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Problems set # 8 | **Physics 541-731** | November 1, 2011

1. One way to measure the muon lifetime is to stop cosmic ray muons in a block of scintillating material, where they decay to electrons and neutrinos, and then measure the time interval between the scintillation light produced by a muon when it enters the scintillator and the light produced when the decay electron is emitted. For a reasonably large sample, you can then extract the lifetime from the distribution of these time intervals. Explain why such an experiment will give the same result irrespective of whether you do it near the top of the atmosphere (near the place where muons are produced and are 'young') or at the surface of the earth (where they have already 'aged' quite a bit).

2. Use the Green's function method to obtain the electron propagator given in (3.4.60) and verify that it is associated with the propagation of positive-energy electrons forward in time and with negative-energy electrons backwards in time.

3. *(i)* Prove the trace theorems

- $\text{Tr}\ \mathbb{1} = 4$
- Trace of an odd number of γ_μ's vanishes.
- $\text{Tr}(\not{a}\ \not{b}) = 4\,a\ .\ b$
- $\text{Tr}(\not{a}\ \not{b}\ \not{c}\ \not{d}) = 4[(a\ .\ b)(c\ .\ d) - (a\ .\ c)(b\ .\ d) + (a\ .\ d)(b\ .\ c)]$
- $\text{Tr}(\gamma_5) = 0$
- $\text{Tr}(\gamma_5\ \not{a}\ \not{b}) = 0$
- $\text{Tr}(\gamma_5\ \not{a}\ \not{b}\ \not{c}\ \not{d}) = 4i\,\epsilon_{\mu\nu\lambda\sigma}\ a^\mu\ b^\nu\ c^\lambda\ d^\sigma,$

where $\epsilon_{\mu\nu\lambda\sigma} = +1\ (-1)$ for $\mu, \nu, \lambda, \sigma$ and even (odd) permutation of $0, 1, 2, 3$; and 0 if two indices are the same.

(ii) Verify the following identities

- $\gamma_\mu \gamma^\mu = 4 \times \mathbb{1} = 4$
- $\gamma_\mu\ \not{a} \gamma^\mu = -2\ \not{a}$
- $\gamma_\mu\ \not{a}\ \not{b} \gamma^\mu = 4a\,.\,b$
- $\gamma_\mu\ \not{a}\ \not{b}\ \not{c} \gamma^\mu = -2\ \not{c}\ \not{b}\ \not{a}.$

4. Show that the differential cross section for $e^-\mu^-$ scattering in the laboratory frame is given by (E.0.12).