Astronomy, Astrophysics, and Cosmology

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Lesson VIII
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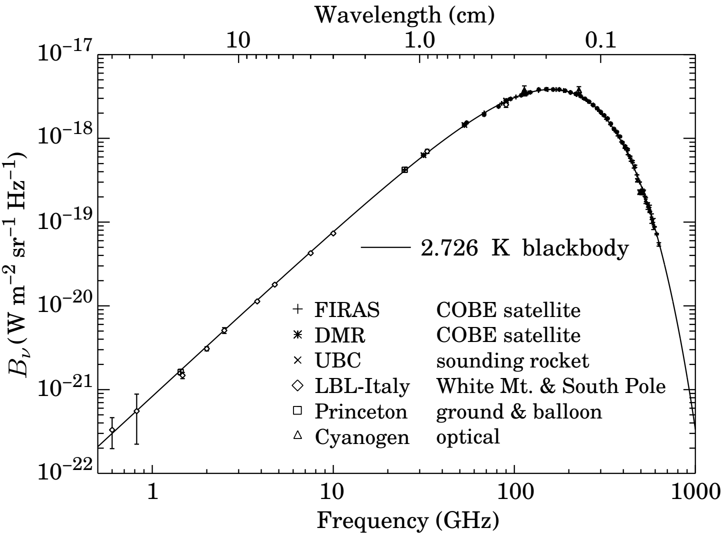
- CMB radiation was discovered in 1964 by Penzias and Wilson
- Radiation was acting as a source of excess noise
  in radio receiver

- Precise measurements at $\lambda = 7.35$ cm ☞ radiation was found:
  - not to vary by day or night or time of year
  - it came from all directions with equal intensity
    to precision of better than 1%

- Blackbody emission of hot dense gas
  $T \sim 3000$ K and $\lambda_{\rm max} \sim 1000$ nm
  redshifted by factor of 1000 ☞ $\lambda_{\rm max} \sim 1$ mm and $T \sim 3$ K

Compilation of experimental measurements



Accurate blackbody spectrum ☞ $T_0 = 2.726 \pm 0.010$ K

- CMB photons we see today interacted with matter for last time some $380\ \mathrm{kyr}$ after the bang
- γ decoupling occurs when T has dropped to point where there are no longer enough high energy photons to keep hydrogen ionized

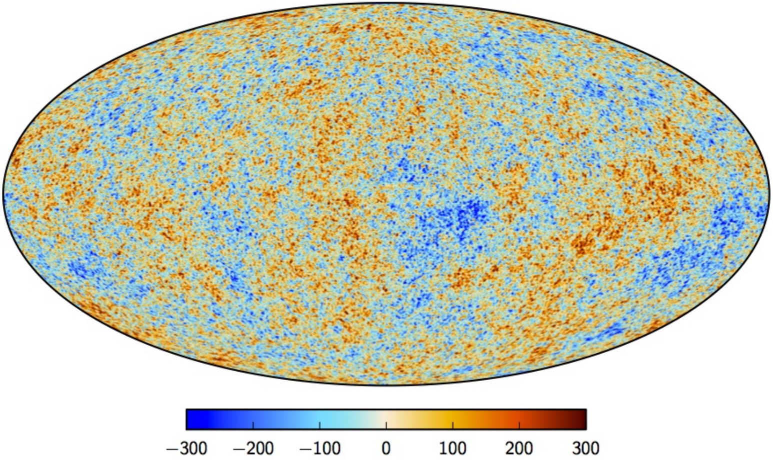
$$\gamma\ {}^1\mathrm{H} \not\rightleftharpoons e^- p^+$$

- Although ionization potential of ^{1}H is 13.6 eV ☞ $T \sim 10^5\ \mathrm{K}$ recombination occurs at $T_{\rm rec} \sim 3000\ \mathrm{K}$
- Low baryon to photon ratio

$$\eta \approx 5 \times 10^{-10}$$

allows high energy tail of Planck distribution to keep small number of hydrogen atoms ionized until this much lower temperature

- B4 recombination epoch universe was opaque "fog" of free electrons and became transparent to radiation afterwards
- When we look at the sky in any direction we can expect to see photons that originated in last-scattering surface
- This hypothesis has been tested very precisely by observed distribution of CMB



- B4 recombination ☞ Compton scattering tightly coupled photons to electrons which in turn coupled to protons via electomagnetic interactions

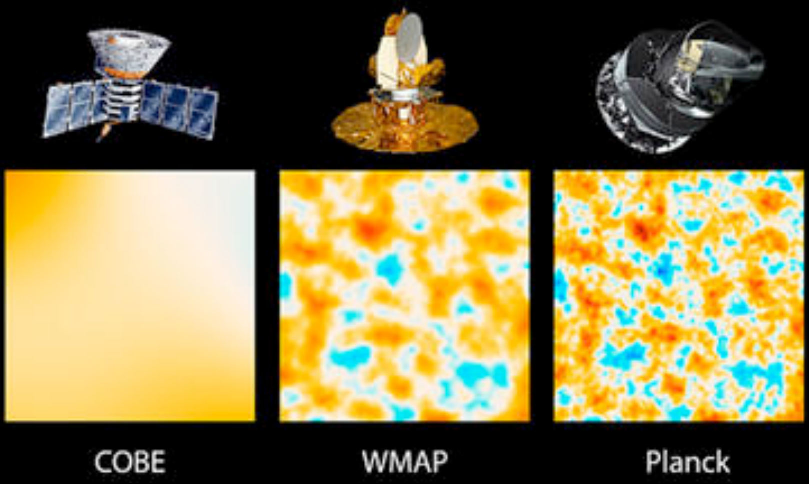
- Consequently ☞ photons and nucleons in early universe behaved as single "photon-nucleon fluid" in gravitational potential well created by primeval variations in the density of matter

- Outward pressure from photons acting against inward force of gravity set up acoustic oscillations that propagated through photon-nucleon fluid exactly like sound waves in air

- Frequencies of these oscillations are now seen imprinted on CMB temperature fluctuations ΔT

- Gravity caused primordial density perturbations across universe to grow with time

- Temperature anisotropies in CMB are interpreted as snapshot of early stages of this growth which eventually resulted in formation of galaxies


COBE
WMAP
Planck

- Convenient to expand difference $\Delta T(\hat{n})$ in spherical harmonics

$$\Delta T(\hat{n}) \equiv T(\hat{n}) - T_0 = \sum_{l=0}^{\infty} \sum_{|m|\leq l} a_{lm} Y_{lm} \tag{1}$$

$$T_0 = \frac{1}{4\pi} \int d^2\hat{n} \; T(\hat{n}) \tag{2}$$

- Set $\{Y_{lm}\}$ is complete and orthonormal ☞ obeying

$$\int d\Omega \; Y_{l_1 m_1}(\Omega) \, Y_{l_2 m_2}(\Omega) = \delta_{l_1 l_2} \, \delta_{m_1 m_2} \tag{3}$$

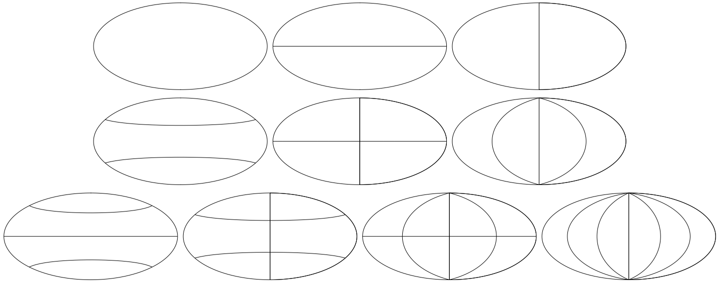
- Since $\Delta T(\hat{n})$ is real

$$Y_{lm}(\theta, \phi) = N(l,m) \begin{cases} P^l_m(x)(\sqrt{2}\cos(m\phi)) & m > 0 \\ P_l(x) & m = 0 \\ P^l_m(x)(\sqrt{2}\sin(m\phi)) & m < 0 \end{cases} \tag{4}$$

- Normalization given by

$$N(l,m) = \sqrt{\frac{(2l+1)(l-m)!}{4\pi\,(l+m)!}} \tag{5}$$

- Lowest multipole ☞ monopole $l = 0$ equal to full-sky average and gets fixed by normalization
- Higher multipoles ($l \geq 1$) and their amplitudes a_{lm} correspond to anisotropies
- A nonzero m ☞ $2\,|m|$ longitudinal "slices" ($|m|$ nodal meridians)
- There are $l + 1 - |m|$ latitudinal "zones" ($l - |m|$ nodal latitudes)

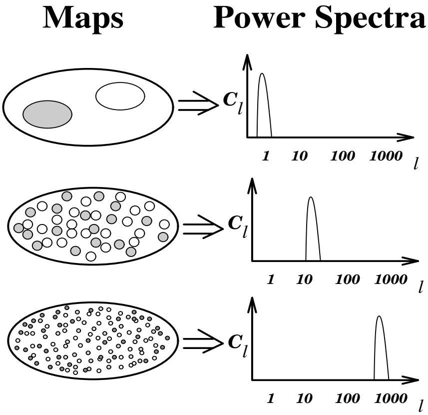


Nodal lines separating excess and deficit regions of sky for various (l, m) pairs
✧ Top row: $(0, 0)$ monopole and partition of sky into two dipoles $(1, 0)$ and $(1, 1)$
✧ Middle row: quadrupoles $(2, 0)$, $(2, 1)$, and $(2, 2)$
✧ Bottom row: $l = 3$ partitions, $(3, 0)$, $(3, 1)$, $(3, 2)$, and $(3, 3)$

- Expansion coefficients $a_{\ell m}$'s ☞ frame-dependent
- Only $\ell = m = 0$ monopole coefficient is coordinate independent
- To combat problem ☞ define power spectrum

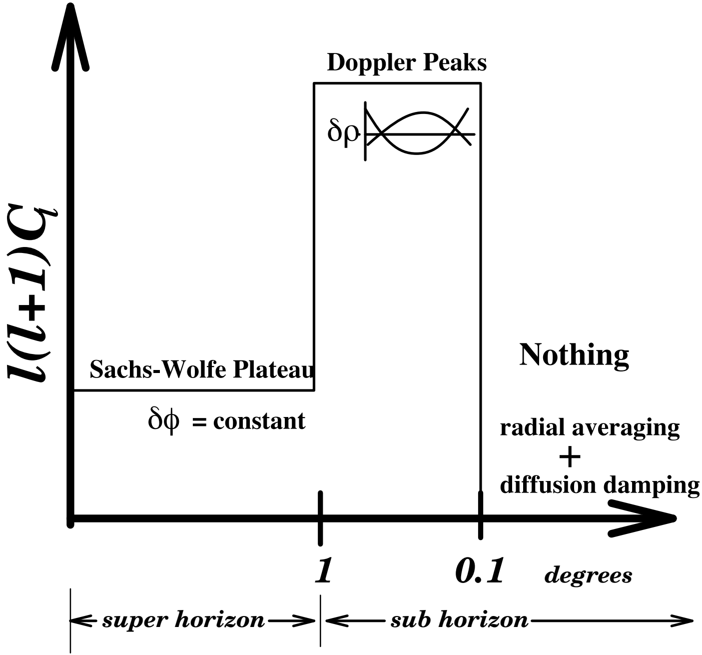
$$C_l \equiv \frac{1}{2l+1} \sum_{m=-l}^{l} a_{lm}^2 \tag{6}$$

- Brief C_l initiation ☞

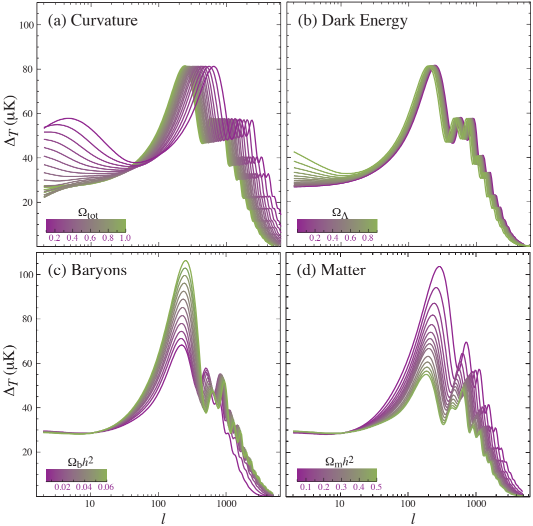


- To get a rough understanding of the power spectra
  divide up plot into super-horizon and sub-horizon regions
- Angular scale corresponding to particle horizon size
  is boundary between super- and sub-horizon scales
- Size of causally connected region on last scattering surface
  is important
- It determines size over which astrophysical processes can occur
- Normal physical processes can act coherently
  only over sizes smaller than the particle horizon
  and could not have produced structure in CMB map
- For historical reasons ☞ quantity usually used in plots

$$\Delta_T \equiv \left[\frac{l(l+1)}{2\pi} C_l\right]^{1/2} \tag{7}$$


Doppler Peaks
δρ
$l(l+1)C_l$
Sachs-Wolfe Plateau
δφ = constant
Nothing
radial averaging
+
diffusion damping
1
0.1
degrees
super horizon
sub horizon

Relative size of peaks and locations of power spectrum



gives information about cosmological parameters

- At recombination universe is already matter-dominated
- Use last class calculations with $z_{\rm ls} \simeq 1100$ to give estimate of horizon distance at CMB epoch

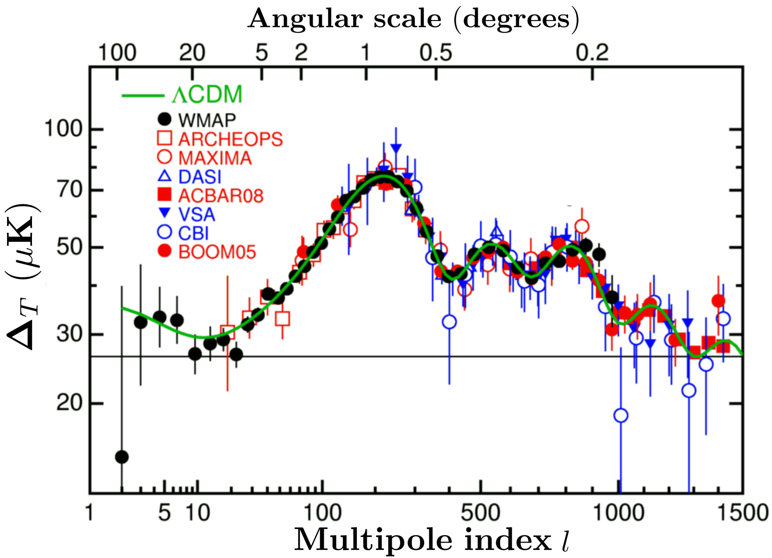
$$d_{\rm h,ls} = \frac{2c}{H_0(1+z)^{3/2}} \approx 0.23 \text{ Mpc} \tag{8}$$

- This is linear diameter of largest causally connected region observed for CMB ☞ $\ell_{\rm ls}$
- Today's angular diameter of this region in the sky is

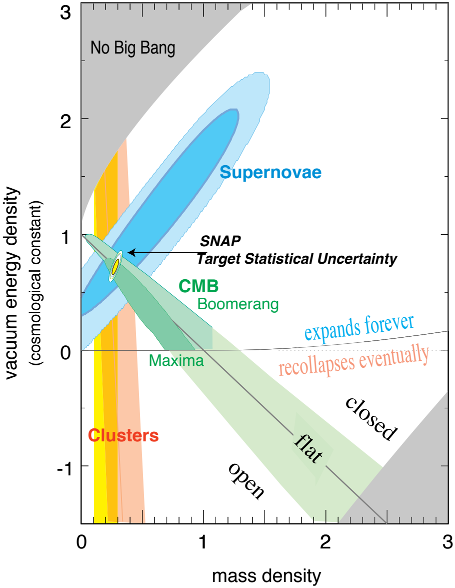
$$\theta = \frac{1}{(1+z)^{1/2} - 1} = 0.03 \approx 1.8^\circ \tag{9}$$

- CMB data point to flat universe

Compilation of measurements of CMB angular power spectrum



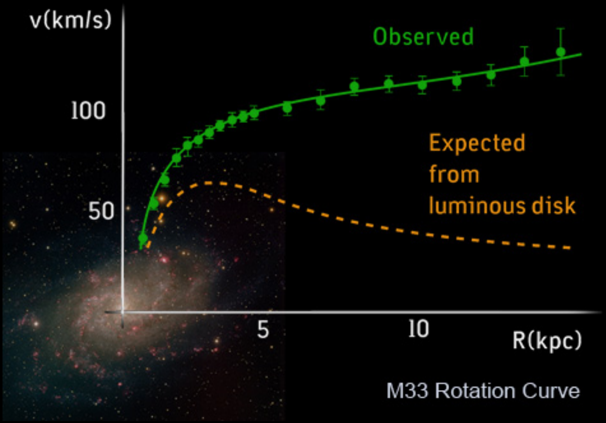
Weighting the Universe



Cold Dark Matter

- There is a strong astrophysical evidence for a significant amount of nonluminous matter in the universe referred to as CDM
- For example ☞ observations of rotation of galaxies suggest that they rotate as they had considerably more mass than we can see
- Similarly ☞ observations of motions of galaxies within clusters also suggest they have considerably more mass than can be seen
- What might this nonluminous matter in the universe be?
- We do not know yet
- It cannot be made of ordinary (baryonic) matter so it must consist of some other sort of elementary particle

Rotational Velocities of Stars in Spiral Galaxies

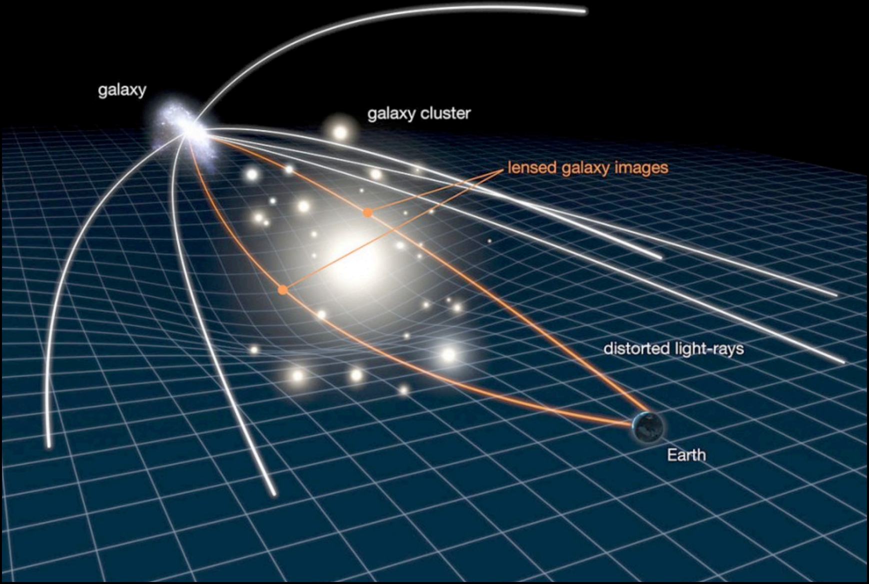


Stars and gas in the disk move in circular orbits

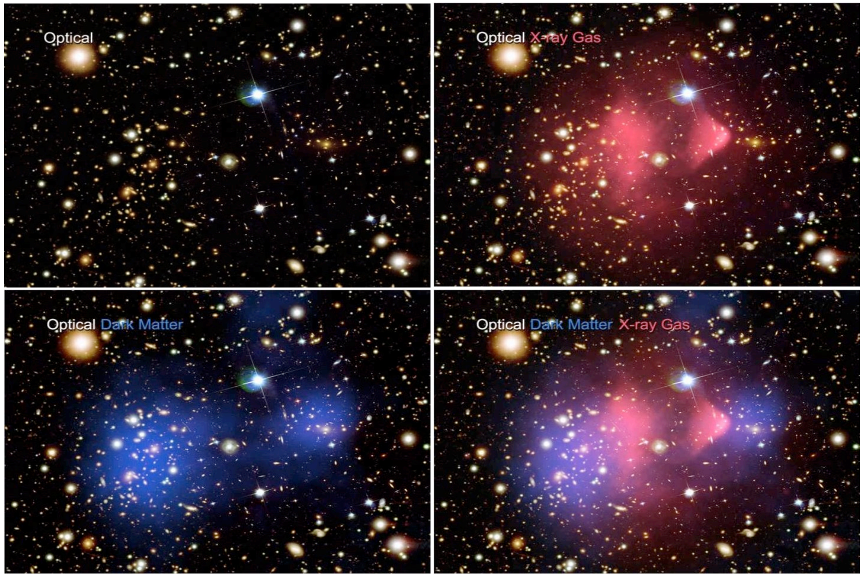
Gravitational field provides inward acceleration

Newtonian approximation ☛ $v^2(R) = G\,M(R)/R$

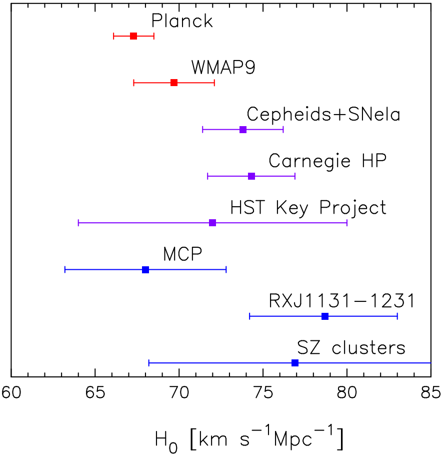
Gravitational Lensing



Bullet Cluster

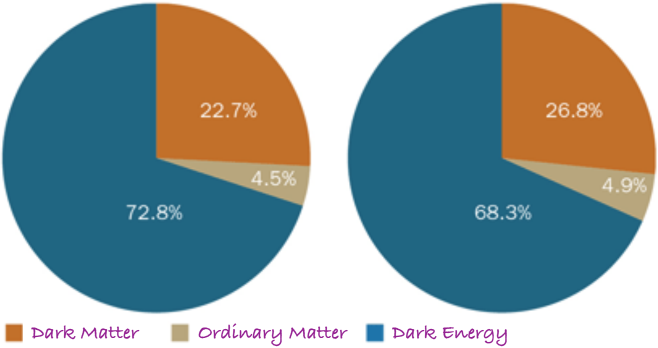


- Best fit to most recent data from Planck satellite yields: $\Omega_{m,0} = 0.308 \pm 0.013$, $\Omega_{b,0}h^2 = 0.02234 \pm 0.00023$, $\Omega_{\rm CDM,0}h^2 = 0.1189 \pm 0.0022$, $h = 0.678 \pm 0.009$, and $\Omega_k < 0.005$
- Unexpectedly ☞ H_0 inference from Planck data deviates by more than 2σ from HST result ☞ $h = 0.738 \pm 0.024$



Estimated Composition of the Universe

Before Planck After Planck



Supernova Cosmology Project Supernova Search Team
Hubble Space Telescope Wilkinson Microwave Anisotropy Probe (WMAP)
Sloan Digital Sky Survey (SDSS) Planck spacecraft

- Consider benchmark model containing only pressure-less matter and cosmological constant ☞ $\Omega_{m,0} + \Omega_\Lambda = 1$
- Multiplying the acceleration equation by 2 and adding it to Friedmann equation ☞ eliminate ρ_m

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = \Lambda c^2 \tag{10}$$

- Using

$$\frac{d}{dt}(a\dot{a}^2) = \dot{a}^3 + 2a\dot{a}\ddot{a} = \dot{a}a^2\left[\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{\ddot{a}}{a}\right] \tag{11}$$

it follows that

$$\frac{d}{dt}(a\dot{a}^2) = \dot{a}a^2\Lambda c^2 = \frac{\Lambda c^2}{3}\frac{d}{dt}(a^3) \tag{12}$$

- Integrating is now trivial

$$a\dot{a}^2 = \frac{\Lambda c^2}{3}a^3 + \mathcal{C} \tag{13}$$

- $\mathcal{C}$ can be determined by setting $a(t_0) = 1$ and comparing Friedmann equation to (13) with $t = t_0$

$$\mathcal{C} = 8\pi G\rho_{m,0}/3$$

- Introducing $x = a^{3/2}$ such that

$$\frac{da}{dt} = \frac{dx}{dt}\frac{da}{dx} = \frac{dx}{dt}\frac{2x^{-1/3}}{3} \tag{14}$$

(13) becomes

$$\dot{x}^2 - \frac{3}{4}\Lambda c^2 x^2 + \frac{9}{4}\mathcal{C} = 0\,. \tag{15}$$

- Inserting solution of homogeneous equation

$$x(t) = A\sinh(\sqrt{3\Lambda}ct/2) \quad \text{fixes} \quad A = \sqrt{3\mathcal{C}/\Lambda c}$$

- Time scale factor is

$$a(t) = A^{2/3}\sinh^{2/3}(\sqrt{3\Lambda}ct/2) \tag{16}$$

- Time-scale of expansion set by $t_\Lambda = 2/\sqrt{3\Lambda c^2}$
- Present age of universe t_0 follows by setting $a(t_0) = 1$

$$t_0 = t_\Lambda \tanh^{-1}(\sqrt{\Omega_\Lambda}) \tag{17}$$

- Deceleration

$$q = -\frac{\ddot{a}}{aH^2} \tag{18}$$

is important parameter for observational tests of ΛCDM model

- We calculate first the Hubble parameter

$$H(t) = \frac{\dot{a}}{a} = \frac{2}{3t_\Lambda} \coth(t/t_\Lambda) \tag{19}$$

and after that

$$q(t) = \frac{1}{2}\left[1 - 3\tanh^2(t/t_\Lambda)\right] \tag{20}$$

- Limiting behavior of q:
  - for $t \to 0$ ☞ $q = 1/2$
  - for $t \to \infty$ ☞ $q = -1$
- For $\Omega_\Lambda = 0.7$
  transition region from decelerating to accelerating universe ☞ $t \approx 0.55 t_0$
- This can be easily converetd to a redshift $z_* = a(t_0)/a(t_*) - 1 \approx 0.7$
  that is directly measured by SNe Ia observations