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Problems set # 1 **Physics 307** September 7, 2016

Complex Analysis I

1. *(i)* Does $z^2 = |z|^2$? If so, prove the equality. If not, decide for what values of z it is true (and prove your answer, of course).
(ii) Describe geometrically the sets of points z in the complex plane defined by the following relations: A- $|z - z_1| = |z - z_2|$ where $z_1, z_2 \in \mathbb{C}$; B- $z^{-1} = z^*$.
(iii) Let $w = \alpha e^{i\beta}$, where $\alpha \geq 0$ and $\beta \in \mathbb{R}$. Solve the equation $z^n = w$ in $\mathbb{C}$, where $n \in \mathbb{N}$.

2. Describe the sets on which the following functions are analytic and compute their derivatives:
(i) $f(z) = e^{iz^2}$;
(ii) $f(z) = \frac{1}{z^*}$;
(iii) $f(z) = e^{1/z}$;
(iv) $f(z) = e^{1/(1-az)}$, $a \in \mathbb{C}$;
(v) $f(z) = \frac{\sin z}{z}$.

3. *(i)* Using $x = r\cos\theta$, $y = r\sin\theta$, and the chain rule show that the Cauchy-Riemann conditions are equivalent to $u_r = v_\theta/r$ and $v_r = -u_\theta/r$.
(ii) Use these equations to show that the logarithm function defined by $\log(z) = \ln(r) + i\theta$, is analytic in the region $-\pi < \theta < \pi$, where $z = r\,e^{i\theta}$.

4. Let $f : \mathbb{C} \to \mathbb{C}$ be a function which is analytic on a region G. Show that if $\Re\mathrm{e}(f)$ is constant on G, then f is constant on G.

5. Find the radius of convergence of: *(i)*

$$\sum_{n=0}^{\infty} \frac{(4iz - 2)^n}{2^n}\,;$$

(ii) the hypergeometric series

$$F(\alpha, \beta, \gamma; z) = 1 + \sum_{n=1}^{\infty} \frac{\alpha(\alpha+1)\dots(\alpha+n-1)\beta(\beta+1)\dots(\beta+n-1)}{n!\gamma(\gamma+1)\dots(\gamma+n-1)}\; z^n\,,$$

where $\alpha, \beta \in \mathbb{C}$ and $\gamma \neq 0, -1, -2, \dots$