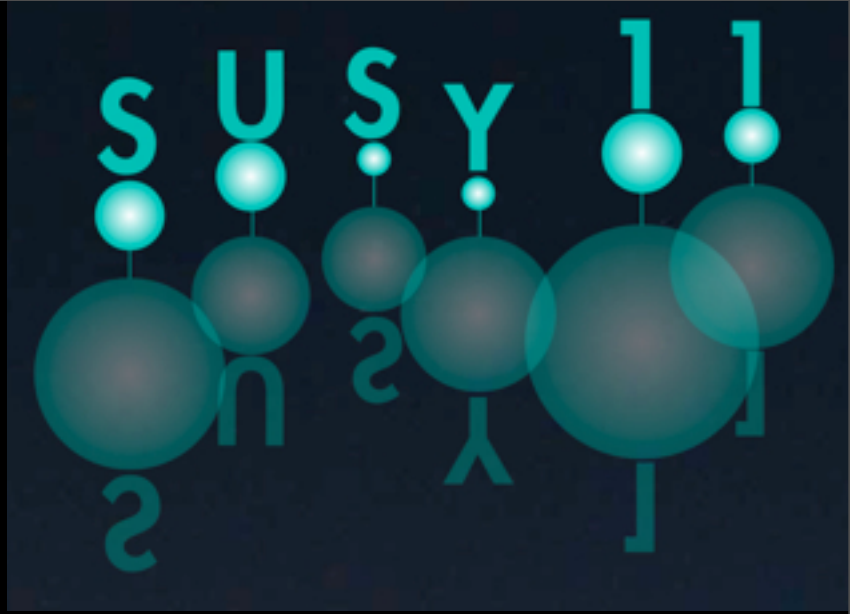

Particle Physics 2011

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A Long Time Ago, in Galaxies Far, Far Away...

BIG-BANG COSMOLOGY

- Elementary particles and cosmology
  seem to be completely different branches of physics
  one concerned with universe's elementary constituents
  and other concerned with universe as a whole
  Most powerful particle accelerators have recreated conditions
  that existed in universe just a fraction of a second after Big-Bang
  opening a window to very early history of universe
- A flood of high-quality data from Supernova Cosmology Project,
  Supernova Search Team, Wilkinson Microwave Anisotropy Probe(WMAP)
  and Sloan Digital Sky Survey (SDSS) pin down cosmological parameters
  to percent-level precision establishing a new paradigm of cosmology
- Standard Big-Bang model assumes homogeneity and isotropy
- A surprisingly good fit to data is provided by
  a simple geometrically flat (expanding) universe
  in which 30% of energy density is in form of non-relativistic matter
  and 70% is in form of a new unknown dark energy component
  (with strongly negative pressure)
- Adding to the puzzle
  baryons represent about 4% of matter-energy budget of universe

FRIEDMANN EQUATIONS

Most general form for metric tensor (consistent with WMAP & SDSS data)
is that of flat Robertson-Walker spacetime
which in co-moving coordinates is given by

$$ds^2 = dt^2 - a^2(t)\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]$$

$a(t)$ distinguishes RW metric from flat Minkowski space

(co-moving volume is a volume where expansion effects are removed)

It is common to assume ☛ matter content of universe is a perfect fluid

Friedmann equations ☛

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N \rho}{3} + \frac{\Lambda}{3} \quad ☣$$

and

$$\frac{\ddot{a}}{a} = \frac{\Lambda}{3} - \frac{4\pi G_N}{3}(\rho + 3p) \quad ☬$$

are result of applying general relativity (with a perfect fluid source)
to a (3+1)-dimensional spacetime that is homogeneous and isotropic

$H(t)$ ☛ Hubble parameter $\qquad G_N = M_{\rm Pl}^{-2}$ ☛ Newton's constant

Λ ☛ cosmological constant

p and ρ ☛ pressure and energy density of matter and radiation

COSMOLOGICAL PARAMETERS

Energy conservation leads to a third useful equation

$$\dot{\rho} = -3H(\rho + p)$$

Expansion rate of universe as a function of time
can be determined by specifying matter or energy content
through an equation of state which relates energy density to pressure

For perfect fluid ☛ eq. of state characterized by dimensionless number

$$\omega = p/\rho$$

Aside from well-known Hubble parameter
it is useful to define several other measurable cosmological parameters

Friedmann equation can be used to define a critical density

such that when $\Lambda = 0$ ☛ $$\rho_c \equiv \frac{3H^2}{8\pi G_N} = 1.05 \times 10^{-5} h^2 \text{ GeV} \text{cm}^{-3}$$

scaled Hubble parameter h defined by ☛ $$H = 100\, h \text{ km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$$

cosmological density parameter
is defined as energy density relative to critical density $$\Omega_{\text{tot}} = \rho/\rho_c$$

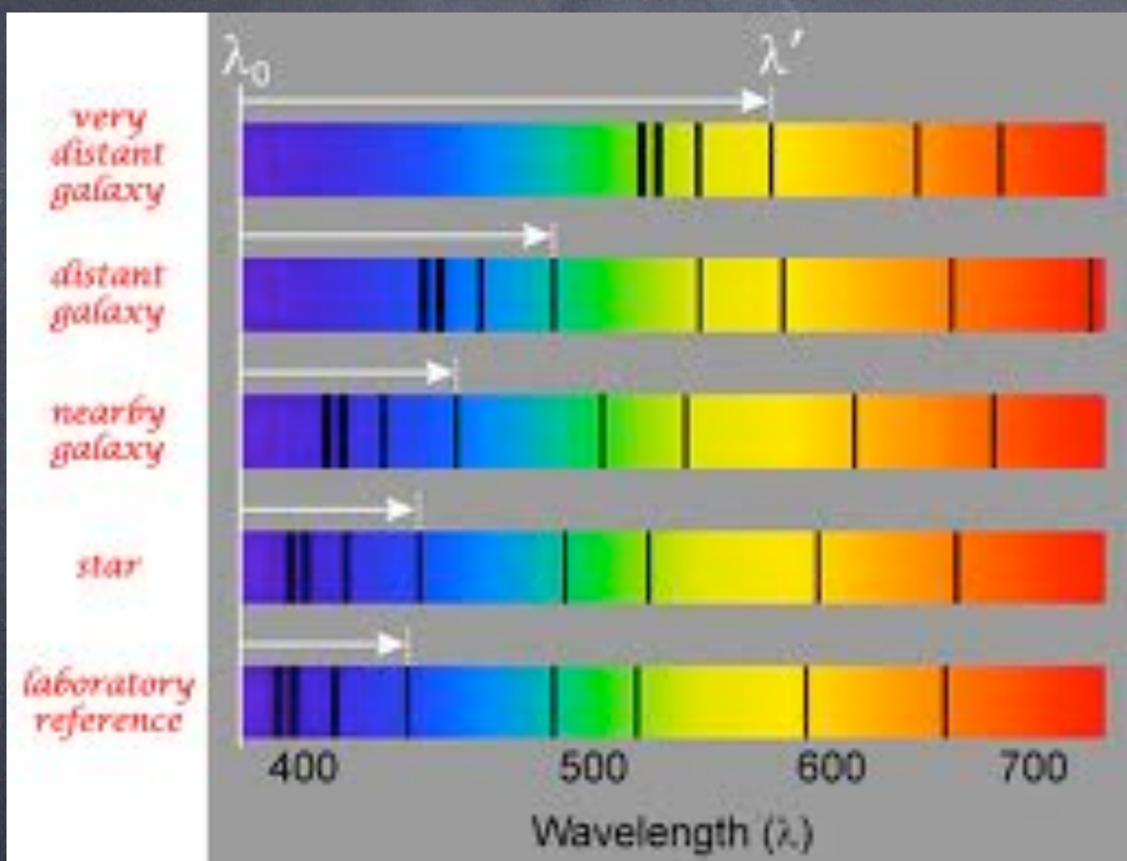
REDSHIFT

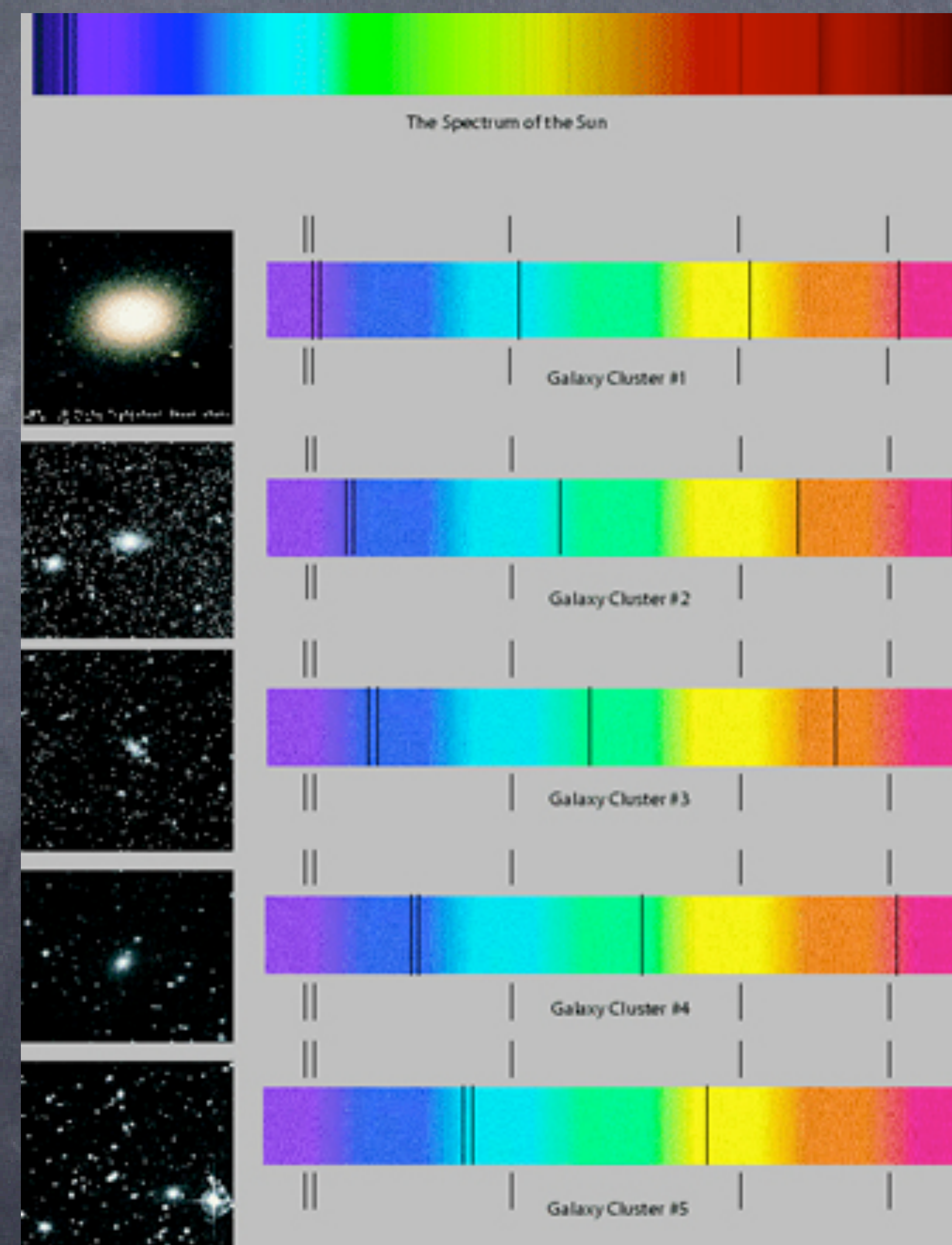
Since universe is expanding galaxies should be moving away from each other

we should observe galaxies receding from us

Recall that wavelength of light emitted from a receding object

is stretched out so that observed wavelength is larger than emitted one



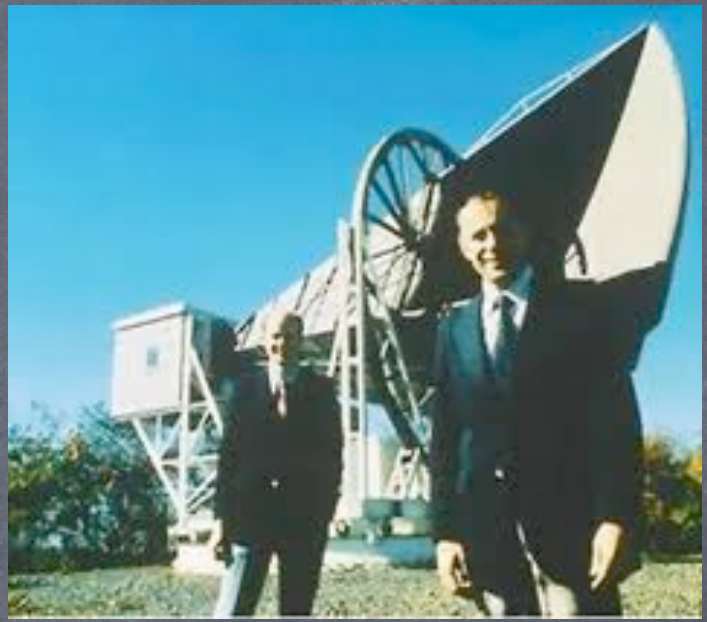


Convenient to define this stretching factor as

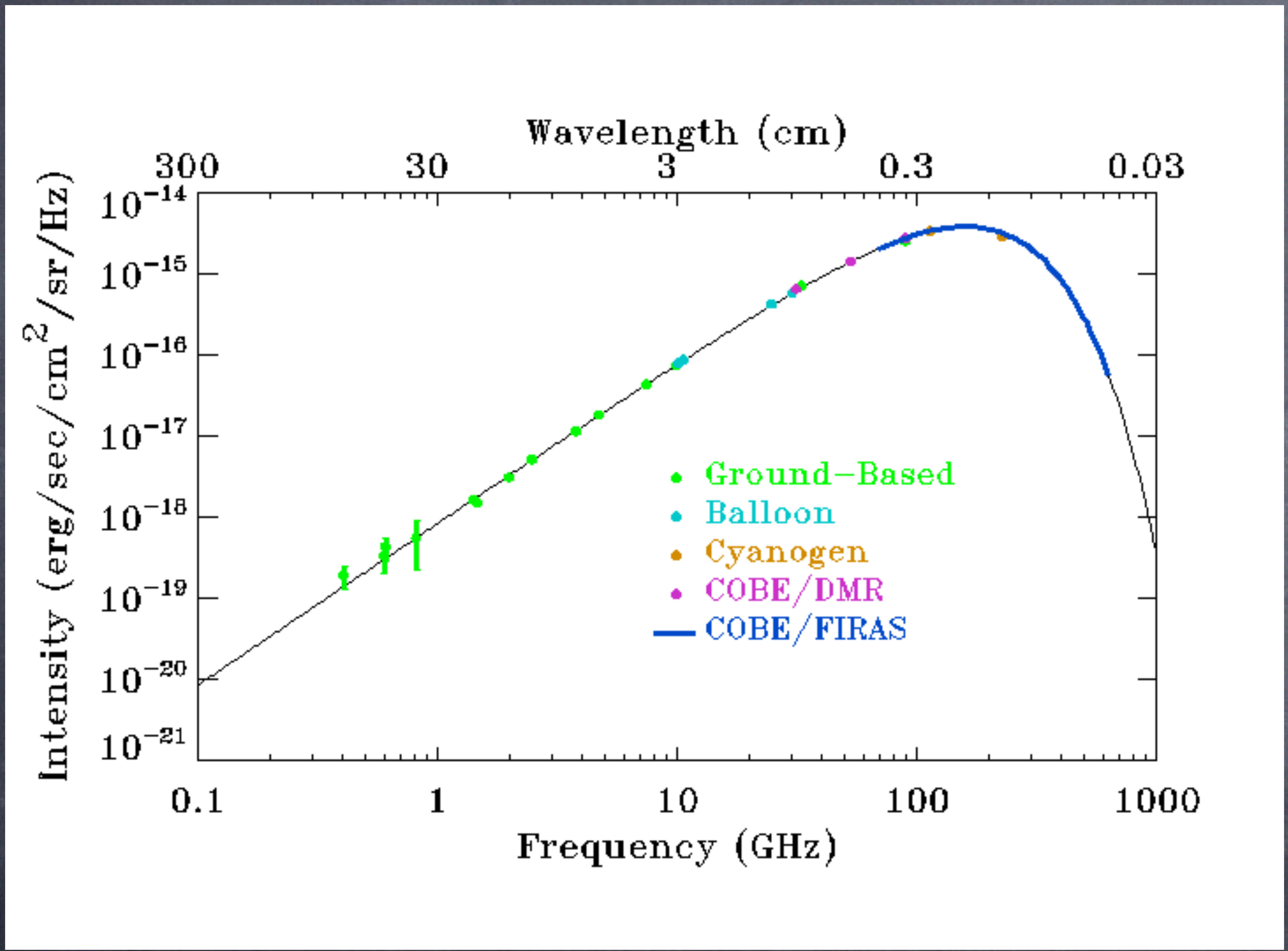
$$1 + z \equiv \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \frac{1}{a}$$

CMB

Perhaps the most conclusive piece of evidence for Big-Bang is the CMB (discovered by chance in 1965)



One fascinating feature of CMB is its Planck spectrum

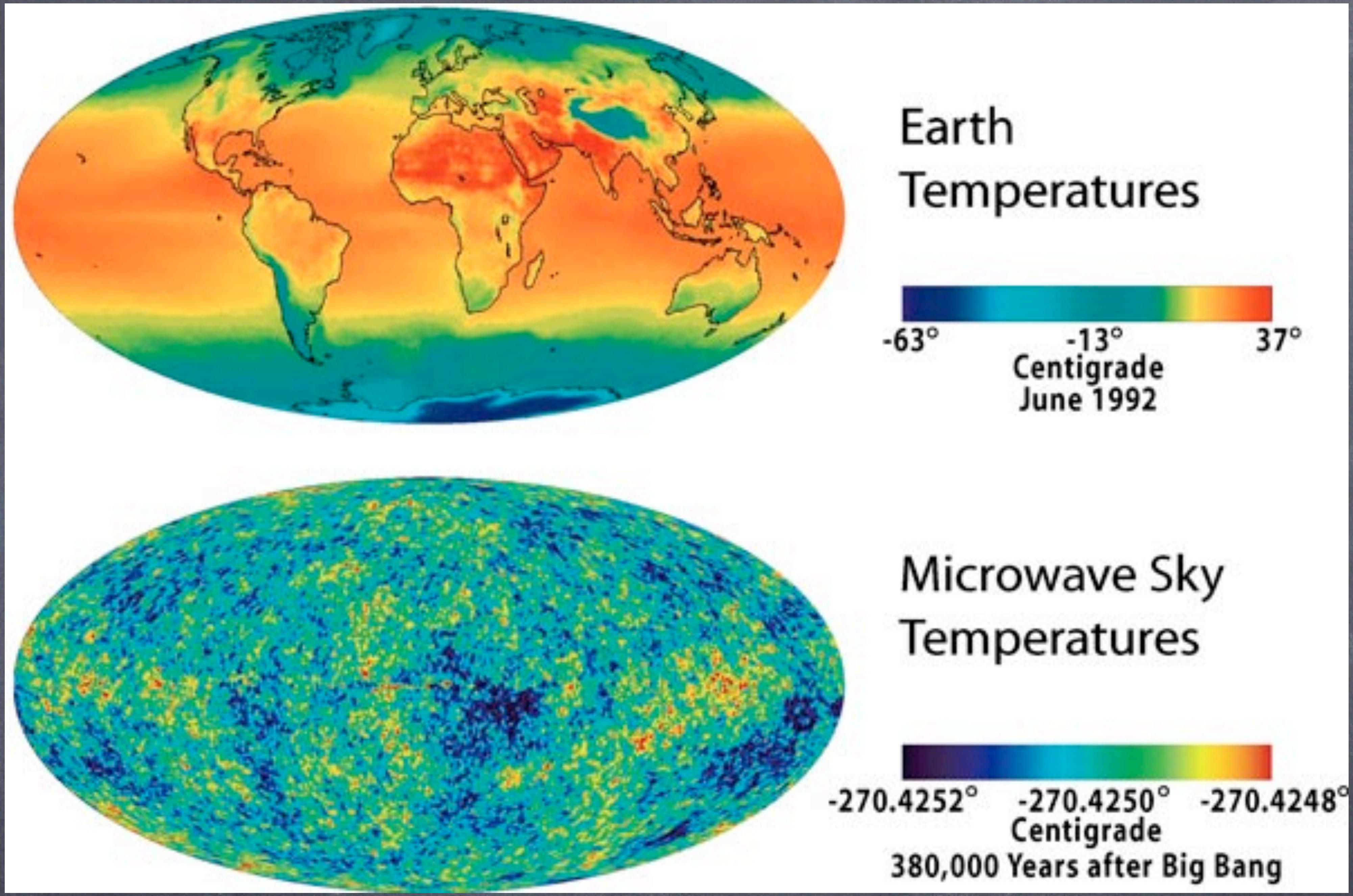


to extremely high precision over more than three decades in frequency

it follows blackbody curve at a temperature $T_\gamma^{\rm CMB} = 2.725 \pm 0.001\ {\rm K}\,(1\sigma)$

universe was in thermal equilibrium when these photons were last scattered

MORE ON CMB



An even more fascinating feature is that ☛ to better than a part in 10^5 CMB temperature is same over entire sky

This strongly suggests that everything in observable universe was in thermal equilibrium at one time in its evolution

EQUILIBRIUM THERMODYNAMICS

Because early universe was to a good approximation in thermal equilibrium particle reactions can be modeled using tools of statistical mechanics

For dilute weakly-interacting gas of particles with g internal d.o.f.

number density
$$n = \frac{g}{(2\pi)^3} \int f(\vec{p}\,) d^3p\,,$$

energy density
$$\rho = \frac{g}{(2\pi)^3} \int E(\vec{p}\,)\, f(\vec{p}\,) d^3p\,,$$

pressure
$$p = \frac{g}{(2\pi)^3} \int \frac{|\vec{p}\,|^2}{3E}\, f(\vec{p}\,)\, d^3p$$

are given in terms of its phase space distribution function $f(\vec{p})$

(or occupancy)

with $E^2 = \vec{p}^{\;2} + m^2$

PHASE SPACE OCCUPANCY f

- For a particle species of type i in kinetic equilibrium

$$\boxed{f(\vec{p}_i\ ) = \frac{1}{e^{(E_i-\mu_i)/T_i} \pm 1}}$$

is given by familiar Fermi-Dirac or Bose-Einstein distributions

T_i is temperature

μ_i is chemical potential (if present)

and $\pm$ corresponds to either Fermi or Bose statistics

- If species of type i is in chemical equilibrium its chemical potential

is related to chemical potentials of other species with which it interacts

e.g. if $\boxed{i + j \leftrightarrow k + l}$

then $\mu_i + \mu_j = \mu_k + \mu_l$ whenever chemical equilibrium holds

FROM EQUILIBRIUM DISTRIBUTIONS...

- it follows that for a particle species of mass m_i

$$\rho_i = \frac{g_i}{2\pi^2}\int_{m_i}^{\infty}\frac{(E_i^2-m_i^2)^{1/2}}{e^{(E_i-\mu_i)/T_i}\pm 1}\,E_i^2\,dE_i\,,$$

$$n_i = \frac{g_i}{2\pi^2}\int_{m_i}^{\infty}\frac{(E_i^2-m_i^2)^{1/2}}{e^{(E_i-\mu_i)/T_i}\pm 1}\,E_i\,dE_i\,,$$

$$p_i = \frac{g_i}{6\pi^2}\int_{m_i}^{\infty}\frac{(E_i^2-m_i^2)^{3/2}}{e^{(E_i-\mu_i)/T_i}\pm 1}\,dE_i$$

where g_i counts total degrees of freedom for type i

- Entropy density is ☛ $s_i = \dfrac{\rho_i + p_i - \mu_i n_i}{T_i}$

- In SM ☛ a chemical potential is often associated with baryon number
  and since net baryon density relative to photon density
  is known to be very small ☛ $\mathcal{O}(10^{-10})$
  we can neglect any such chemical potential
  when computing total thermodynamic quantities

BOSONS & FERMIONS STATISTICS

❄ For a nondegenerate $(T_i \gg \mu_i)$ relativistic species $(T_i \gg m_i)$ we have

$$
\begin{aligned}
n_i &= \begin{cases} \frac{1}{\pi^2}\,\zeta(3)\, g_i T_i^3 & \text{for bosons} \\ \frac{3}{4}\frac{1}{\pi^2}\,\zeta(3)\, g_i T_i^3 & \text{for fermions} \end{cases}, \\
\rho_i &= \begin{cases} \frac{\pi^2}{30}\, g_i\, T_i^4 & \text{for bosons} \\ \frac{7}{8}\,\frac{\pi^2}{30}\, g_i\, T_i^4 & \text{for fermions} \end{cases}, \\
p_i &= \rho_i/3
\end{aligned}
$$

♜

where $\zeta(3) = 1.20206\ldots$ is Riemann Zeta function of 3

❄ On the other hand ☛ for a nonrelativistic particle species $(T_i \ll m_i)$ relevant statistical quantities follow a Maxwell-Boltzmann distribution and thus there is no difference between fermions and bosons

$$
\begin{aligned}
n_i &= g_i \left(\frac{m_i T_i}{2\pi}\right)^{3/2} e^{-m_i/T_i}, \\
\rho_i &= m_i\, n_i\,, \\
p_i &= n_i T_i \ll \rho_i
\end{aligned}
$$

❥

MORE ON BOSONS & FERMIONS STATISTICS

❄ For a nongenerate relativistic species average energy per particle is

$$\langle E_i \rangle = \rho_i / n_i \begin{cases} \dfrac{\pi^4}{30\zeta(3)} T_i & \simeq 2.701\, T_i \text{ for bosons} \\ \dfrac{7\pi^4}{180\zeta(3)} T_i & \simeq 3.151\, T_i \text{ for fermions} \end{cases},$$

whereas for a non-relativistic species $\langle E_i \rangle = m_i + \dfrac{3}{2} T_i$

❄ For photons

we can compute all of thermodynamic quantities rather easily

$$\rho_\gamma = \frac{\pi}{15} T_\gamma^4; \quad p_\gamma = \frac{1}{3}\rho_\gamma; \quad s_\gamma = \frac{4\rho_\gamma}{3T_\gamma}; \quad n_\gamma = \frac{2\zeta(3)}{\pi^2} T_\gamma^3$$

❄ In limit $T \gg m_i$ total energy density can be conveniently expressed by

$$\rho_R \quad = \quad \left(\sum_B g_B + \frac{7}{8} \sum_F g_F \right) \frac{\pi^2}{30} T^4 \equiv \frac{\pi^2}{30} N(T)\, T^4 \quad ♣$$

where $g_{B(F)}$ is total number of boson (fermion) degrees of freedom and sum runs over all boson (fermion) states with $m_i \ll T$

(factor of $7/8$ is due to difference between Fermi and Bose integrals)

SM EFFECTIVE NUMBER OF DEGREES OF FREEDOM

$$\rho_R = \frac{\pi^2}{30} N(T)\, T^4$$ defines effective number of d.o.f. $N(T)$ by taking account new particle d.o.f. as temperature is raised

Change in $N(T)$ (ignoring mass effects) is given in

Temperature	New particles	$4N(T)$
$T < m_e$	γ's + ν's	29
$m_e < T < m_\mu$	$e^\pm$	43
$m_\mu < T < m_\pi$	$\mu^\pm$	57
$m_\pi < T < T_c^*$	π's	69
$T_c < T < m_{\rm charm}$	- π's + $u, \bar{u}, d, \bar{d}, s, \bar{s}$ + gluons	247
$m_c < T < m_\tau$	$c, \bar{c}$	289
$m_\tau < T < m_{\rm bottom}$	$\tau^\pm$	303
$m_b < T < m_{W,Z}$	$b, \bar{b}$	345
$m_{W,Z} < T < m_{\rm Higgs}$	$W^\pm, Z$	381
$m_H < T < m_{\rm top}$	H^0	385
$m_t < T$	$t, \bar{t}$	427

T_c ☛ confinement-deconfinement transition between quarks and hadrons

At higher temperatures $N(T)$ will be model dependent

TEMPERATURE TIME RELATION

At early times $t < 10^5$ yr

universe is thought to have been dominated by radiation

equation of state can be given by ☛ $\omega = 1/3$

If we neglect contributions to H from Λ

(this is always a good approximation for small enough a)

then we find that $a \sim t^{1/2}$ and $\rho_R \sim a^{-4}$

Substituting ♣ into ☣

we can rewrite expansion rate as a function of temperature in plasma

$$H = \left(\frac{8\pi G_N \rho_R}{3}\right)^{1/2} = \left(\frac{8\pi^3}{90} N(T)\right)^{1/2} T^2/M_{\rm Pl} \sim 1.66\sqrt{N(T)}T^2/M_{\rm Pl}$$ ☁

Neglecting T-dependence of N

(i.e. away from mass thresholds and phase transitions)

integration of ☁ yields useful commonly used approximation

$$t \simeq \left(\frac{3M_{\rm Pl}^2}{32\pi\rho_R}\right)^{1/2} \simeq 2.42 \frac{1}{\sqrt{N(T)}} \left(\frac{T}{\rm MeV}\right)^{-2} \text{ s}$$

MATTER-RADIATION EQUALITY

Universe made transition between radiation and matter domination

when $\rho_R = \rho_m$ or when $T \simeq$ few $\times\ 10^3$ K at $z_{\rm eq} \sim 3300$

For a matter or dust dominated universe $\omega = 0$

and therefore $a(t) \sim t^{2/3}$ and $\rho_m \sim a^{-3}$

DARK ENERGY

In a vacuum or Λ dominated universe

(which we are approaching today)

$\omega = -1$ yielding $a \sim e^{\sqrt{\Lambda/3}\,t}$

Current best measurement of equation of state (assumed constant) is

$$\omega_{z=0} = -1.006^{+0.067}_{-0.068}$$

ISENTROPIC DYNAMICS

For a system in thermodynamic equilibrium

$$\boxed{\dot{\rho} = -3H(\rho + p)}$$

can be converted into eq. for conservation of entropy per co-moving V

Recognizing that $\dot{p} = s\dot{T}$ becomes

$$\boxed{\frac{d}{dt}(sa^3) = 0}$$

A non-evolving system
would stay at constant s and n in co-moving coordinates
even though number or entropy density is in fact decreasing
due to the expansion of universe

For radiation
this corresponds to relationship between expansion and cooling
$T \propto a^{-1}$ in an adiabatically expanding universe

Note that both s and n scale as T^3

BBN

- Nucleosynthesis taking place in primordial plasma
  is undoubtedly an observational pillar of standard cosmological model
  indeed known simply as big-bang nucleosynthesis (BBN)

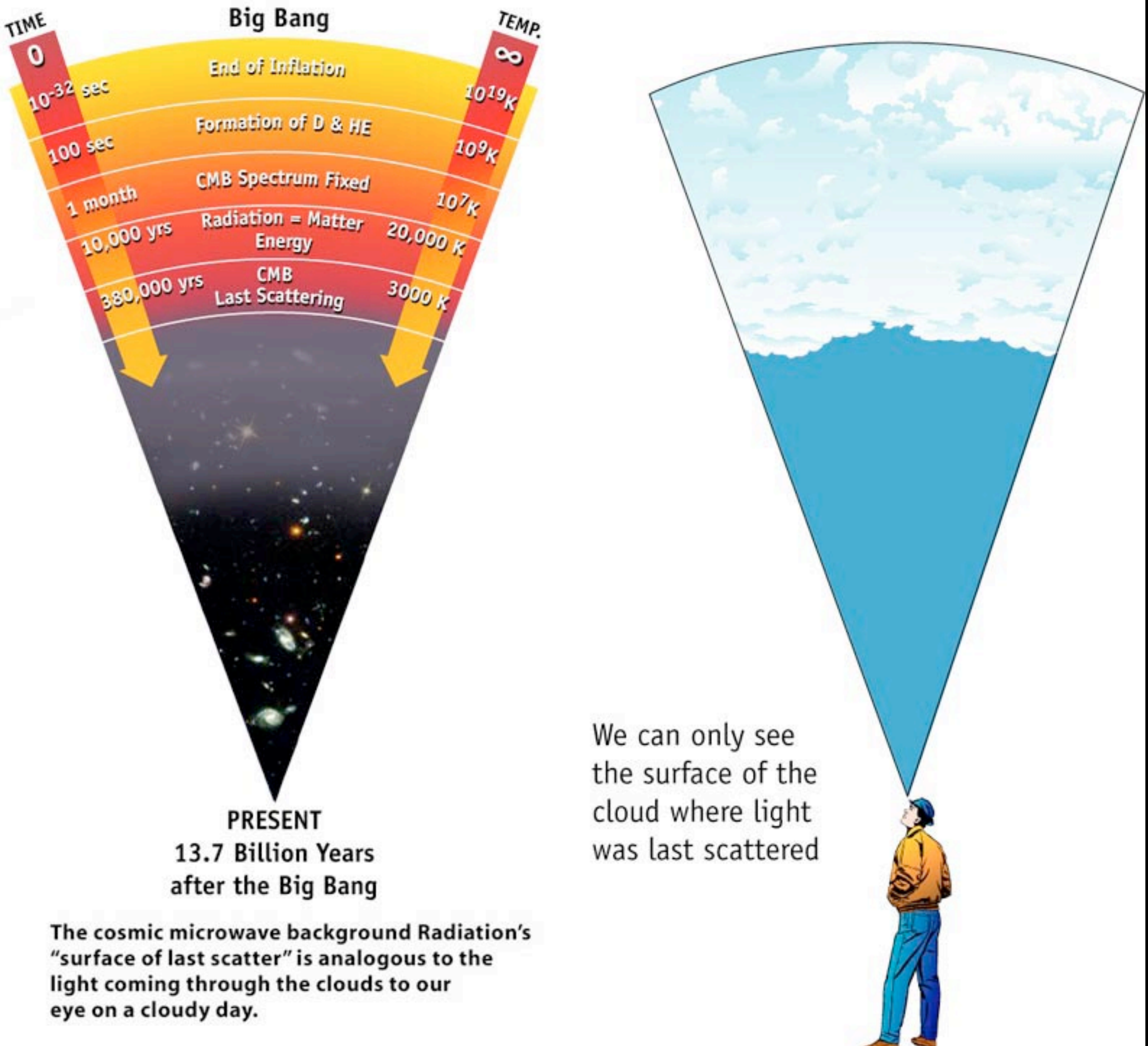
- BBN probes evolution of universe during its first few minutes
  providing a glimpse into its earliest epochs $(z \sim 10^8)$

- Physical processes involved
  (which have been well-understood for some time)
  interrelate four fundamental interactions:
  gravity sets dynamics of expanding cauldron
  weak interactions determine neutrino decoupling
  and neutron-proton equilibrium freeze-out

  and
  electromagnetic & nuclear processes regulate nuclear reaction network

- Final abundance of synthesized elements
  is sensitive to a variety of parameters and physical constants
  allowing many interesting probes on physics beyond SM

Lookback Time



The cosmic microwave background Radiation's "surface of last scatter" is analogous to the light coming through the clouds to our eye on a cloudy day.

NEUTRINO COSMOLOGY

- Extrapolating present state of cosmos backwards in time we infer that at a temperature of say a few tens of MeV universe was filled with a plasma of $(p, n, \gamma, e^-, e^+, \nu,$ and $\overline{\nu})$

  (baryons are nonrelativistic while all other particles are relativistic)

- Introducing ratio of baryon number density to photon number density

$$\eta = n_{\rm b}/n_\gamma \sim 5 \times 10^{-10}$$

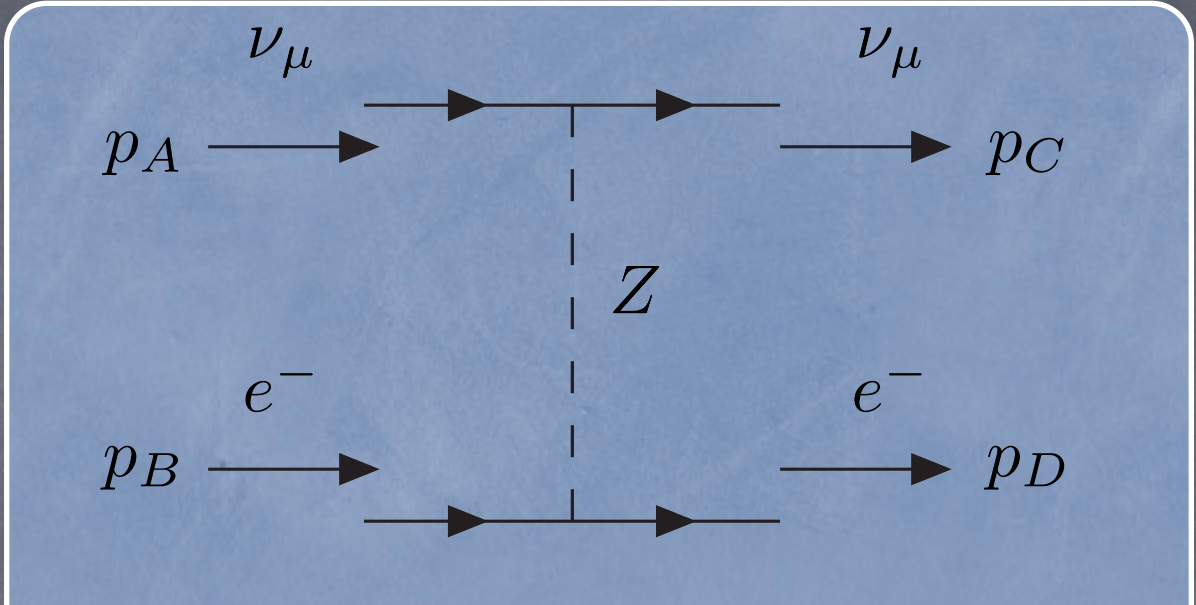
  we see that $\eta m_N/T \sim 10^{-8}$

  and thus nucleons contribute a negligible fraction to ρ_R

- These particles are kept in thermal equilibrium by various electromagnetic and weak processes of the sort

$$\bar{\nu}\nu \rightleftharpoons e^+e^-, \nu e^- \rightleftharpoons \nu e^-, n\nu_e \rightleftharpoons pe^-, \gamma\gamma \rightleftharpoons e^+e^-, \gamma p \rightleftharpoons \gamma p$$

$\nu_\mu e^-$ and $\bar{\nu}_\mu e^-$ scattering processes can only proceed via NC interaction



t-channel

For process $\nu_\mu e^- \to \nu_\mu e^-$

current-current form of invariant amplitude

$$\mathfrak{M}^{\rm NC}(\nu e \to \nu e) = \frac{\rho G_F}{\sqrt{2}} \left[\bar{\nu}\gamma^\mu(1-\gamma^5)\nu\right] \left[\bar{e}\gamma_\mu(c_V^e - c_A^e\gamma^5)e\right]$$

we take $\rho = 1$ and define the momenta according to

$$\nu_\mu(\omega, \vec{k}) + e^-(E, \vec{p}\,) \to \nu_\mu(\omega', \vec{k}') + e^-(E', \vec{p}\,')$$

since mean energies of interacting particles are of order of temperature $T \simeq \text{ MeV} \ll m_Z$

we can express averaged square amplitude (for massless electrons) as

$$|\mathfrak{M}^{\rm NC}|^2 = 16 G_F^2 [(c_V^e + c_A^e)^2 (p^\alpha k_\alpha)(p'^\alpha k'_\alpha) + (c_V^e - c_A^e)^2 (p'^\alpha k_\alpha)(p^\alpha k'_\alpha)]$$

INVARIANT AMPLITUDE & CROSS SECTION

Using $$\overline{|\mathfrak{M}|^2} = \frac{8e^4}{(k-k')^4}[(k' \,.\, p')(k \,.\, p) + (k' \,.\, p)(k \,.\, p')]$$ we rewrite Ψ as

$$= 2e^4 \frac{s^2+u^2}{t^2}$$

$$\boxed{\begin{aligned} |\mathfrak{M}^{\rm NC}|^2 &= 4G_F^2[(c_V^e + c_A^e)^2 s^2 + (c_V^e - c_A^e)^2 u^2] \\ &= G_F^2\, s^2 \left[4(c_V^e + c_A^e)^2 + (c_V^e - c_A^e)^2(1+\cos\theta)^2\right] \end{aligned}}$$ ☃

Integration over phase space $\left.\frac{d\sigma}{d\Omega}\right|_{\rm c.m.} = \frac{|\mathfrak{M}|^2}{64\pi^2 s}$ is straightforward

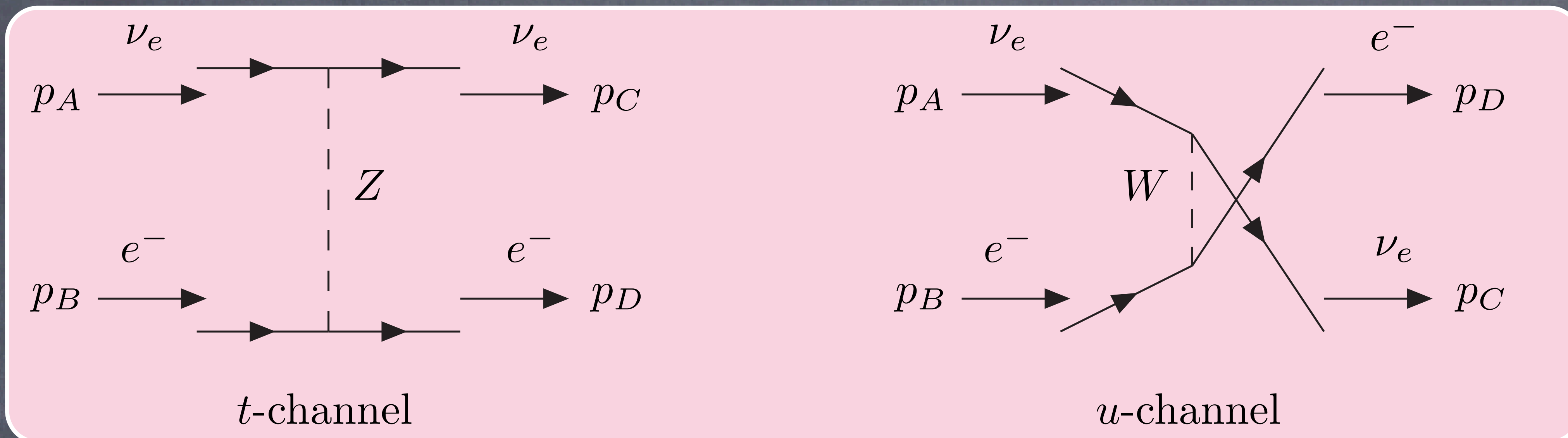
$$\boxed{\sigma(\nu_\mu e^- \to \nu_\mu e^-) = \frac{G_F^2}{3\pi}\, s\, ({c_A^e}^2 + c_A^e\, c_V^e + {c_V^e}^2)}$$ ♝

For $\bar{\nu}_\mu e^-$ elastic scattering $c_A \to -c_A$ in ☃ and so

$$\boxed{\sigma(\bar{\nu} e^- \to \bar{\nu} e^-) = \frac{G_F^2}{3\pi}\, s\, ({c_A^e}^2 - c_A^e\, c_V^e + {c_V^e}^2)}$$ ♞

CC AND NC ITERFERENCE

For $\nu_e e^- \to \nu_e e^-$ ☛ scattering amplitude comes from two diagrams
Z in t-channel and W in u-channel



Amplitude for t-channel process is $\mathfrak{M}^{\rm NC}$ of ♟ with $\nu = \nu_e$

For u-channel we have

$$\mathfrak{M}^{\rm CC} = -\frac{G_F}{\sqrt{2}} \left[\bar{\nu}_e \gamma^\mu (1-\gamma^5)\nu_e\right] \left[\bar{e}\gamma_\mu (1-\gamma^5) e\right]$$

To obtain $\mathfrak{M}(\nu_e e^- \to \nu_e e^-)$ ☛ we add amplitudes $(\mathfrak{M}^{\rm NC}$ and $\mathfrak{M}^{\rm CC})$

$\mathfrak{M} = \mathfrak{M}^{\rm NC} + \mathfrak{M}^{\rm CC}$ is given by ♟ with $\boxed{c_V \to c_V + 1, \qquad c_A \to c_A + 1}$

$\nu_e e^-$ and $\bar{\nu}_e e^-$ elastic scattering cross sections are given by ♝ and ♞

with $\boxed{c_V \to c_V + 1, \qquad c_A \to c_A + 1}$

INTERACTION RATE

From we first obtain number density of massless particles

$$n_{e^-}(T) = 0.182\, T^3$$

and then compute weak interaction rate (per neutrino species)

$$\boxed{\Gamma_{\nu_\alpha} \sim n_{e^-}\ \sigma(\nu e^- \to \nu e^-)\, v}$$

$v = p^\alpha k_\alpha/(E\omega) = (1 - \cos\theta)$ is Moller velocity

we adopt a thermal average followed by angular average on this factor

$$\boxed{\langle v\sigma\rangle_\alpha = \frac{1}{2}\int_1^1 \frac{G_F^2}{3\pi} s \mathcal{Z}_{\nu_\alpha}\,(1-\cos\theta)\, d(\cos\theta) = \frac{8}{9\pi} G_F^2\, \mathcal{Z}_{\nu_\alpha}\, \langle E\rangle\, \langle\omega\rangle}$$

$$s = 2E\omega(1-\cos\theta)$$

- $\mathcal{Z}_{\nu_\mu} = \mathcal{Z}_{\nu_\tau} = {c_V^e}^2 + c_V^e c_A^e + {c_A^e}^2$
- $\mathcal{Z}_{\nu_e} = (1 + c_V^e)^2 + (1 + c_A^e)(1 + c_V^e) + (1 + c_A^e)^2$

Electron neutrino interaction rate is $\boxed{\Gamma_{\nu_e} = 1.16 \times 10^{-22} \left(\frac{T_{\nu_e}}{\text{MeV}}\right)^5}$

NEUTRINO DECOUPLING

Comparing ☯ with expansion rate ☁ calculated for $N(T) = 10.75$

$$\boxed{H \simeq 4.46 \times 10^{-22} \left(\frac{T}{\text{MeV}}\right)^3}$$

we see that at high T weak interaction processes are fast enough

But as T drops below some characteristic $T^{\text{dec}}_{\nu_\alpha}$

neutrinos decouple - they lose thermal contact with electrons

The condition $$\boxed{\Gamma_{\nu_\alpha}(T^{\text{dec}}_{\nu_\alpha}) = H(T^{\text{dec}}_{\nu_\alpha})}$$

sets decoupling temperature for neutrinos

$$T^{\text{dec}}_{\nu_e} \approx 1.56 \text{ MeV} \quad \text{and} \quad T^{\text{dec}}_{\nu_\mu} \simeq T^{\text{dec}}_{\nu_\tau} \approx 2.88 \text{ MeV}$$

Roughly speaking all neutrino species decouple at $T^{\text{dec}}_{\nu} \approx 2 \text{ MeV}$

KINETIC & CHEMICAL EQUILIBRIUM

Much stronger electromagnetic interaction

continues to keep p, n, e^-, e^+, γ in equilibrium

Using dimensional analysis $\sigma \sim \alpha^2/m_N^2$

Then ☛ reaction rate per nucleon $\boxed{\Gamma_N \sim T^3\alpha^2/m_N^2}$

is larger than expansion rate as long as

$$\boxed{T > \frac{m_N^2}{\alpha^2 M_{\rm Pl}} \sim \text{a very low temperature}}$$

Nucleons are thus mantianed in kinetic equilibrium

Average kinetic energy per nucleon is $\frac{3}{2}T$

One must distinguish between kinetic and chemical equilibrium

Reactions like $\gamma\gamma \to p\bar{p}$ have long been suppressed

as there are essentially no anti-nucleons around

ISENTROPIC HEATING

- If a is separation between any pair of typical particles then $sa^3 \propto N(T) T^3 a^3 =$ constant
- For $T \gtrsim m_e$ particles in thermal equilibrium with photons include photon $(g_\gamma = 2)$ and $e^\pm$ pairs $(g_{e^\pm} = 4)$
- Effective # of particle species before annihilation is $N_{\rm before} = 11/2$
- After annihilation of electrons and positrons only remaining abundant particles in equilibrium are photons effective # of particle species is $N_{\rm after} = 2$
- It follows from conservation of entropy that

$$\boxed{\frac{11}{2}\,(T_\gamma a)^3\Big|_{\rm before} = 2\,(T_\gamma a)^3\Big|_{\rm after}}$$

- That is ☛ heat produced by annihilation of electrons and positrons increases quantity $T_\gamma a$ by a factor of $\boxed{\dfrac{(T_\gamma a)|_{\rm after}}{(T_\gamma a)|_{\rm before}} = \left(\dfrac{11}{4}\right)^{1/3} \simeq 1.4}$

ISENTROPIC HEATING (cont'd)

- Before annihilation of electrons and positrons neutrino temperature T_ν is same as photon temperature T_γ

- But from then on ☛ T_ν simply dropped like a^{-1} so for all subsequent times $T_\nu a$ equals value before annihilation

$$(T_\nu a)|_{\rm after} = (T_\nu a)|_{\rm before} = (T_\gamma a)|_{\rm before}$$

- We conclude therefore that after annihilation process is over photon temperature is higher than neutrino temperature by a factor of

$$\left(\frac{T_\gamma}{T_\nu}\right)\bigg|_{\rm after} = \frac{(T_\gamma a)|_{\rm after}}{(T_\nu a)|_{\rm after}} \simeq 1.4$$

EFFECTIVE NUMBER OF NEUTRINO SPECIES

Energy density stored in relativistic species is customarily given in terms of so-called effective number of light neutrino species $N_\nu^{\rm eff}$

through relation

$$\rho_R = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_\nu^{\rm eff}\right]\rho_\gamma$$

Without a doubt,

$$\begin{aligned} N_\nu^{\rm eff} &\equiv \left(\frac{\rho_R - \rho_\gamma}{\rho_\nu}\right) \\ &\simeq \frac{8}{7}\sum_B{}' \frac{g_B}{2}\left(\frac{T_B}{T_\nu}\right)^4 + \sum_F{}' \frac{g_F}{2}\left(\frac{T_F}{T_\nu}\right)^4 \end{aligned}$$

ρ_ν denotes energy density of a single species of massless neutrinos

$T_{B(F)}$ is effective temperature of boson (fermion) species

and primes indicate that electrons and photons are excluded from sums

PROBES OF $\Delta N_\nu^{\rm eff} = N_\nu^{\rm eff} - 3$

- For a good part of the past two decades
  BBN provided best inference of radiation content of the universe
  Time-dependent quantity being the neutron abundance at $t \gtrsim \tau_n$
  which regulates the primordial fraction of baryonic mass in ${}^4\mathrm{He}$

$$Y_{\rm p} \simeq 0.251 + 0.014\ \Delta N_\nu^{\rm eff} + 0.0002\ \Delta\tau_n + 0.009\ \ln\left(\frac{\eta}{5\times 10^{-10}}\right)$$

- More recently
  observations of CMB anisotropies and large-scale structure distribution
  probe $N_\nu^{\rm eff}$ at CMB decoupling epoch with unprecedented precision

$$\frac{\Delta N_\nu^{\rm eff}}{N_\nu^{\rm eff}} \simeq 2.45\ \frac{\Delta(\Omega_m h^2)}{\Omega_m h^2} - 2.45\ \frac{\Delta z_{\rm eq}}{1+z_{\rm eq}}$$

- Though significant uncertainties remain
  most recent cosmological observations show a consistent preference
  additional relativistic d.o.f. during BBN and CMB epochs

$$\Delta N_\nu^{\rm eff} = \begin{cases} 0.68^{+0.40}_{-0.35} & (1\sigma) \quad \text{BBN} \\ 1.34^{+0.86}_{-0.88} & (1\sigma) \quad \text{WMAP + BAO} + H_0 \end{cases}$$

$(\Omega_\Lambda, \Omega_m)$ PLANE

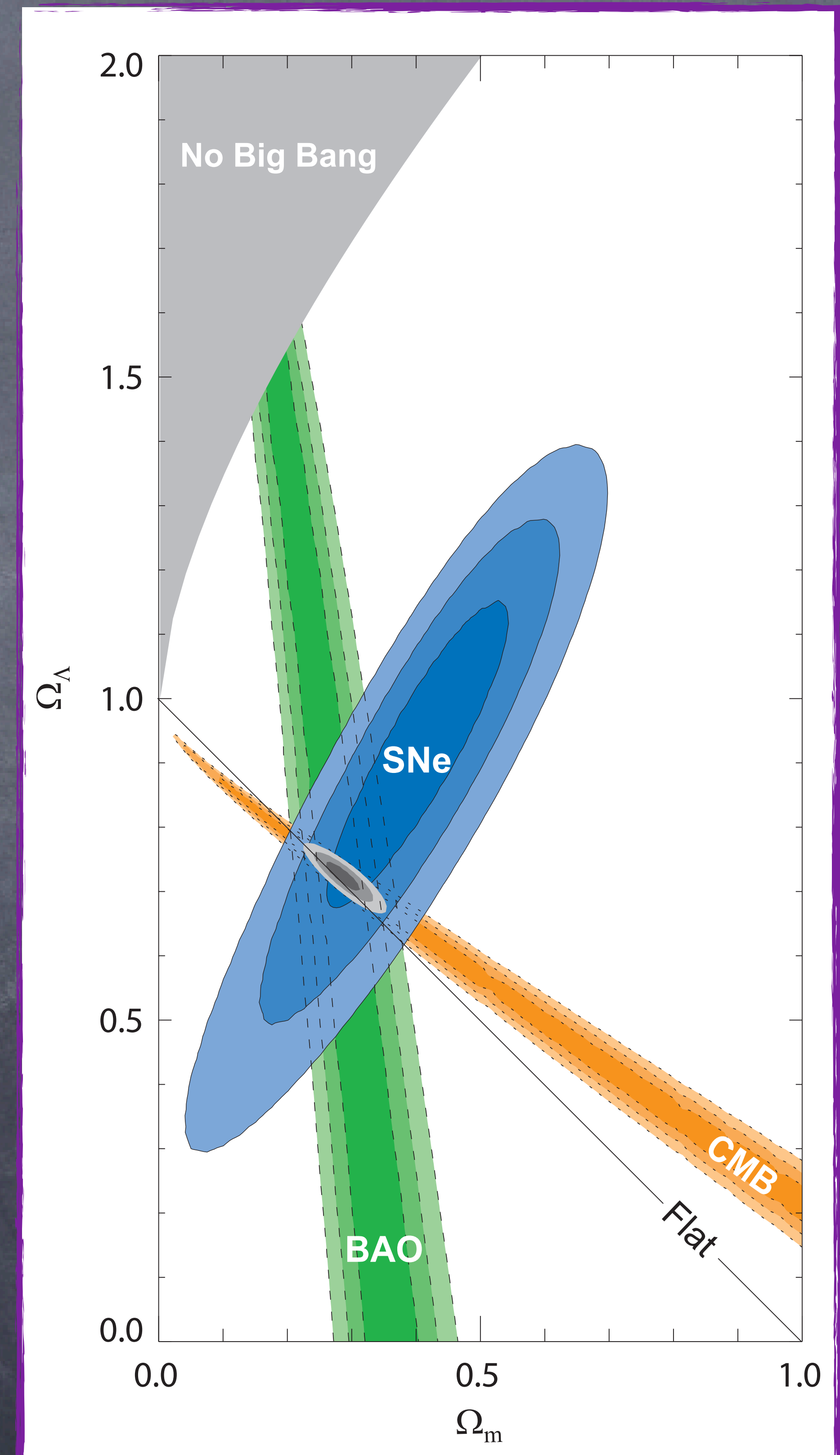
- Observations of SN CMB and BAO have provided three stringent constraints on $\Omega_{\rm M}$ and Ω_Λ

- Results favor $(\Omega_{\rm M}, \Omega_\Lambda) \approx (0.3, 0.7)$

- Baryonic matter constrained by CMB and BBN

$$\Omega_{\rm B} = 4\% \pm 0.4\% \Rightarrow \Omega_{\rm DM} = 23\% \pm 4\%$$

$$\Omega_{\rm DM} h^2 = 0.113 \pm 0.003$$



WIMPs

Particle (or particles) that make up most of dark matter must be stable
at least on cosmological time scales
and non-baryonic so that they do not disturb subprocesses of BBN
Must also be cold or warm to properly seed structure formation
and their interactions must be weak enough
to avoid violating current bounds from dark matter searches

Among plethora of dark matter candidates
weakly interacting massive particles (WIMPs)
represent a particularly attractive and well-motivated class of possibilities

This is because they combine virtues of weak scale masses and couplings
and their stability often follows as a result of discrete symmetries
that are mandatory to make electroweak theory viable
(independent of cosmology)

WIMPs are naturally produced with cosmological densities required of DM

SUSY ESSENTIALS

Rotations ☛ angular momentum operators L_i

Spacetime symmetries → Boosts ☛ boosts operators K_i

Translations ☛ momentum operators P_μ

SUSY is the symmetry that results when these 10 generators are further supplemented by fermionic operators Q_α

$$\boxed{Q_\alpha|\mathrm{Boson}\rangle = |\mathrm{Fermion}\rangle, \qquad Q_\alpha|\mathrm{Fermion}\rangle = |\mathrm{Boson}\rangle}$$

No particle of SM is superpartner of another

SUSY therefore predicts a plethora of superpartners

none of which has (yet) been discovered

MSSM PARTICLE SPECTRUM

Boson Fields	Fermionic Partners	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
g	$\widetilde{g}$	8	0	0
W^a	$\widetilde{W}^a$	1	3	0
B	$\widetilde{B}$	1	1	0
leptons $\begin{cases} \widetilde{L}^j = (\tilde{\nu}, \tilde{e}^-)_L \end{cases}$	$(\nu, e^-)_L$	1	2	$-1/2$
$\widetilde{E} = \tilde{e}^+_R$	e^c_L	1	1	1
quarks $\begin{cases} \widetilde{Q}^j = (\tilde{u}_L, \tilde{d}_L) \end{cases}$	$(u, d)_L$	3	2	$1/6$
$\widetilde{U} = \tilde{u}^*_R$	u^c_L	3^*	1	$-2/3$
$\widetilde{D} = \tilde{d}^*_R$	d^c_L	3^*	1	$1/3$
Higgs $\begin{cases} H^i_1 \end{cases}$	$(\widetilde{H}^0_1, \widetilde{H}^-_1)_L$	1	2	$-1/2$
H^i_2	$(\widetilde{H}^+_2, \widetilde{H}^0_2)_L$	1	2	$1/2$

Normal particles/fields		Supersymmetric partners			
		Interaction eigenstates		Mass eigenstates	
Symbol	Name	Symbol	Name	Symbol	Name
$q = d, c, b, u, s, t$	quark	$\tilde{q}_L,\ \tilde{q}_R$	squark	$\tilde{q}_1,\ \tilde{q}_2$	squark
$l = e, \mu, \tau$	lepton	$\tilde{l}_L,\ \tilde{l}_R$	slepton	$\tilde{l}_1,\ \tilde{l}_2$	slepton
$\nu = \nu_e, \nu_\mu, \nu_\tau$	neutrino	$\tilde{\nu}$	sneutrino	$\tilde{\nu}$	sneutrino
g	gluon	$\tilde{g}$	gluino	$\tilde{g}$	gluino
$W^\pm$	W-boson	$\tilde{W}^\pm$	wino		
H^-	Higgs boson	$\tilde{H}^-_1$	higgsino	$\tilde{\chi}^\pm_{1,2}$	chargino
H^+	Higgs boson	$\tilde{H}^+_2$	higgsino		
B	B-field	$\tilde{B}$	bino		
W^3	W^3-field	$\tilde{W}^3$	wino		
H^0_1	Higgs boson			$\tilde{\chi}^0_{1,2,3,4}$	neutralino
		$\tilde{H}^0_1$	higgsino		
H^0_2	Higgs boson				
		$\tilde{H}^0_2$	higgsino		
A^0	Higgs boson				

SUSY BREAKING

Novel feature of SUSY ☛ its boson-fermion symmetry

also posses one important drawback:

Bose-Fermi symmetry has not been observed in nature !!!

If SUSY can serve as a theory of low energy interactions

it must be a broken symmetry

If SUSY were unbroken

SM particle and its superpartner

would have same mass and quantum numbers (except for spin)

From phenomenological perspective

most interesting mechanisms responsible for SUSY breaking

are those with low-energy (or weak-scale) SUSY

in which effective scale of SUSY breaking

is tied to scale of electroweak symmetry breaking

(natural solution of hierarchy problem)

LSP

R-parity is defined by $\boxed{R_p = (-1)^{3(B-L)+2S}}$

All SM particles have $R_p = 1$ and all superpartners have $R_p = -1$

Conservation of R-parity implies $\Pi R_p = 1$ at each vertex

☛ both B and L violating processes are forbidden

An immediate consequence of R-parity conservation
is that LSP cannot decay to SM particles and is therefore stable

Particle physics constraints
naturally suggest a symmetry
that provides a new stable particle
that may contribute significantly to present energy density of universe

Lightest neutralino in R-parity conserving models of SUSY
is by far the best studied candidate for dark matter

Detecting it has become benchmark
by which experiments are evaluated and mutually compared

We will follow this tradition here

WIMP RELIC DENSITY

Generic WIMPs were once in thermal equilibrium
but decoupled while strongly non-relativistic

Consider a particle of mass m_χ in thermal equilibrium in early universe
Evolution of n_χ as universe expands is driven by Boltzmann's equation

$$\frac{dn_\chi}{dt} + 3H(T)\, n_\chi = -\langle \sigma v \rangle (n_\chi^2 - n_\chi^{\mathrm{eq}\,2})$$

n_χ^{eq} is equilibrium number density

$\langle \sigma v \rangle$ is thermally averaged annihilation cross section of χ particles
multiplied by their relative velocity

At equilibrium ❥ gives number density of a non-relativistic species

$$n_\chi^{\mathrm{eq}} = g_\chi \left(\frac{m_\chi T_\chi}{2\pi} \right)^{3/2} e^{-m_\chi/T_\chi}$$

g_χ is number of internal degrees of freedom of WIMP particle

WIMP RELIC DENSITY (cont'd)

In very early universe when $n_\chi \simeq n_\chi^{\rm eq}$

right hand side of $\dfrac{dn_\chi}{dt} + 3H(T)\, n_\chi = -\langle \sigma v \rangle (n_\chi^2 - n_\chi^{\rm eq\,2})$ is small

and evolution of density is dominated by Hubble expansion

As temperature falls below m_χ

$n_\chi^{\rm eq}$ becomes suppressed and the annihilation rate increases

rapidly reducing number density n_χ

When number density falls enough

rate of depletion due to expansion becomes greater than annihilation rate

and particles freeze-out of thermal equilibrium

Defining freeze-out temperature to be time when $n_\chi \langle \sigma v \rangle = H$ we have

$$\frac{T_\chi^{\rm FO}}{m_\chi} \equiv \frac{1}{x_{\rm FO}} \simeq \left[\ln \left(\sqrt{\frac{45}{8}} \frac{g_\chi}{2\pi^3} \frac{m_\chi M_{\rm Pl} \langle \sigma v \rangle}{\sqrt{x_{\rm FO}\, N(T_\chi^{\rm FO})}} \right) \right]^{-1}$$

USING DIMENSIONAL ANALYSIS...

$$\boxed{\sigma \sim \left(\frac{g^2}{4\pi}\right)\frac{1}{M_{\rm W}^2} \sim 10^{-8}\ {\rm GeV}^{-2}}$$

($g \simeq 0.65$ and $M_{\rm W} = (G_F)^{-1/2} \simeq 300\ {\rm GeV}$)

Solving for $x_{\rm FO}$ by numerical integration we obtain $x_{\rm FO} \simeq 20 - 30$

(for weak scale cross sections and masses)

Recall that $m_\chi v^2/2 = 3T/2$ and so WIMPs freeze-out with $v \sim 0.3$

Freeze-out temperatures $5\ {\rm GeV} < T_\chi^{\rm FO} < 80\ {\rm GeV}$

correspond to WIMPs with $100\ {\rm GeV} < m_\chi < 1500\ {\rm GeV}$

Adding up SM d.o.f. lighter than $80\ {\rm GeV}$ leads to $N(T_\chi^{\rm FO}) = 92$

For a very heavy or very light WIMP this number may change

but is not expected to significantly modify final result

WIMP MIRACLE

Altogether ☛ $$\langle \sigma v \rangle \sim 3 \times 10^{-9}\ \mathrm{GeV}^{-2} \simeq 3 \times 10^{-26}\,\mathrm{cm}^3/\mathrm{s}$$

After freeze-out ☛ density of χ particles that remain is given by

$$\Omega_\chi h^2 = \frac{\rho_\chi}{\rho_c} = \frac{m_\chi\, n_\chi}{\rho_c} \simeq \frac{10^9\ \mathrm{GeV}^{-1}}{M_{\mathrm{Pl}}} \frac{x_{\mathrm{FO}}}{\sqrt{N(T_{\mathrm{FO}})}} \frac{1}{\langle \sigma v \rangle}$$

Numerically ☛ this expression yields

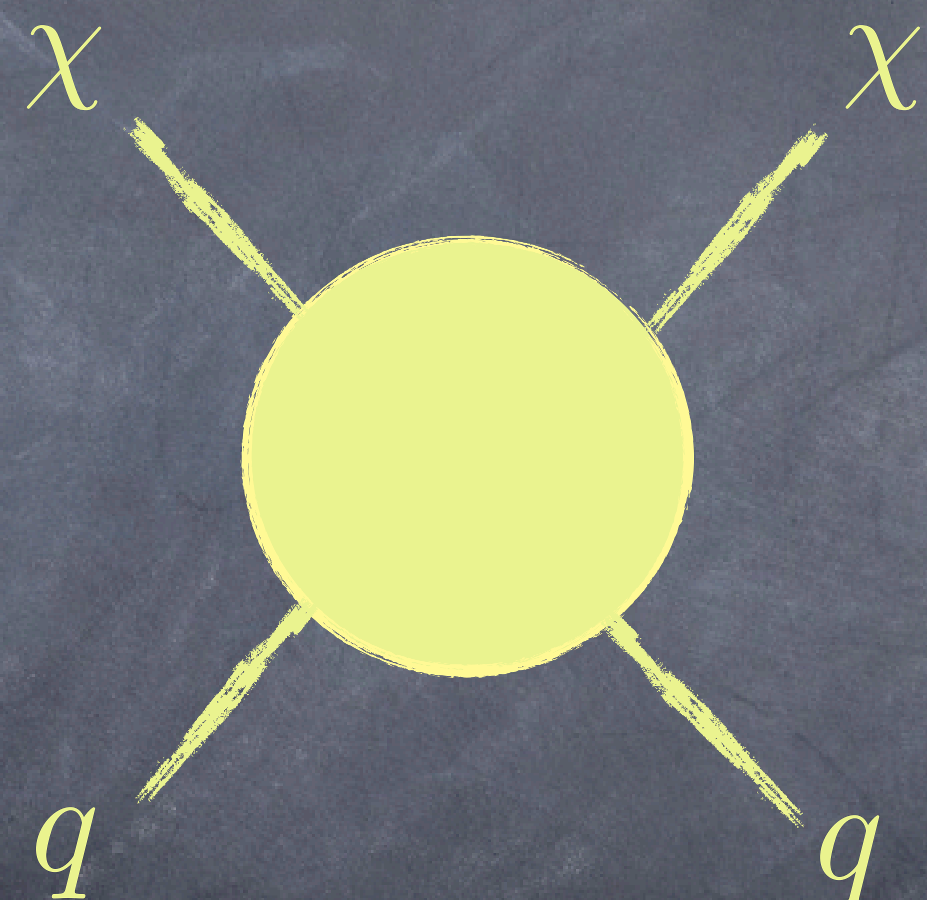
$$\Omega_\chi h^2 \sim 0.1 \times \frac{3 \times 10^{-26}\,\mathrm{cm}^3/\mathrm{s}}{\langle \sigma v \rangle}$$

Thus we see that observed cold dark matter density $(\Omega_{\mathrm{CDM}} h^2 \simeq 0.1)$ can be obtained for a thermal relic with weak scale interactions

EXPERIMENTAL PROBES

Correct relic density ⟶ Efficient annihilation then

Efficient annihilation now (Indirect detection)



Efficient production now (Particle colliders)

Efficient scattering now (Direct detection)

DARK MATTER HALO

- When our Galaxy was formed
cold dark matter inevitably clustered with luminous matter
to form a sizeable fraction of $$\boxed{\rho_\chi = 0.4\ \mathrm{GeV/cm^3}}$$
galactic matter density implied by observed rotation curves

- Unlike baryons ☛ dissipationless WIMPs fill galactic halo
which is believed to be an isothermal sphere of WIMPs
with average velocity $$\boxed{v_\chi = 300\ \mathrm{km/s}}$$

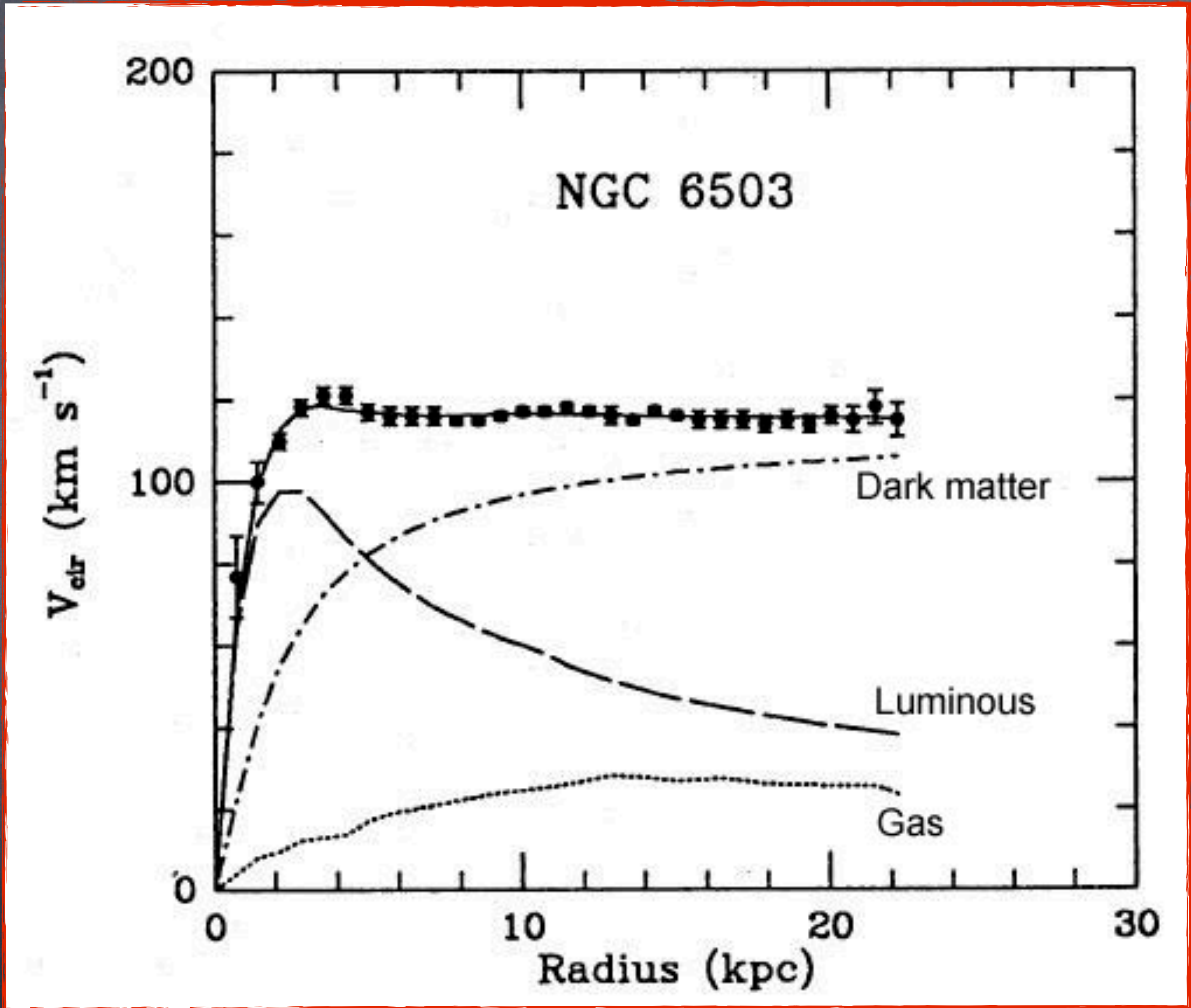
- In summary ☛ we know everything about these particles
(except whether they really exist!)

- We know that their mass is of order of weak boson mass

- We know that they interact weakly

- We also know their density and average velocity in our Galaxy
given assumption that they constitute dominant component
of density of our galactic halo as measured by rotation curves

GALAXY ROTATION CURVES



SI and SD interactions

Elastic scattering and annihilation cross sections of lightest neutralino depend on its couplings

and on mass spectrum of Higgs bosons and superpartners

Couplings (in turn) depend on neutralino's composition

Spin-dependent (SD) axial-vector scattering is mediated

by t-channel exchange of a Z-boson

or s-channel exchange of a squark

Spin-independent (SI) scattering occurs:

at tree level

through t-channel squark exchange and s-channel Higgs exchange

and at one-loop level

through diagrams involving loops of quarks and/or squarks

Cross sections for these processes can vary dramatically with parameters

even for case of MSSM

WIMP DETECTION SCHEMES

- For a first look at experimental problem of how to detect χ
  it is sufficient to recall that they are weakly interacting

  with masses in range

$$\boxed{\text{tens of GeV} < m_\chi < \text{several TeV}\,.}$$

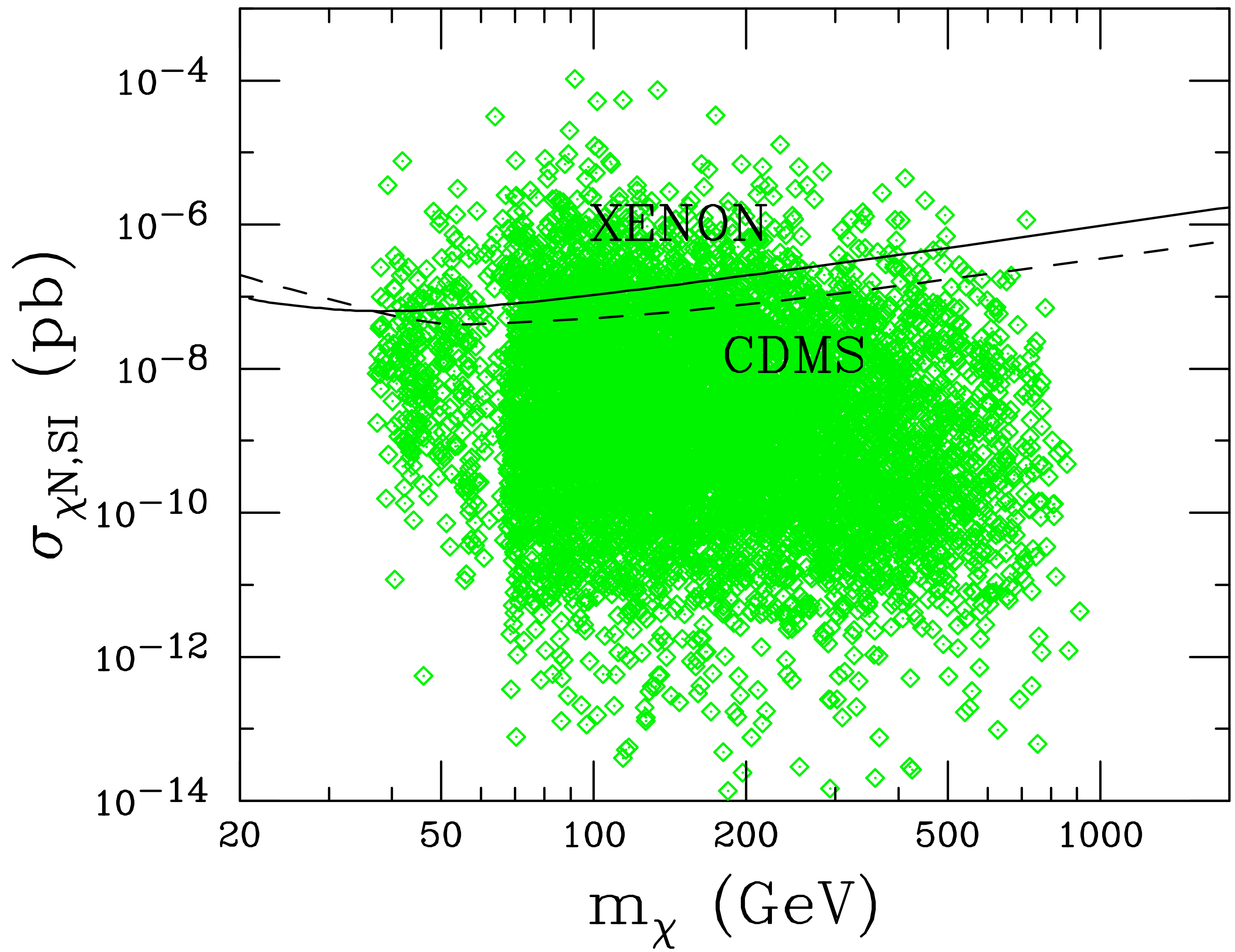
- Lower masses are excluded by accelerator and (in)direct searches
  while masses beyond several TeV are excluded by cosmology

- Two general techniques referred to as direct (D) and indirect (ID)
  are pursued to demonstrate existence of WIMPs

- In direct detectors
  we observe energy deposited when WIMPs elastically scatter off nuclei

- Indirect method infers existence of WIMPs
  from observation of their annihilation products

SPIN INDEPENDENT BOUNDS



Lightest neutralino's SI scattering cross sections for a range of MSSM parameters

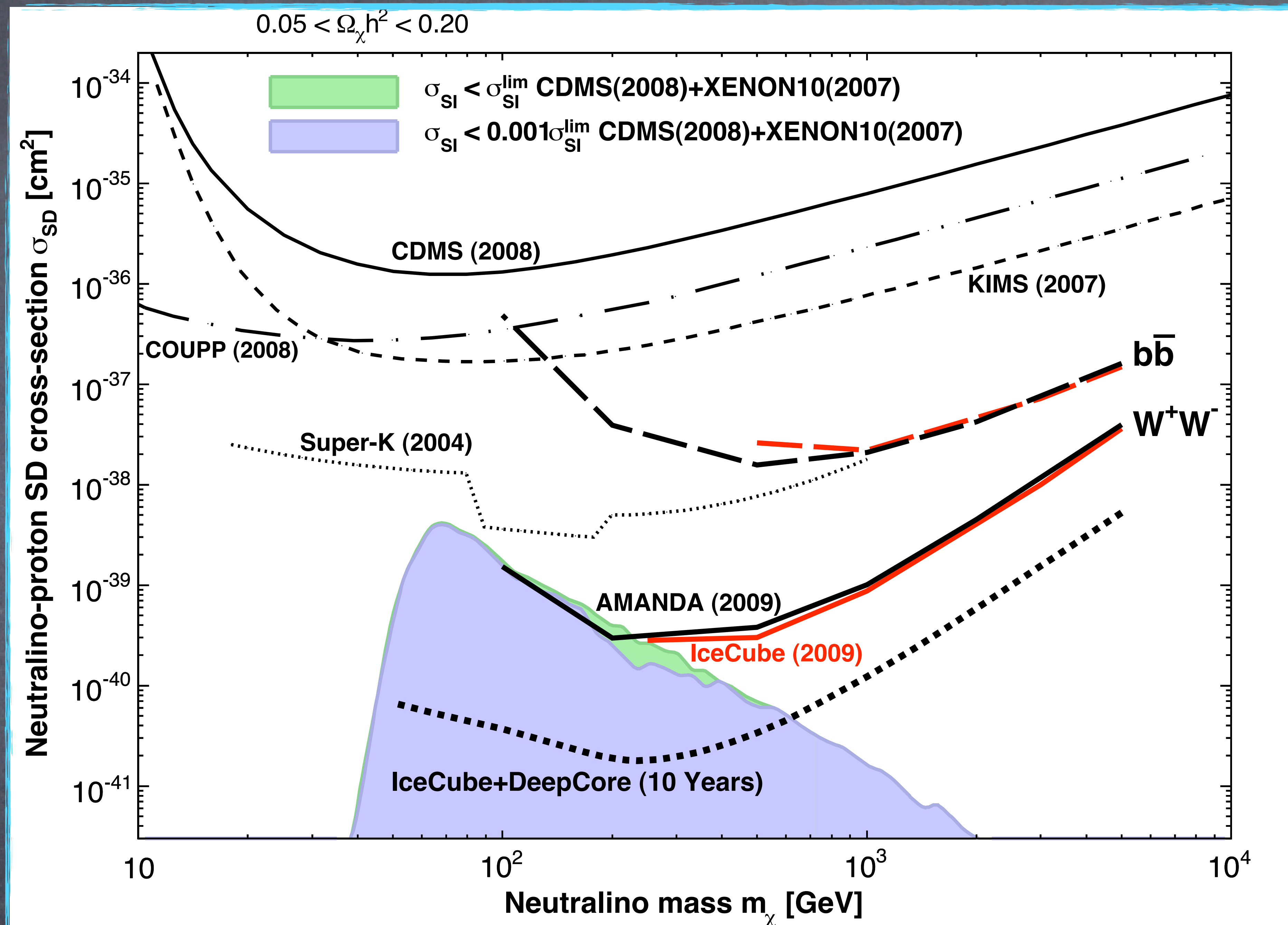
Scan varies mass of CP-odd Higgs boson m_A up to 1 TeV

all other mass parameters up to 10 TeV

and ratio of other Higgs bosons VEV $1 < \tan\beta < 60$

Also shown are current limits from direct detection experiments

SPIN DEPENDENT BOUNDS



90% CL upper limits on SD neutralino-proton cross section

Blue shaded region indicates MSSM parameter space allowed by existing limits on corresponding SI cross section

Green shaded region would be rule out if existing limits on $\sigma_{\chi N,\mathrm{SI}}$ are improved by a factor of 10^3

Higgs rumors fly as December 13 press conference approaches

Rumors say that ATLAS' peak near 126 GeV has 3.5 standard deviations and CMS' peak near 124 GeV has 2.5 standard deviations

If you are really optimistic you can add these two results in quadrature to get an overall result with a significance of 4.3σ

That is roughly 99.998 confidence level


That's all Folks!