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Problems set # 3 **Physics 541-735** September 27, 2011

1. Prove that α_i and β in (1.5.34) are hermitian, traceless matrices of even dimensionality, with eigenvalues ± 1.

2. Show that as long as the particles are massive, there is no 2×2 set of matrices that satisfy the anti-commutator relationships

$$\{\alpha_i, \alpha_j\} = 2\delta_{ij}, \qquad \{\alpha_i, \beta\} = 0, \qquad \beta^2 = \mathbb{1}.$$

Hence, the Dirac matrices must be of dimension 4 or higher. First show that the set of matrices $(\mathbb{1}; \vec{\sigma})$ can be used to express any 2×2 matrix; that is coefficients c_0, c_i always exist such that any 2×2 matrix can be written as

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = c_0 \, \mathbb{1} + c_i \, \sigma_i \, . \tag{1}$$

Having shown this, you can pick up an intelligent choice for the α_i in terms of the Pauli matrices, *e.g.* $\alpha_i = \sigma_i$ which automatically obeys $\{\alpha_i, \alpha_j\} = 2\delta_{Ij}$, and express β in terms of $(\mathbb{1}; \vec{\sigma})$ using (1). Show then that there is no 2×2 matrix that satisfies $\{\alpha_i, \beta\} = 0$.

3. Calculate the $\lambda = +\frac{1}{2}$ helicity eigenspinor of an electron of momentum $\vec{p}' = (p \sin\theta, 0, p\cos\theta)$.

4. Confirm the desired result that the Dirac equation provides a description of "intrinsic" angular momentum ($\equiv$ spin)-$\frac{1}{2}$ elementary particles.

5. For a massive fermion, show that handedness is not a good quantum number. That is show that γ^5 does not commute with the Hamiltonian. However, verify that helicity is conserved but is frame dependent. In particular, show that the helicity is reversed by overtaking the particle concerned.