Prof. Anchordoqui

Problems set # 10 **Physics 168**

1. Jupiter's mean orbital radius is 5.2 AU? What is the period of Jupiter's orbit around the Sun?

2. Show that the total energy of a satellite in a circular orbit about the Earth is equal to half of the potential energy.

3. The rings of Saturn are composed of chunks of ice that orbit the planet. The inner radius of the ring is 73,000 km, while the outer radius is 170,000 km. Find the period of an orbiting chunk of ice at the inner radius and the period of a chunk of ice at the outer radius. Compare your numbers with Saturn's mean rotation period of 10 h and 39 minutes. The mass of Saturn is 5.7×10^{26} kg.

6. What is the apparent weight of a 75 kg astronaut 4200 km from the center of the Moon in a space vehicle (a) moving at constant velocity, and (b) accelerating toward the Moon at 2.9 m/s^2? State the direction in each case.

4. (a) Show that if a satellite orbits very near the surface of a planet with period τ, the density of the planet is

$$\rho \equiv \frac{M}{V} = \frac{3\pi}{G\tau^2}\,.$$

(b) Estimate the density of the Earth, given that the satellite near the surface orbits with a period of about 85 min.

5. Halley's comet orbits the Sun roughly once every 76 years. It comes very close to the surface of the Sun on its closest approach. Estimate the greatest distance of the comet from the Sun. It is still in the solar system? What planet's orbit is nearest when it is out there? [Hint: The mean distance s in Kepler's thrid law is half the sum of the nearest and farthest distance from the Sun. (See Fig. 1)

6. The table below gives the mass, period, and mean distance for the four largest moons of Jupiter (those discovered by Galileo in 1609). (a) Determine the mass of Jupiter using the data for Io. (b) Determine the mass of Jupiter using the data from each of the other three moons. Are the results consistent? (See Fig. 1.)

7. [As in Larry Niven's Ringworld] A science-fiction tale describes an artificial "planet" in the form of a band completely encircling a sun (Fig. 2). The inhabitants live on the inside surface (where it is always noon). Imagine that this Sun is exactly like our own, that the distance to the band is the same as the Earth-Sun distance (to make the climate temperate), and that the ring rotates quickly enough to produce an apparent gravity of g as on Earth. What will be the period of revolution, this planet's year, in Earth days?

8. Two point particles, each of mass M, are fixed in position on the y axis at $y = +a$ and $y = -a$. Find the gravitational field at all points in the x axis as a function of x.

9. A planet that has a hollow core consists of a uniform spherical shell with mass M, outer radius R, and inner radius $R/2$. (a) What amount of mass is closer than $3/4R$ to the center of the planet? (b) What is the gravitational field a distance $3/4R$ from the center.

10. A solid sphere of radius R and mass M is spherically symmetric but not uniform. Its density ρ, defined as its mass per unit volume, is proportional to the distance r from the center for $r \leq R$.

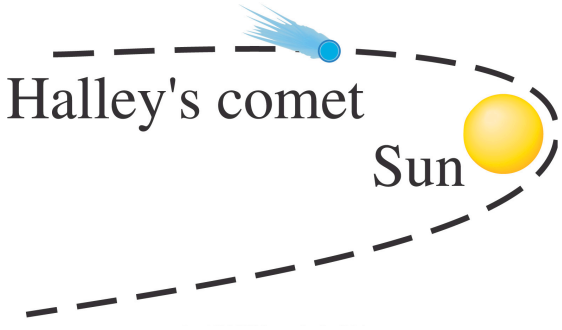

TABLE 5–3 Principal Moons of Jupiter

Moon	Mass (kg)	Period (Earth days)	Mean distance from Jupiter (km)
Io	8.9×10^{22}	1.77	422×10^3
Europa	4.9×10^{22}	3.55	671×10^3
Ganymede	15×10^{22}	7.16	1070×10^3
Callisto	11×10^{22}	16.7	1883×10^3

Figure 1: The situations in problems 5 (left) and 6 (right).

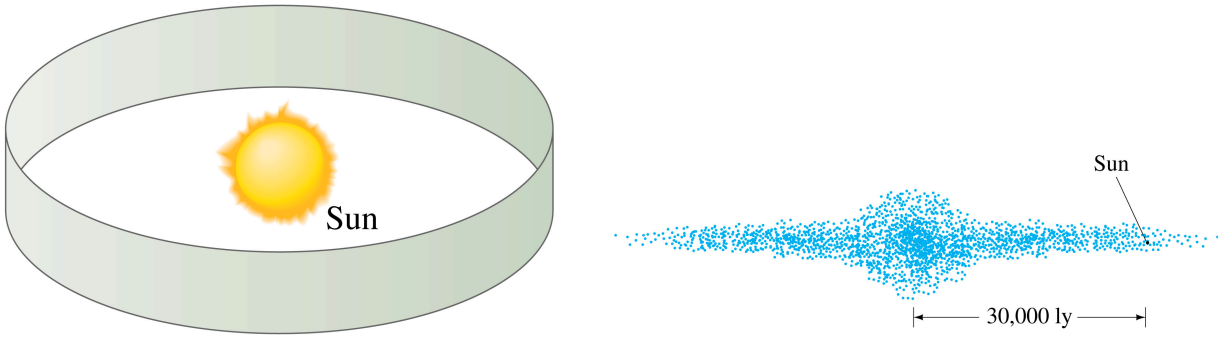

Figure 2: The situations in problems 7 (left) and 12 (right).

That is, $\rho = Cr$, for $r \leq R$, where C is a constant. (a) Find C. (b) Find $\vec{g}$ for all $r \geq R$. (c) Find $\vec{g}$ at $r = R/2$.

11. A thin uniform rod of mass M and length L is centered at the origin and lies along the x axis. Find the gravitational field due to the rod at all points on the x axis in the region $x > L/2$.

12. The Sun rotates around the center of the Milky Way at a distance of about 30,000 light-years from the center. If it takes about 200 million years to make one rotation, estimate the mass of our Galaxy. Assume that the mass distribution of our Galaxy is concentrated mostly in a central uniform sphere. If all the stars had about the mass of our Sun ($M_\odot = 2 \times 10^{30}$ kg), how many stars would there be in our Galaxy. (See Fig. 2.) [Hint: 1 ly $= 9.5 \times 10^{15}$ m.]