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Problems set # 12 | **Physics 541-735** | November 29, 2011

1. *(i)* Show that the muon decay rate is given by (G.0.14).

(ii) What is the decay rate of the semileptonic process $\tau^- \to \pi^- \nu_\tau$. [Hint: see Y. S. Tsai, Phys. Rev. D **4**, 2821 (1971); Erratum-ibid. D **13**, 771 (1976)].

2. *(i)* Estimate the relative rates for the following three decay modes of the $D^0(c\bar{u})$ meson: $D^0 \to K^-\pi^+,\ \pi^-\pi^+,\ K^+\pi^-$.

(ii) Given the partial rate

$$\Gamma(K^+ \to \pi^0 e^+ \nu) = 4 \times 10^{-6}\ \mathrm{s}^{-1}$$

calculate the rate for $D^0 \to K^- e^+ \nu$. Hence, estimate the lifetime of the D^0 meson.

3. *(i)* Assuming the standard model coupling, show that

$$\Gamma(Z \to \nu_e \bar{\nu}_e) = \frac{g^2}{96\pi \cos^2\theta_w M_Z}.$$

Given that $\sin^2\theta_w = 0.25$ and $M_Z = 90$ GeV, predict the numerical value of the partial decay width.

(ii) Calculate the partial widths of the three decay modes $Z \to e^+e^-,\ \bar{u}u,\ \bar{d}d$. Hence predict the total width of the Z in the standard model, assuming $\sin^2\theta_w = 1/4$ and $M_Z = 90$ GeV. Do not forget color.

4. *(i)* Predict the numerical value of the partial decay width $W^+ \to e^+\nu_e$.

(ii) Calculate the partial widths of the two decay modes $W^+ \to \bar{d}u,\ \bar{s}u$. Predict the total width of the W^+ in the standard model.

5. *(i)* Integrate (5.4.74) over Ω and find an expression for the ratio $R_\mu = \sigma(e^+e^- \to \mu^+\mu^-)/\sigma_0$, where σ_0 is the QED cross section of (3.3.53).

(ii) Using the couplings in the standard model, calculate R_μ at $s = M_Z^2$.

6. It is relevant to ask what electroweak effects occur in $e^+e^- \to \bar{q}q$. They are not the same as in $e^+e^- \to \mu^+\mu^-$, since $c^q_{V,A} \neq c^\mu_{V,A}$. However, the calculation proceeds in a similar fashion. For example, the counterpart to (5.4.71) is

$$\left.\frac{d\sigma}{d\Omega}\right|_{e^-_L e^+_R \to q^-_L q^+_R} = 3\frac{\alpha^2}{4s}(1+\cos\theta)^2\left[e_q + r c^q_L c^e_L\right]^2 ,$$

where e_q is the charge of the quark and the factor 3 is for color. Following through the calculation obtain an expression for the ratio $R_q = \sigma(e^+e^- \to \mu^+\mu^-)/\sigma_0$, and calculate R_u and R_d at $s = M_Z^2$. Hence, calculate $\sigma(e^+e^- \to \text{hadrons})$ at the Z resonance.