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**Problems set # 1** **Physics 411** September 13, 2007

1. A point $P$ is represented in the $(x_1,\ x_2,\ x_3)$ system by $P(2,\ 1,\ 3)$. Another coordinate system represents the same point as $P(x_1',\ x_2',\ x_3')$ but in a system where $x_2$ has been rotated towards $x_3$ around the $x_1$-axis by an angle of 30°. *(i)* Find the rotation matrix. *(ii)* Determine $P(x_1',\ x_2',\ x_3')$. *(iii)* Verify that the rotation operator preserves the length of the position vector.

2. For the two vectors, $\vec{a} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $\vec{b} = -2\hat{\imath} + 3\hat{\jmath} + \hat{k}$, find: *(i)* $\vec{a} - \vec{b}$, *(ii)* $|\vec{a} - \vec{b}|$, *(iii)* Component of $\vec{b}$ along $\vec{a}$, *(iv)* angle between $\vec{a}$ and $\vec{b}$, *(v)* $\vec{a} \times \vec{b}$, and *(vi)* $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})$.

3. Consider the following matrices:

$$\boldsymbol{A} = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 3 & 1 \\ 2 & 0 & 1 \end{pmatrix}$$

$$\boldsymbol{B} = \begin{pmatrix} 2 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 1 & 3 \end{pmatrix}$$

$$\boldsymbol{C} = \begin{pmatrix} 2 & 1 \\ 4 & 3 \\ 1 & 0 \end{pmatrix}$$

find the following:

*(i)* $|\boldsymbol{AB}|$, *(ii)* $\boldsymbol{AC}$, *(iii)* $\boldsymbol{ABC}$, and *(iv)* $\boldsymbol{AB} - \boldsymbol{B}^T\boldsymbol{A}^T$.

4. Show that the product of 2 orthogonal matrices is also an orthogonal matrix.

5. Show that:

*(i)* $\vec{a}\ .\ (\vec{b} \times \vec{c}) = \vec{c}\ .\ (\vec{a} \times \vec{b})$

*(ii)* $(\vec{a} \times \vec{b})\ .\ (\vec{c} \times \vec{d}) = (\vec{a}\ .\ \vec{c})\ (\vec{b}\ .\ \vec{d}) - (\vec{b}\ .\ \vec{c})(\vec{a}\ .\ \vec{d})$

*(iii)* $\vec{a}\ .\ (\vec{a} \times \vec{c}) = 0$

6. Show that $\vec{\nabla}(\ln|\vec{r}|) = \vec{r}/r^2$

7. Show that $\vec{\nabla}(\phi\,\psi) = \phi\,\vec{\nabla}\psi + \psi\vec{\nabla}\phi$

8. Evaluate the integral

$$\int \vec{a} \times \ddot{\vec{a}}\, dt$$