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Problems set # 14 **Physics 411** December 18, 2007

1. *(i)* Use the Hamilton method to find the equations of motion of a particle of mass m constrained to move on the surface of a cylinder defined by $x^2 + y^2 = R^2$. The particle is subject to a force directed toward the origin and proportional to the distance of the particle from the origin: $\vec{F} = -kr$. *(ii)* Construct the phase diagram for the particle.

2. Using the Hamiltom method, find the equations of motion for a spherical pendulum of mass m and length b

3. Consider an ideal gas containing N atoms in a container of volume V, pressure P, and absolute temperature T_1. Use the virial theorem to derive the equation of state for a perfect gas.

4. Consider any two continuous functions of the generalized coordinates and momenta $g(q_k, p_k)$ and $h(q_k, p_k)$. The Poisson brackets are defined by:

$$[g, h] \equiv \sum_k \left(\frac{\partial g}{\partial q_k} \frac{\partial h}{\partial p_k} - \frac{\partial g}{\partial p_k} \frac{\partial h}{\partial q_k} \right)$$

Verify the following properties of the Poisson brackets:
(a) $\frac{dg}{dt} = [g, H] + \frac{\partial g}{\partial t}$
(b) $\dot{q}_j = [q_j, H]$ and $\dot{p}_j = [p_j, H]$
(c) $[p_i, p_j] = [q_i, q_j] = 0$
(d) $[q_i, p_j] = \delta_{ij}$
where H is the Hamiltonian. If the Poisson brackets of the two quantities vanishes, the quantities are said to commute. If the Poisson bracket of the two quantities equals unity, the two quantities are said to be canonically conjugate. *(e)* Show that any quantity that does not depend explicitely on time and that commutes with the Hamiltonian is a constant of motion of the system.

5. Determine the Hamiltonian and Hamilton's equation of motion for the double Atwood machine.

6. A particle moves in a circular orbit in a force field given by $F(r) = -k/r^2$. Show that if k suddenly decreases to half its original value, the particle's orbit becomes parabolic.

7. Discuss the implications of Liouville's theorem on focusing a beam of charged particles by considering the following simple case. An electron beam of circular cross section (radius R_0 is directed along the z-axis. The density of electrons across the beam is constant, but the momentum components transverse to the beam (p_x and p_y) are distributed uniformly over a circle of radius p_0 in momentum space. If some focusing system reduces the beam radius from R_0 to R_1, find the resulting distribution of the transverse momentum components. What is the physical meaning of this result? (Consider the angular divergence of the beam)

8. Four particles are directed upward in a uniform gravitational field with the following initial conditions: *(i)* $z(0) = z_0, p_z(0) = p_0$; *(ii)* $z(0) = z_0 + \Delta z_0, p_z(0) = p_0$; *(iii)* $z(0) = z_0, p_z(0) = p_0 + \Delta p_0$; *(iv)* $z(0) = z_0 + \Delta z_0, p_z(0) = p_0 + \Delta p_0$. Show by direct calculation that the representative points corresponding to these particles always define an area in phase space equal to $\Delta z_0 \Delta p_0$. Sketch the phase paths, and show for several times $t > 0$ the shape of the region whose area remains constant.

9. A communication satellite is in a circular orbit around the Earth at radius R and velocity v. A rocket accidentally fires quite suddenly giving the satellite an outward radial velocity v in addition to its original velocity. (a) Calculate the ratio of the new energy and angular momentum to the old. (b) Describe the subsequent motion of the satellite and plot $T(r)$, $V(r)$, $U(r)$, and $E(r)$ after the rocket fires.