Classical Mechanics

Planetary Motion

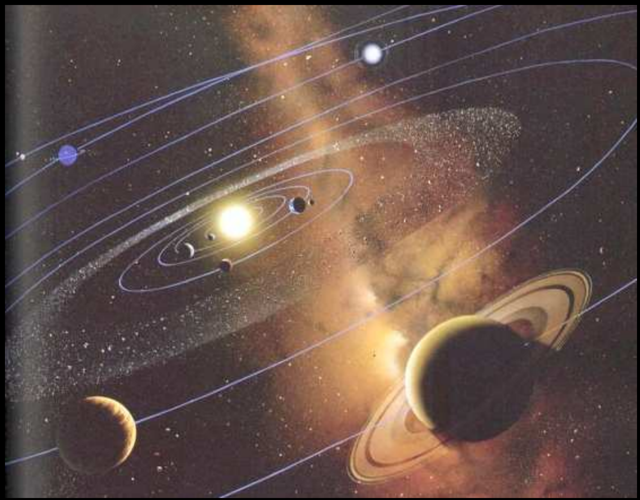
- ☞ Kepler's Laws
- ☞ Orbital Energies
- ☞ Orbital Dynamics

Summary


Nice store. How do you keep the floors so clear?
Oh, we hired this dude named Kepler, he's really good. Hard worker. Doesn't mind the monotony. Sweeps out the same area every night.

Kepler's Laws

Johannes Kepler was the first astronomer to correctly describe planetary motion in the Solar System
(in works published between 1609 and 1619)



The motion of the Planets is summed up in three simple laws

☞ **1. The planetary orbits are all ellipses which are confocal with the Sun**
(i.e., the Sun lies on one of the focii of the ellipses)

☞ **2. Each planet sweeps out an equal area in an equal time interval**

☞ **3. The squares of the orbital periods of the planets are proportional to the cubes of their orbital major radii**

Let us now see if we can derive Kepler's laws from Newton's laws of motion

Conservation Theorems: First Integrals of Motion

Gravity is a conservative force

$\Downarrow$

the gravitational force can be written

$$\vec{F} = -\vec{\nabla} U$$

potential energy of our planet in the Sun's gravitational field

$$U(\vec{r}) = -\frac{G\,M\,m}{r}$$

$\Downarrow$

the total energy of our planet is a conserved quantity
(i.e., is constant in time)

$$\mathcal{E} = \frac{v^2}{2} - \frac{G\,M}{r}$$

$\mathcal{E}$ **is actually the planet's total energy per unit mass and** $\vec{v} = d\vec{r}/dt$.

Conservation Theorems: First Integrals of Motion (cont'd)

Gravity is also a central force

$\Downarrow$

the angular momentum of our planet is a conserved quantity

$$\vec{h} = \vec{r} \times \vec{v}$$

$\Downarrow$

the planet's angular momentum per unit mass is constant in time

Taking the scalar product of the above equation with $\vec{r}$

$$\vec{h} \cdot \vec{r} = 0$$

This is the equation of a plane
which passes through the origin and whose normal is parallel to $\vec{h}$

$\vec{h}$ is a constant vector ☞ always points in the same direction

the motion of our planet is two-dimensional in nature
☛ it is confined to some fixed plane which passes through the origin

Without loss of generality we can let this plane coincide with the x-y plane

Polar Coordinates

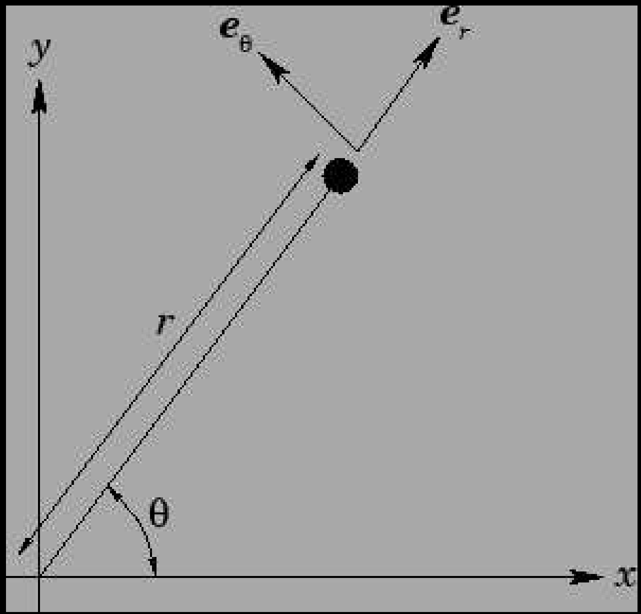
☞ in terms of standard Cartesian coordinates $(x,\, y)$

We can determine the instantaneous position of our planet

☞ in terms of plane polar coordinates $(r,\, \theta)$

It is helpful to define 2 unit vectors at the instantaneous position of the planet

- **one always points radially away from the origin** ☞ $\boldsymbol{e}_r \equiv \boldsymbol{r}/r$
- **the other is normal to $\boldsymbol{e}_r$ in the direction of increasing θ** ☞ $\boldsymbol{e}_\theta \equiv \hat{\mathbf{z}} \times \boldsymbol{e}_r$



The Cartesian components of $\boldsymbol{e}_r$ and $\boldsymbol{e}_\theta$ are

$$\boldsymbol{e}_r = (\cos\theta,\, \sin\theta) \qquad \boldsymbol{e}_\theta = (-\sin\theta,\, \cos\theta)$$

Polar Coordinates (cont'd)

We can write the position vector of our planet as

$$\vec{r} = r\,\boldsymbol{e}_r$$

$$\Downarrow$$

the planet's velocity becomes

$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{r}\,\boldsymbol{e}_r + r\,\dot{\boldsymbol{e}}_r$$

Note that e_r has a non-zero time-derivative
(unlike a Cartesian unit vector)
because its direction changes as the planet moves around

Differentiating $\boldsymbol{e}_r$ with respect to time

$$\dot{\boldsymbol{e}}_r = \dot{\theta}\,(-\sin\theta,\,\cos\theta) = \dot{\theta}\;\boldsymbol{e}_\theta$$

$$\Downarrow$$

$$\boldsymbol{v} = \dot{r}\;\boldsymbol{e}_r + r\,\dot{\theta}\;\boldsymbol{e}_\theta$$

Polar Coordinates (cont'd)

The planet's acceleration is written as

$$\boldsymbol{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2} = \ddot{r}\boldsymbol{e}_r + \dot{r}\dot{\boldsymbol{e}}_r + (\dot{r}\,\dot{\theta} + r\,\ddot{\theta})\,\boldsymbol{e}_\theta + r\,\dot{\theta}\;\dot{\boldsymbol{e}}_\theta$$

Again ☛ e_θ has a non-zero time-derivative
because its direction changes as the planet moves around

Differentiation of e_θ with respect to time yields

$$\dot{\boldsymbol{e}}_\theta = \dot{\theta}\,(-\cos\theta,\, -\sin\theta) = -\dot{\theta}\,\boldsymbol{e}_r$$

$$\Downarrow$$

$$\boldsymbol{a} = (\ddot{r} - r\,\dot{\theta}^{\,2})\,\mathbf{e}_r + (r\,\ddot{\theta} + 2\,\dot{r}\,\dot{\theta})\,\boldsymbol{e}_\theta$$

Polar Coordinates (cont'd)

It follows that the equation of motion of our planet
(derived from Newton's Law)

$$\frac{d^2\mathbf{r}}{dt^2} = -\frac{G\,M}{r^2}\,\boldsymbol{e}_r$$

can be re-written as

$$\mathbf{a} = (\ddot{r} - r\,\dot{\theta}^{\,2})\,\mathbf{e}_r + (r\,\ddot{\theta} + 2\,\dot{r}\,\dot{\theta})\,\mathbf{e}_\theta = -\frac{G\,M}{r^2}\,\mathbf{e}_r$$

e_r and e_θ are mutually orthogonal

$\Downarrow$

we can separately equate the coefficients to give a radial equation of motion

$$\ddot{r} - r\,\dot{\theta}^{\,2} = -\frac{G\,M}{r^2}$$

and a tangential equation of motion

$$r\,\ddot{\theta} + 2\,\dot{r}\,\dot{\theta} = 0$$

Conic Sections

- **the ellipse**
- **the parabola ⇒ are collectively known as conic sections**
- **the hyperbola**

(these 3 types of curve can be obtained by taking various plane sections of a cone)

Solutions of the radial and tangential equations of motion are all conic sections

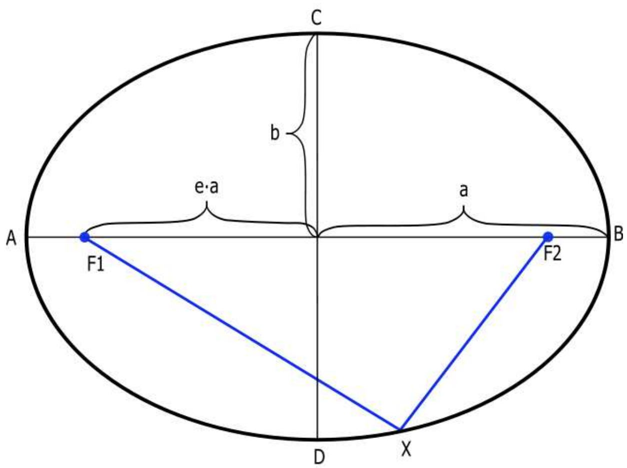
⇓

appropriate to briefly review these curves

Conic Sections (cont'd)

An ellipse centered on the origin of major radius a and minor radius b
(aligned along the x- and y-axes)
satisfies the following well-known equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



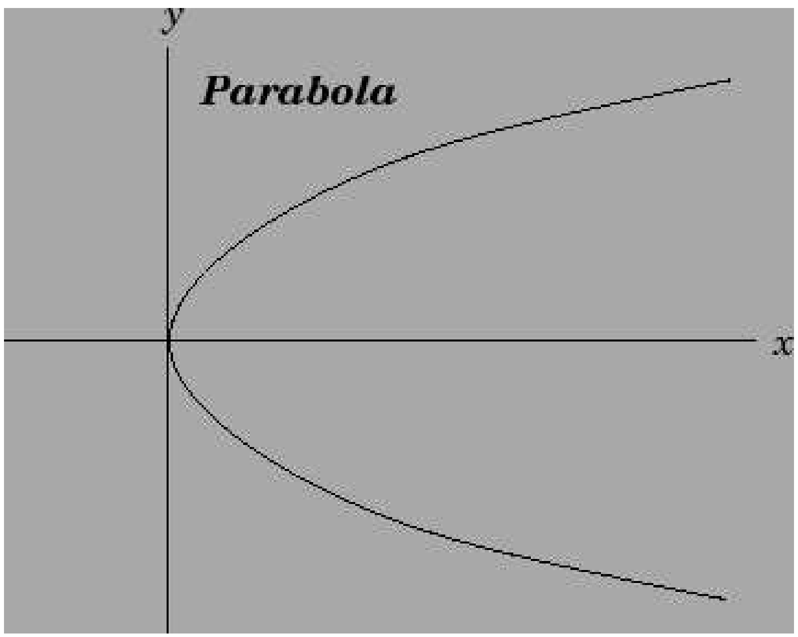
Ellipse

locus of points on a plane where
sum of distances from any point on the curve to two fix points is constant
the two fix points are called foci

Conic Sections (cont'd)

A parabola which is aligned along the $+x$-axis and passes through the origin satisfies

$$y^2 - b\,x = 0$$



☞ $b > 0$

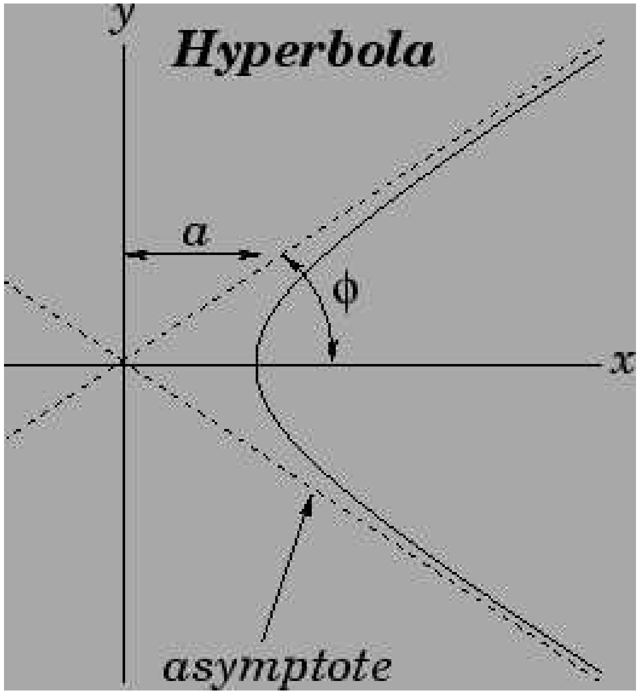
Parabola

locus of points in a plane which are equidistant from a given point (the focus) **and a given line** (the directrix)

Conic Sections (cont'd)

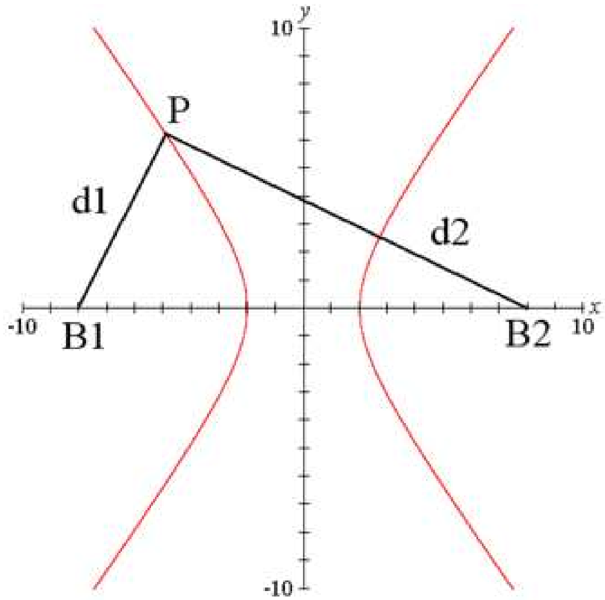
A hyperbola which is aligned along the $+x$-axis and whose asymptotes intersect at the origin satisfies

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



- ☞ a is the distance of closest approach to the origin
- ☞ The asymptotes subtend an angle $\phi = \tan^{-1}(b/a)$ with the x-axis

Conic Sections (cont'd)



Hyperbola

locus of points where

the difference in the distance to two fixed points (called the foci) **is constant**

- That fixed difference in distance is two times a
- a is the distance from the center of the hyperbola to the vertex of the nearest branch of the hyperbola
- a is also known as the semi-major axis of the hyperbola
- The foci lie on the transverse axis and their midpoint is called the center

Conic Sections (cont'd)

It is not clear at this stage what

- **the ellipse**
- **the parabola**
- **the hyperbola**

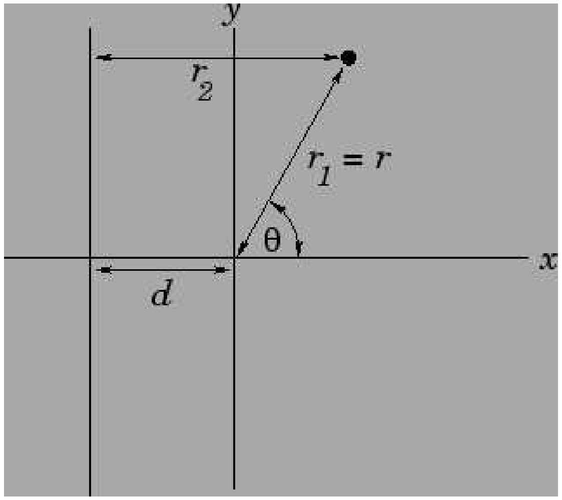
have have in common

the 3 curves can be represented as the locus of a movable point whose distance from a fixed point is in a constant ratio to its perpendicular distance to some fixed straight-line

Let the fixed point

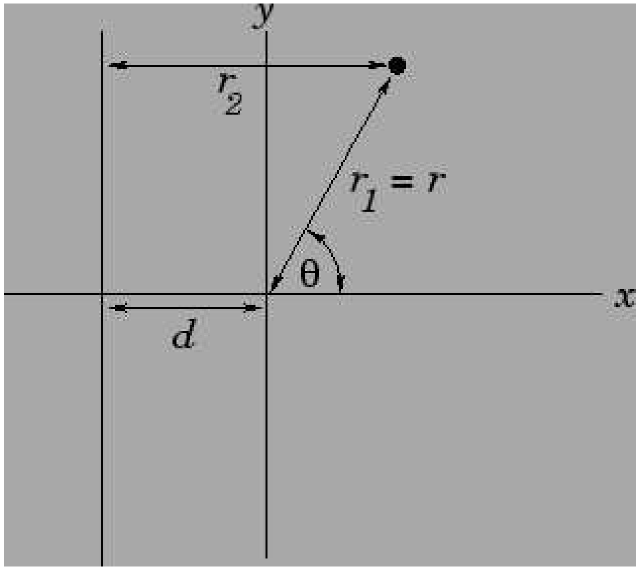
(which is termed the focus of the ellipse/parabola/hyperbola)

lie at the origin and let the fixed line correspond to $y = -d$ (with $d > 0$)



Conic Sections (cont'd)

The distance of a general point (x, y) from the origin is $r_1 = \sqrt{x^2 + y^2}$
(which lies to the right of the line $y = -d$)
the perpendicular distance of the point from the line $y = -d$ is $r_2 = x + d$



In plane polar coordinates ☞ $r_1 = r$ and $r_2 = r\cos\theta + d$

$\Downarrow$

the locus of a point for which r_1 and r_2 are in a fixed ratio satisfies

$$\frac{r_1}{r_2} = \frac{\sqrt{x^2 + y^2}}{x + d} = \frac{r}{r\cos\theta + d} = e \Leftrightarrow e \geq 0 \quad \text{is a constant}$$

Conic Sections (cont'd)

When expressed in terms of plane polar coordinates the above equation can be rearranged to give

$$r = \frac{r_c}{1 - e\,\cos\theta} \Leftrightarrow r_c = e\,d$$

and when expressed in terms of Cartesian coordinates the same equation can be rearranged to give

$$\frac{(x - x_c)^2}{a^2} + \frac{y^2}{b^2} = 1 \Leftrightarrow e < 1$$

This equation can be recognized as the equation of an ellipse

whose center lies at $(x_c, 0)$

and whose major and minor radii a and b are aligned along the x- and y-axes

In changing variables ☞ we have taken

$$a = \frac{r_c}{1 - e^2}$$

$$b = \frac{r_c}{\sqrt{1 - e^2}} = \sqrt{1 - e^2}\,a$$

$$x_c = \frac{e\,r_c}{1 - e^2} = e\,a$$

Conic Sections (cont'd)

When expressed in terms of Cartesian coordinates

$$\frac{r_1}{r_2} = \frac{\sqrt{x^2+y^2}}{x+d} = \frac{r}{r\,\cos\theta + d} = e$$

can be rearranged to give

$$\frac{(x-x_c)^2}{a^2} - \frac{y^2}{b^2} = 1 \Leftrightarrow e > 1$$

equation of a hyperbola

whose asymptotes intersect at $(x_c,\,0)$ and which is aligned along the $+x$-direction

In changing variables ☞ we have taken

$$a = \frac{r_c}{e^2-1}$$

$$b = \frac{r_c}{\sqrt{e^2-1}} = \sqrt{e^2-1}\,a$$

$$x_c = -\frac{e\,r_c}{e^2-1} = -e\,a$$

The asymptotes subtend an angle with the x-axis

$$\phi = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}(\sqrt{e^2-1})$$

Conic Sections (cont'd)

When again expressed in terms of Cartesian coordinates

$$\frac{r_1}{r_2} = \frac{\sqrt{x^2+y^2}}{x+d} = \frac{r}{r\,\cos\theta + d} = e \qquad (\dagger)$$

can be rearranged to give

$$y^2 - 2\,r_c\,(x - x_c) = 0 \Leftrightarrow e = 1$$

This is the equation of a parabola

passing through the point $(x_c,\, 0)$ and aligned along the $+x$-direction

In changing variables ☞ we have taken

$$x_c = -r_c/2$$

Summing up

(†) is the polar equation of a general conic section confocal with the origin

- for $e < 1$ ☞ the conic section is an ellipse
- for $e = 1$ ☞ the conic section is a parabola
- for $e > 1$ ☞ the conic section is a hyperbola

Kepler's Second Law

Multiplying our planet's tangential equation of motion by r we obtain

$$r^2\,\ddot{\theta} + 2\,r\,\dot{r}\,\dot{\theta} = 0$$

The above equation can be also written

$$\frac{d(r^2\,\dot{\theta})}{dt} = 0$$

which implies that

$$h = r^2\,\dot{\theta}$$

is constant in time

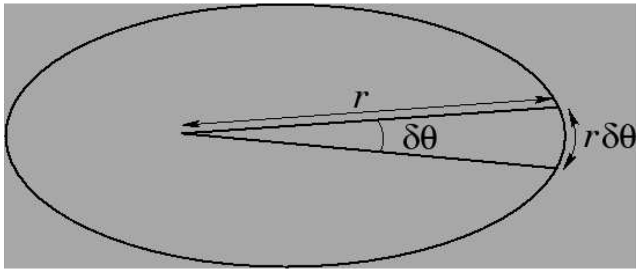
$$\Downarrow$$

the angular momentum of our planet is a constant of its motion

this is the case because gravity is a central force

Kepler's Second Law (cont'd)

Suppose that the radius vector connecting our planet to the origin sweeps out an angle $\delta\theta$ between times t and $t + \delta t$



The approximately triangular region swept out by the radius vector has the area

$$\delta A \simeq \frac{1}{2} r^2 \, \delta\theta,$$

(because the area of a triangle is half its base ☞ $r\,\delta\theta$ times its height ☞ r)

$$\Downarrow$$

the rate at which the radius vector sweeps out area is

$$\frac{dA}{dt} = \frac{1}{2} \lim_{\delta t \to 0} \frac{r^2\,\delta\theta}{\delta t} = \frac{r^2}{2}\frac{d\theta}{dt} = \frac{h}{2}$$

$$\Downarrow$$

h is constant in time ☞ the r-vector sweeps out area at a constant rate
Kepler's Second Law ☞ consequence of angular momentum conservation

Kepler's First Law

Our planet's radial equation of motion can be combined with

$$h = r^2\,\dot\theta$$

to give

$$\ddot r - \frac{h^2}{r^3} = -\frac{G\,M}{r^2} \qquad (\ddagger)$$

Define $r = u^{-1}$

$$\Downarrow$$

$$\dot r = -\frac{\dot u}{u^2} = -r^2\,\frac{du}{d\theta}\,\frac{d\theta}{dt} = -h\,\frac{du}{d\theta}$$

Likewise

$$\ddot r = -h\,\frac{d^2u}{d\theta^2}\,\dot\theta = -u^2\,h^2\,\frac{d^2u}{d\theta^2}$$

$$\Downarrow$$

($\ddagger$) can be written as

$$\frac{d^2u}{d\theta^2} + u = \frac{G\,M}{h^2}$$

Kepler's First Law (cont'd)

The general solution to the above equation takes the form

$$u(\theta) = \frac{G\,M}{h^2}\,[1 - e\,\cos(\theta - \theta_0)]$$

☛ e and θ_0 are arbitrary constants

Without loss of generality ☞ we can set $\theta_0 = 0$

(by rotating our coordinate system about the z-axis)

$\Downarrow$

$$r(\theta) = \frac{r_c}{1 - e\,\cos\theta} \Leftrightarrow r_c = \frac{h^2}{G\,M}$$

This the equation of a conic section which is confocal with the origin (i.e., with the Sun)

for $e < 1$ ☛ equation of an ellipse which is confocal with the Sun

the orbit of our planet around the Sun is a confocal ellipse

Of course a planet cannot have a parabolic or a hyperbolic orbit

$\downarrow$

such orbits are only appropriate to objects
which are ultimately able to escape from the Sun's graviational field

Kepler's Third Law

We have seen that

- **the radius vector connecting our planet to the origin sweeps out area at the constant rate $dA/dt = h/2$**
- **the planetary orbit is an ellipse**

Suppose that the major and minor radii of the ellipse are a and b

$$\Downarrow$$

the area of the ellipse is $A = \pi\, a\, b$

radius vector sweeps out the whole area of the ellipse in a single orbital period T

$$\Downarrow$$

$$T = \frac{A}{(dA/dt)} = \frac{2\,\pi\,a\,b}{h}$$

$$\Downarrow$$

$$T^2 = \frac{4\,\pi^2\,a^3}{G\,M}$$

the square of the orbital period of our planet is proportional to the cube of its orbital major radius

Kepler's Third Law (cont'd)

For an elliptical orbit

☛ **the closest distance to the Sun (perihelion distance) is**

$$r_p = \frac{r_c}{1+e} = a\,(1-e)$$

☛ **the furthest distance from the Sun (aphelion distance) is**

$$r_a = \frac{r_c}{1-e} = a\,(1+e)$$

⇓

the major radius a is simply the mean of the perihelion and aphelion distances

$$a = \frac{r_p + r_a}{2}$$

The eccentricity ☞ $e = (r_a - r_p)/(r_a + r_p)$
measures the deviation of the orbit from circularity

⇓

$e = 0$ corresponds to a circular orbit
$e \to 1$ corresponds to an infinitely elongated elliptical orbit

Halley

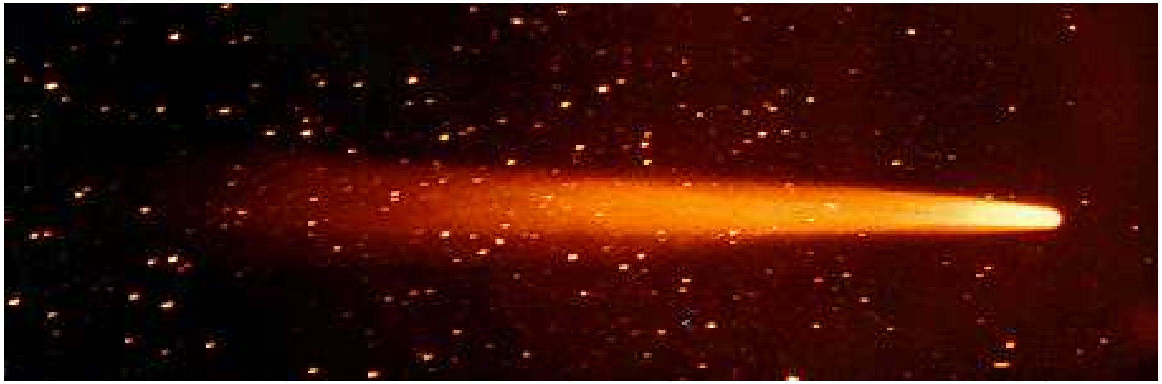
- **The English astronomer Edmund Halley was a good friend of Newton**
- **In 1705 he used Newton's theory of gravitation to determine orbits of comets from their recorded positions in sky** (as a function of time)
- **He found that the bright comets of**
  - **1531**
  - **1607**
  - **1682**

  had almost the same orbits
- **when accounting for gravitational perturbation on the cometary orbits** (from Jupiter and Saturn) **he concluded that these were different appearances of the same comet**

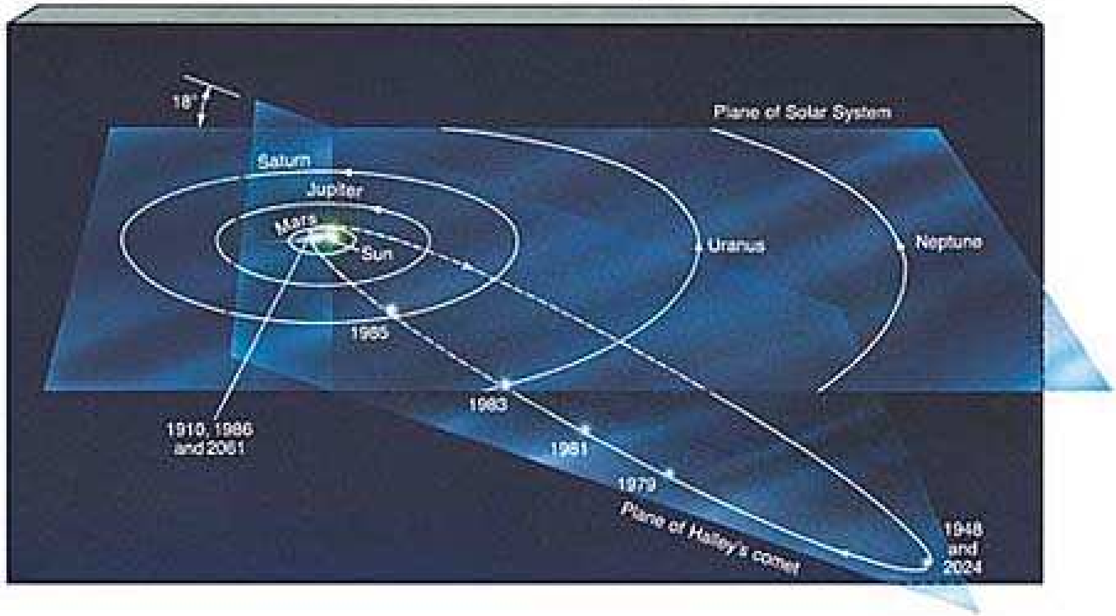
  he predicted the return of this comet in 1758

- **Halley did not live to see his prediction tested because he died in 1742**
- On Christmas night 1758 the comet destined everafter to bear Halley's name reappeared in a spectacular vindication of Newton's gravitational theory

Halley (cont'd)



Comet Halley from Mont Wilson

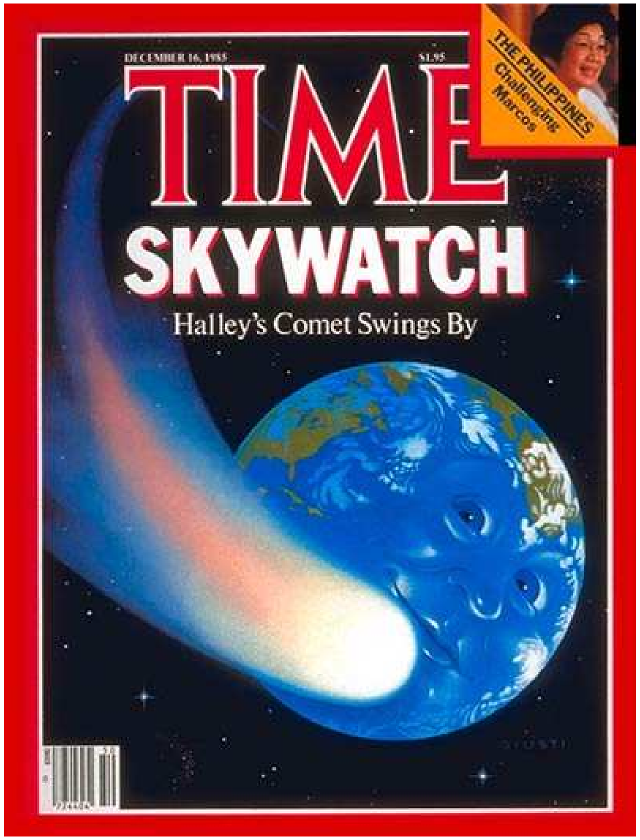


Halley (cont'd)

Tracing back in the historical records

it was concluded ☞ Halley had been observed periodically as far back as 240 B.C.

the most recent return was in 1986

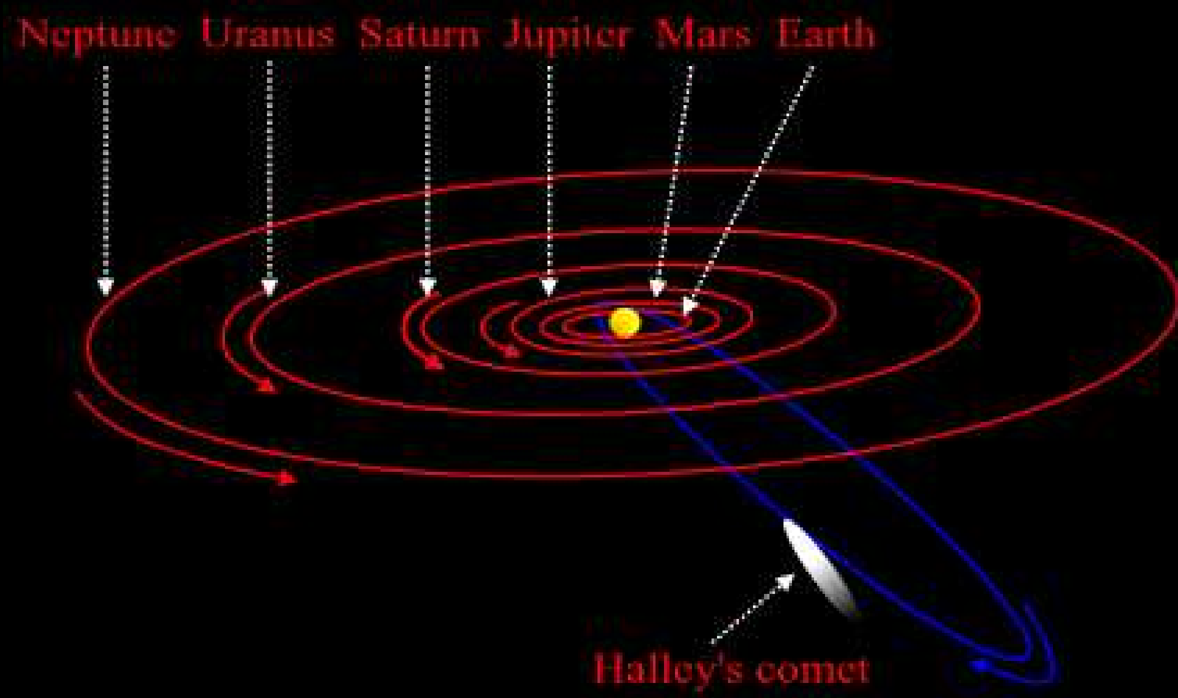


the predicted next appearance of Halley in the inner Solar System will be in 2061

Halley (cont'd)

The comet moves in a highly eliptical orbit

- eccentricity of 0.967
- period of 76 yr



CLASSWORK

Calculate its minimum and maximum distances from the Sun

Halley (cont'd)

According to third Kepler's law

$$a = \left(\frac{GM_\odot \tau^2}{4\pi^2}\right)^{1/3}$$
$$= \left[\frac{(6.67\times 10^{-11}\ \mathrm{Nm^2/kg^2})(1.99\times 10^{30}\ \mathrm{kg})(76\ \mathrm{yr}\ \pi\times 10^7\ \mathrm{s/yr})^2}{4\pi^2}\right]^{1/3}$$
$$= 2.68\times 10^{12}\ \mathrm{m}$$

Perihelion distance

$$r_p = a(1-e) = 2.68\times 10^{12}\ \mathrm{m}\,(1-0.967) = 8.8\times 10^{10}\ \mathrm{m}$$

Aphelion distance

$$r_a = a(1+e) = 2.68\times 10^{12}\ \mathrm{m}\,(1+0.967) = 5.27\times 10^{12}\ \mathrm{m}$$

Orbital Energies

Recall that

$$\mathcal{E} = \frac{v^2}{2} - \frac{G\,M}{r}$$

and that

$$\vec{v} = \dot{r}\ \boldsymbol{e}_r + r\,\dot{\theta}\ \boldsymbol{e}_\theta$$

$$\Downarrow$$

the total energy per unit mass of an object in orbit around the Sun is

$$\mathcal{E} = \frac{\dot{r}^{\,2} + r^2\,\dot{\theta}^{\,2}}{2} - \frac{G\,M}{r}$$

using

- $h = r^2\,\dot{\theta}$
- $\dot{r} = -h\,\frac{du}{d\theta}$
- $r_c = \frac{h^2}{G\,M}$
- $u = r^{-1}$
- r_c ☞ latus rectum of the orbit
- $u_c = r_c^{-1}$

$$\Downarrow$$

$$\mathcal{E} = \frac{h^2}{2}\left[\left(\frac{du}{d\theta}\right)^2 + u^2 - 2\,u\,u_c\right]$$

Orbital Energies (cont'd)

Recall that

$$u(\theta) = u_c\,(1 - e\,\cos\theta)$$

$$\Downarrow$$

$$\mathcal{E} = \frac{u_c^2\,h^2}{2}\,(e^2 - 1) = \frac{G\,M}{2\,r_p}\,(e - 1)$$

We conclude that

- **elliptical orbits ($e < 1$) have negative total energies**
- **parabolic orbits ($e = 1$) have zero total energies**
- **hyperbolic orbits ($e > 1$) have positive total energies**
- This makes sense ☞ in a conservative system with $U(\infty) = 0$ we expect
  - bounded orbits to have negative total energies
  - unbounded orbits to have positive total energies
- elliptical orbits (which are bounded) should have negative total energies
- hyperbolic orbits (which are unbounded) should have positive total energies
- Parabolic orbits are marginally bounded
  (objects executing parabolic orbits just escapes from the Sun's $\vec{g}$-field)
  ☞ have zero total energy

Orbital Dynamics

Consider an artificial satellite in an elliptical orbit around the Sun
(the same considerations also apply to satellites in orbit around the Earth)

at perihelion

$$\dot{r} = 0 \Rightarrow \frac{v_t}{v_c} = \sqrt{1+e}$$

$v_t = r\,\dot{\theta}$ ☞ satellite's tangential velocity

$v_c = \sqrt{G\,M/r_p}$ ☞ tangential velocity needed to maintain circular orbit at the perihelion distance

at aphelion

$$\frac{v_t}{v_c} = \sqrt{1-e}$$

$v_c = \sqrt{G\,M/r_a}$ ☞ tangential velocity that the satellite would need in order to maintain a circular orbit at the aphelion distance

Orbital Dynamics (cont'd)

Suppose that our satellite is initially in a circular orbit of radius r_1
we wish to transfer it into a circular orbit of radius r_2 ☞ $r_2 > r_1$
We can achieve this by temporarily placing the satellite in an elliptical orbit whose perihelion distance is r_1 and whose aphelion distance is r_2

$$\Downarrow$$

the required eccentricity of the elliptical orbit is

$$e = \frac{r_2 - r_1}{r_2 + r_1}$$

we can transfer our satellite from its initial circular orbit
into the temporary elliptical orbit by increasing its tangential velocity by a factor
(by briefly switching on the satellite's rocket motor)

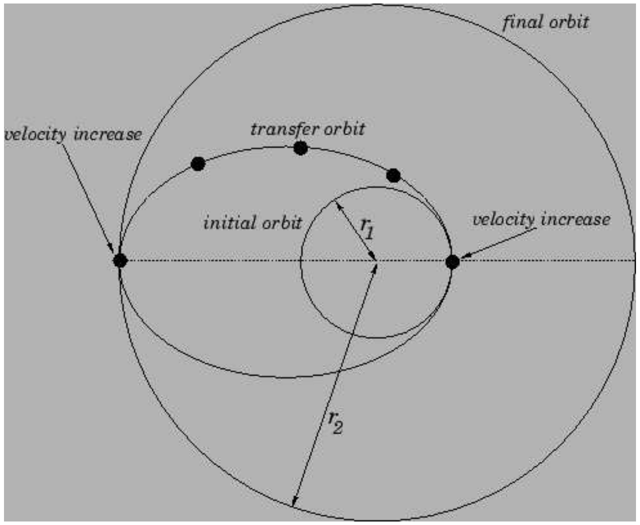
$$\alpha_1 = \sqrt{1 + e}$$

We must allow the satellite to execute half an orbit to attain its aphelion distance
and then boost the tangential velocity by a factor

$$\alpha_2 = \frac{1}{\sqrt{1 - e}}$$

The satellite will now be in a circular orbit at the aphelion distance r_2

Orbital Dynamics (cont'd)



we can transfer our satellite from a larger to a smaller circular orbit by performing the above process in reverse

Since

$$\frac{v_t}{v_c} = \sqrt{1+e}$$

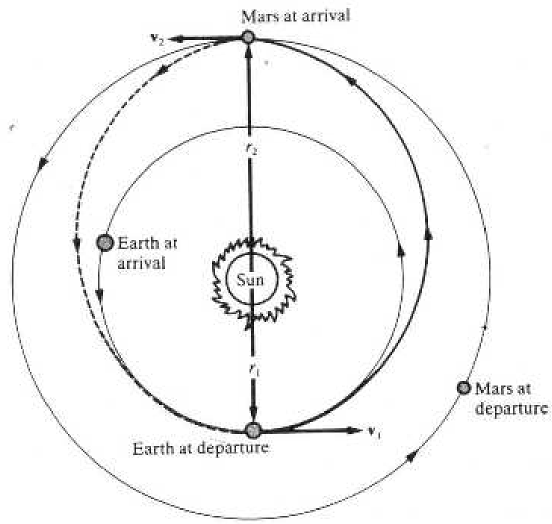
if we increase the tangential velocity of a satellite in a circular orbit about the Sun by a factor greater than $\sqrt{2}$

$$\Downarrow$$

we will transfer it into a hyperbolic orbit ☞ $e > 1$
and it will eventually escape from the Sun's gravitational field

Mission to Mars

Mission to Mars (cont'd)



Walter Hohmann ☚ pioneer in space travel research propose in 1925
most energy efficient method of transferring between coplanar eliptical orbits
using only two velocity changes
The semimajor axis of the transfer ellipse

$$2a_{\rm t} = r_1 + r_2$$

the energy transfer is then

$$\mathcal{E}_{\rm t} = -\frac{GM_\odot}{r_1 + r_2} = \frac{1}{2} v_{{\rm t},1}^2 - \frac{GM_\odot}{r_1}$$

Mission to Mars (cont'd)

$$v_{\mathrm{t},1} = \sqrt{\frac{2GM_\odot}{r_1}\left(\frac{r_2}{r_1 + r_2}\right)}$$

$$\Delta v_1 = v_{\mathrm{t},1} - v_1$$

Likewise ☞ to come back

$$\Delta v_2 = v_2 - v_{\mathrm{t},2}$$

$$v_{\mathrm{t},2} = \sqrt{\frac{2GM_\odot}{r_2}\left(\frac{r_1}{r_1 + r_2}\right)}$$

The total time required to make the transfer T_t is half a period of the transfer orbit

$$T_\mathrm{t} = \pi\sqrt{\frac{1}{GM_\odot}}a_\mathrm{t}^{3/2}$$

Mission to Mars (cont'd)

Example

Calculate the time needed for a spacecraft to make a Hohmann transfer from Earth to Mars and the heliocentric transfer speed required (assume both planets are in coplanar orbits)

$$
\begin{aligned}
a_{\rm t} &= \frac{1}{2}(R_{\rm Earth-sun} + R_{\rm Mars-Sun}) \\
&= \frac{1}{2}(1.5 \times 10^{11}~{\rm m} + 2.28 \times 10^{11}~{\rm m}) \\
&= 1.89 \times 10^{11}~{\rm m}
\end{aligned}
$$

$$
\begin{aligned}
T_{\rm t} &= \pi(7.53 \times 10^{-21}{\rm s}^2/{\rm m}^3)^{1/2}(1.89 \times 10^{11}{\rm m})^{1/2} \\
&= 259~{\rm days}
\end{aligned}
$$

$$
\begin{aligned}
v_{\rm t,1} &= \left[\frac{2(1.33 \times 10^{20}{\rm m}^3/{\rm s}^2)(2.28 \times 10^{11}{\rm m})}{(1.5 \times 10^{11}{\rm m})(3.78 \times 10^{11}{\rm m}}\right]^{1/2} \\
&= 3.27~{\rm km/s}
\end{aligned}
$$

Mission to Mars (cont'd)

the orbital speed of the Earth is

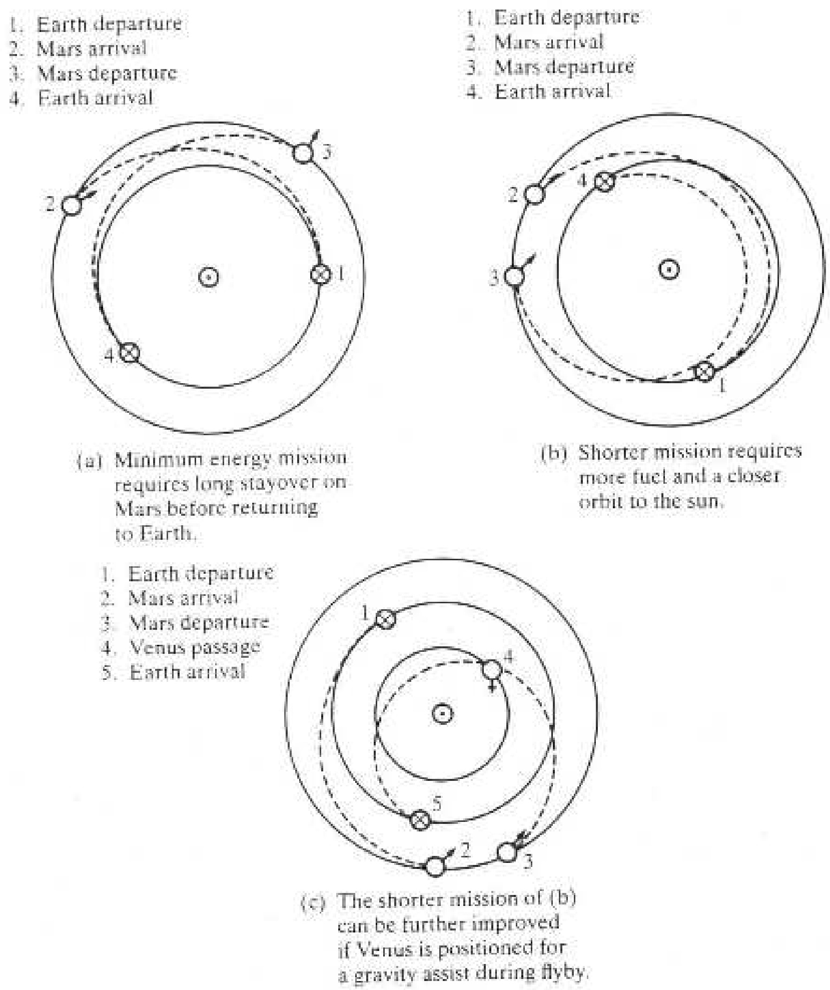
$$v_1 = \left[\frac{1.33 \times 10^{20} m^3/s^2}{1.5 \times 10^{11} m}\right]^{1/2}$$
$$= 29.8 \text{ km/s}$$

Hohmann transfer path represents the least energy expediture
it does not represent the shortest time
for a roundtrip to Mars the spacecraft would have to remain on Mars for 460 days
until the Earth and Mars were position correctly for the return trip

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

the total trip ☞ 259 + 460 + 259 = 978 = 2.7 yr

Mission to Mars (cont'd)



(a) Minimum energy mission requires long stayover on Mars before returning to Earth.

(b) Shorter mission requires more fuel and a closer orbit to the sun.

(c) The shorter mission of (b) can be further improved if Venus is positioned for a gravity assist during flyby.