_{0_x} = 16$ m/s. The acceleration vector is the same at the heighest point as it is throughout the height, which is 9.8 m/s^2 downward.

8. Choose upward to be the positive direction. The origin is the point from which the pebbles are released. In the vertical direction $a_y = -9.8$ m/s^2, the velocity at the window has $v_y = 0$, and the vertical displacement is 4.5 m. The equations of motion are: $x = v_{0_x} t$ and $y = v_{0_y} t - \frac{1}{2} g t^2$, supplemented by $v_y = v_{0_y} - g t$. From the supplementary equation we obtained $t_{\rm max} = v_{0_y}/g$, where we have

imposed the $v_y = 0$ condition. Now, we substitute $t_{\rm max}$ in the vertical component of the displacement to obtain $y = v_{0_y}(v_{0_y}/g) - \frac{1}{2}g(v_{0_y}/g)^2 = \frac{v_{0_y}^2}{2g}$. Thus, $v_{0_y} = \sqrt{2gy} = 9.39$ m/s and $t_{\rm max} = 0.96$ s. We find the horizontal speed from the horizontal equation of motion $v_{0_x} = x/t_{\rm max} = 5.2$ m/s, where $x = 5$ m is the distance from the base of the wall.

9. Choose the origin to be the location on the ground directly below the airplane at the time the supplies are dropped, and choose upward as the positive direction. For the supplies, $y_0 = 235$ m, $v_{0_y} = 0$, $a_y = -g$, and the final vertical location is at $y = 0$ m. The initial and constant velocity of the supplies in the horizontal direction is $v_{0_x} = 69.4$ m/s. The time for the supplies reaching the ground is obtained from the vertical component of the displacemnet, $y(t) = y_0 - \frac{1}{2}gt^2$, using the condition $y(t_{\rm ground}) = 0$. This leads to $t_{\rm ground} = 6.93$ s. Then the horizontal distance travelled by the package is found from the horizontal motion at constant velocity, $\Delta x = v_{0_x} t_{\rm ground} = 481$ m.

10. Now the supplies have to travel a horizontal distance of only $\Delta x = 425$ m. Thus, the time of flight will be less and is found from the horizontal motion at constant velocity $\Delta x = v_{0_x} t_{\rm flight}$, yielding $t_{\rm flight} = 6.12$ s. The motion in the y direction is described by $y = y_0 + v_{0_y} t - \frac{1}{2}gt^2$, and so the intial velocity is obtained by substituting $t = t_{\rm flight}$ in this expression. We obtain $v_{0_y} = (y - y_0 - \frac{1}{2}gt_{\rm flight}^2)/t_{\rm flight} = -8.37$ m/s. Note that this is a negative velocity, the object must be projected down. The horizontal component of the speed of the supplies upon landing is the constant horizontal speed of 6.94 m/s. The vertical speed is $v_y = v_{0_y} - gt_{\rm flight} = -68.4$ m/s. Therefore, the speed is $v = \sqrt{v_x^2 + v_y^2} = 97.4$ m/s.

11. See slides

12. Choose the origin to be at ground level, under the place where the projectile is launched and upwards to be the positive y-directon. For the projectile, $v_0 = 65.0$ m/s, $\theta_0 = 37°$, $y_0 = 125$ m, and $v_{0_y} v_0 \sin\theta_0$. Then, using the equation of motion for the vertical direction, $y = y_0 + v_{0_y} t - \frac{1}{2}gt^2$, we obtain $t = 10.4$ s. The horizontal range is found from the horizontal motion at constant velocity, $\Delta x = v_{0_x} t = 541$ m. At the instant just before the particle reaches the ground, the horizontal component of the velocity is constant $v_x = v_0 \cos\theta_0 = 51.9$ m/s and $v_y = v_0 \sin\theta_0 - gt = -63.1$ m/s. The magnitude of the velocity is $v = \sqrt{v_x^2 + v_y^2} = 81.7$ m/s. The direction of the velocity is obtained from $\tan\theta v_y/v_x$, yielding $\theta = -50.6°$. This means the object is moving with velocity 50.6° degrees below the horizon. The maximum height $y_{\rm max}$ above the cliff top reached by the projectile will occur when $v_y = 0$. This leads to $v_0^2 \sin^2\theta_0 - 2gy_{\rm max} = 0$, yielding $y_{\rm max} = 78.1$ m.

13. Choose downward to be the positive direction for this problem. The vertical component of her acceleration is directed downward and its magnitude will be given by $a_y = a\sin\theta = 0.9$ m/s^2, where $\theta = 30°$. The time to reach the bottom of the hill is calculated from $y = y_0 + v_{0_y} t - \frac{1}{2}a_y t^2$, with $y = 335$ m, $v_{0_y} = 0$, $y_0 = 0$, and $a_y = 0.9$ m/s^2. This gives $t = 27.3$ s.

14. Choose the origin to be the location on the ground directly underneath the ball when served, and choose upwards as the positive y direction. Then for the ball $y_0 = 2.5$ m, $v_{0_y} = 0$, $a_y = -g$,

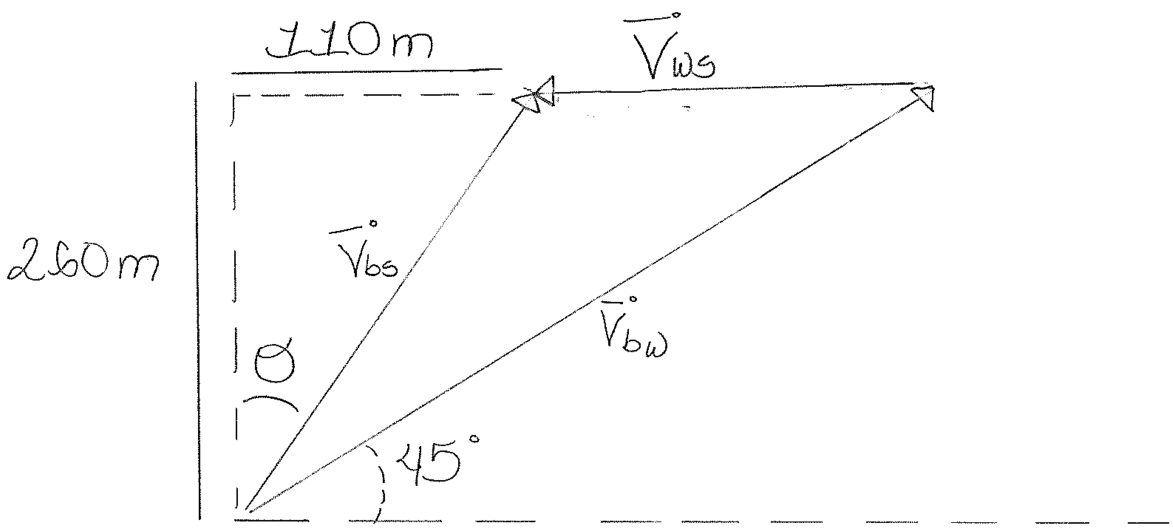

Figure 1: The situation in problem 3.

and the ball location when the ball just clears the net is $y = 0.9$ m. The time for the ball to reach the net is calculated from $y = y_0 + v_{0_y}t - \frac{1}{2}gt^2$, yielding $t_{\text{net}} = 0.57$ s. The x velocity is found from the horizontal motion at constant velocity, $\Delta x = v_x t$, and thus $v_x = 26.6$ m/s. This is the minimum speed required to clear the net. To find the full time of flight of the ball, set the final location to be $y = 0$, and so $y = y_0 + v_{0_y}t - \frac{1}{2}gt^2$, yielding $t = 0.71$ s. The horizontal position where the ball lands is found from the horizontal motion at constant velocity, $\Delta x = v_x t = 18.8$ m. Since this is between 15.0 m and 22.0 m the ball lands in the "good" region.

15. The equations of motion are $x = x_0 + v_{0_x}t$ and $y = y_0 + v_{0_y}t - \frac{1}{2}gt^2$, with $v_{0_x} = v_0 \cos\theta_0$, $v_{0_y} = v_0 \sin\theta_0$, $\theta_0 = 60°$, $y_0 = 4$ m, $y = 8.5$ m, and $x = 38$ m. From the equation for the horizontal displacement we get $t = x/(v_0 \cos\theta_0$. We substitute this expression in the equation for the vertical displacement to obtain $y = y_0 + x\tan\theta_0 - \frac{1}{2}gx^2/(v_0^2 \cos\theta_0^2)$. This means that $v_0^2 = \frac{gx^2}{(y - y_0 - x\tan\theta_0)2\cos^2\theta_0}$, yielding $v_0 = 22.3$ m/s. We obtain the time the spaetzle was in the air using $t = x/(v_0 \cos\theta_0) = 3.5$ s. We obtain the vertical component of the projectile's velocity when it hits the wall using $v_y = v_0 \sin\theta_0 - gt = -15.7$ m/s. The horizontal velocity of the projectile is $v_x = 10.7$ m/s.