Modern Physics

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Historical Overview

- In 19th century ☞ it was known that:
  - water waves must have medium to move across (water)
  - audible sound waves require medium to move through (e.g. air)
- It was thought that just as in previous examples ☞ light waves require medium called "luminiferous" (light-bearing) "æther"
- If this were the case ☞ as Earth moves in its orbit around Sun flow of æther across Earth could produce detectable "æther wind"
- Unless æther were always stationary with respect to Earth speed of beam of light emitted from source on Earth would depend on magnitude of æther wind and on beam direction
- 1881 Michelson-Morley experiment to measure speed of light in different directions became most famous failed experiment to date and first strong evidence against luminiferous æther

Historical Overview (cont'd)

(1892 -1909)

- To explain nature's apparent conspiracy to hide æther drift Lorentz developed theory based on two *ad hoc* hypotheses:
  - Longitudinal contraction of rigid bodies
  - slowing down of clocks (time dilation)

  when moving through æther at speed v ☞ both by $(1 - v^2/c^2)^{1/2}$
- This would so affect every aparatus designed to measure the æther drift as to neutralize all expected effects

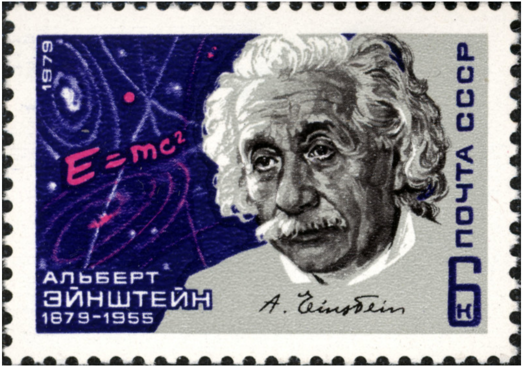
(1898)

- Poincare arugued that æther might be unobservable and suggested concept would be thrown aside as useless BUT he continued to use concept in later papers of 1908

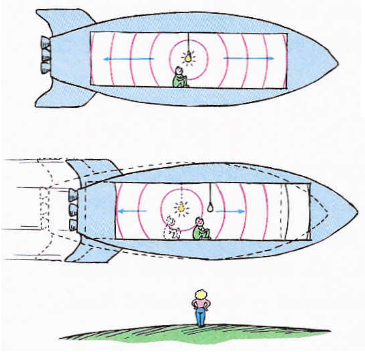
(1905)

- Einstein advanced principle of relativity

1. *The laws of physics are identical in all inertial frames*
   or equivalently
   the outcome of any experiment is the same when performed with identical initial conditions relative to any inertial frame
2. *There exists an inertial frame in which light signals in vacuum always travel rectilinearly at constant speed c (in all directions) independently of the motion of the source*



Vinnie and Brittany investigate relativity of simultaneity

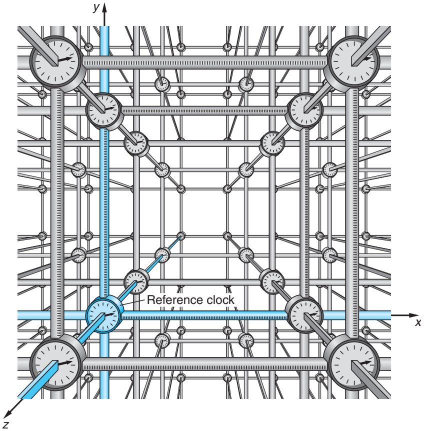


- From Vinnie's viewpoint light from travels equals distances to both ends of rocket ☞ striking both ends simultaneously
- Events of striking front and the end of spacecraft are not simultaneous in Brittany's reference frame
- Because of rocket's motion light strikes back end sooner than front end

How does observer in inertial reference frame describe event?

- Event ☞ an occurrence characterized by: three space coordinates and one time coordinate
- Events are described by observers who do belong to particular inertial frames of reference
- Different observers in different inertial frames would describe same event with different spacetime coordinates
- Observer's rest frame is also known as proper frame
- Two ordinary methods to construct coordinate system
  - use of confederates at each place
  - single observer method

Confederate scheme for coordinatizing any event



- Observer establishes lattice of confederates
  with identical synchronized clocks
- label of any event in spacetime
  is reading of clock and location of nearest confederate to event

Single clock at spatial origin

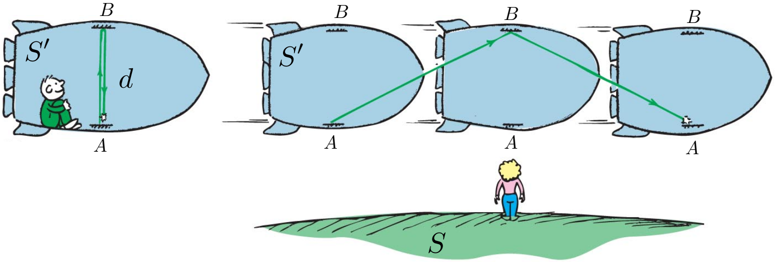
- Observer continuously sends out light rays in all directions keeping track of emission time
- At any event ☞ incoming light ray is reflected back to observer
- Observer has 2 times + a direction associated with any event
- To yield spatial coordinatizing consistent with confederate scheme:

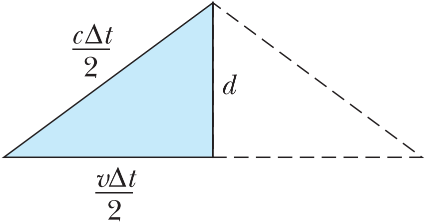
$$|\vec{r}\,| = c\,(\tau_2 - \tau_1)/2 \tag{1}$$

$$t = (\tau_1 + \tau_2)/2 \tag{2}$$

Einstein's thought experiments

- Idealized clock
  light wave is bouncing back and forth between two mirrors
- Clock "ticks" when light wave makes a round trip
  from mirror A to mirror B and back
- Assume mirrors A and B are separated by distance d in rest frame
- Llight wave will take $\Delta t' = 2d'/c$ for round trip $A \to B \to A$





Time dilation

Since light has velocity c in all directions

$$d^2 + \left(v\,\frac{\Delta t}{2}\right)^2 = \left(\frac{c\Delta t}{2}\right)^2 \tag{3}$$

$$\Delta t = \frac{2d'}{\sqrt{c^2 - v^2}} = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}} \tag{4}$$

Ticking of clock in Vinnie's frame
which moves @ v in direction $\perp$ to separation of mirrors
is slower by $\gamma = (1 - v^2/c^2)^{-1/2}$

Twin Paradox

- Consider two synchronized standard clocks A and B
  at rest at point P of intertial frame S
- Let A remain @ P while B is briefly accelerated to some velocity v with which it travels to distant point Q
- There it is decelerated and made to return with velocity v to P
- If one of two twins travels with B while other remains with A
  ☞ B twin will be younger than A twin when meet again

Can't B claim with equal right it was her who remained where she was while A went on round-trip ☞ A should be younger when meet again?

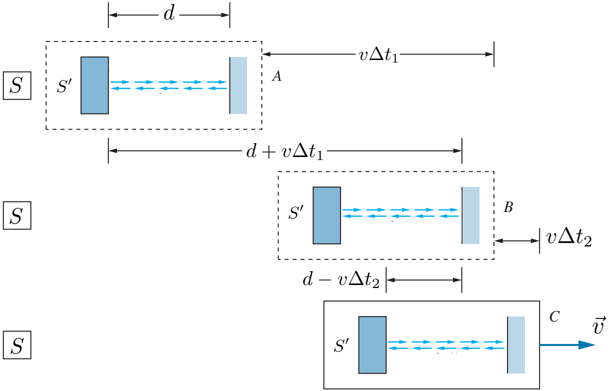
Answer is NO ☞ this solves paradox

- A remained at rest in single inertial frame while B accelerated out of his rest frame: @P, @Q, and once again @P
- Accelerations recorded on B's accelerometer she can be under no illusion that it was her who remain at rest
- Two accelerations at P are not essential (age comparison could be made in passing) but acceleration in Q is vital

Answer is NO ☞ this solves paradox

- Eperimet involves 3 inertial frames:
  1. earth-bound frame S
  2. S' of outbound rocket
  3. S'' of returning rocket
- Experiment not symmetrical between twins:
  - A stays at rest in single inertial frame S
  - but B occupies at least two different frames
- This allows result to be unsymmetrical

Rotate clock by 90° before setting it in motion



A-B

$$d + v\,\Delta t_1 = c\,\Delta t_1 \quad (5)$$

$$\Delta t_1 = \frac{d}{c - v} \quad (6)$$

B-C

$$d - v\Delta t_2 = c\Delta t_2 \quad (7)$$

$$\Delta t_2 = \frac{d}{c + v} \quad (8)$$

Length contraction

1. Interval between two consecutive ticks in the moving frame is

$$\begin{aligned} \Delta t &= \Delta t_1 + \Delta t_2 = \frac{2d}{c(1 - v^2/c^2)} \\ &= \left(\frac{d}{d'}\right) \frac{\Delta t'}{1 - v^2/c^2} \end{aligned} \tag{9}$$

2. Because of time dilation

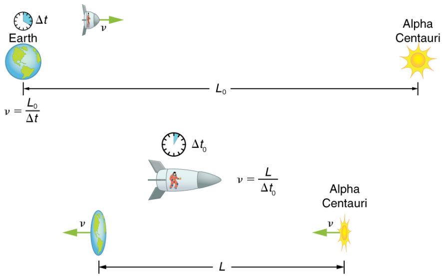
$$\Delta t' = \Delta t \, \sqrt{1 - v^2/c^2} \tag{10}$$

we get

$$d = \left(1 - \frac{v^2}{c^2}\right)^{1/2} d' \tag{11}$$

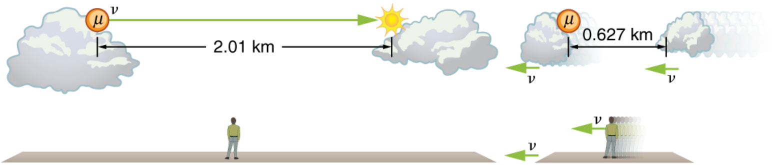
A Trip to Alpha Centauri

- One thing all observers agree upon is relative speed
- Even though clocks measure different elapsed times for same process ☞ they still agree that relative speed is the same
- Distance too depends on observer's relative motion!
- If two observers see different times ☞ they must also see different distances for relative speed to be same to each of them



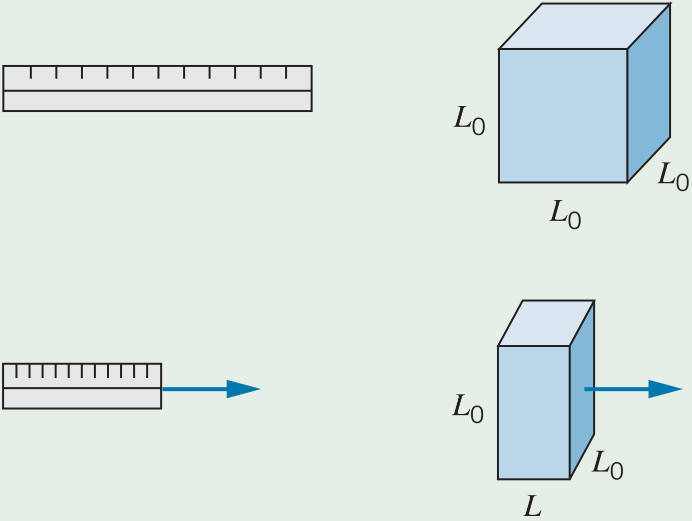
Life of a Muon

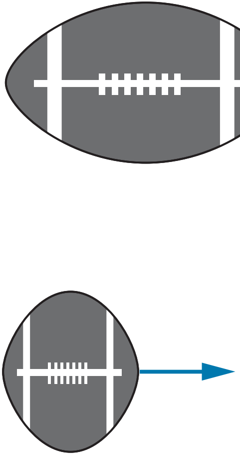
- Earth-bound observer sees muon travels at $0.95c$ for $7.05\ \mu s$ from time it is produced until it decays
- It travels distance ☞ $L_0 = v\,\Delta t = 2.1\ \mathrm{km}$ relative to Earth
- In muon's rest frame ☞ its lifetime is only $2.20\ \mu s \Rightarrow$ it has enough time to travel only $L = v\Delta t_0 = 0.627\ \mathrm{km}$
- Distance between same two events (muon production and decay) depends on who measures it + how they are moving relative to it



Einstein time dilation factor agrees with experiment with fractional error of 2×10^{-3} at 95% confidence!

Example



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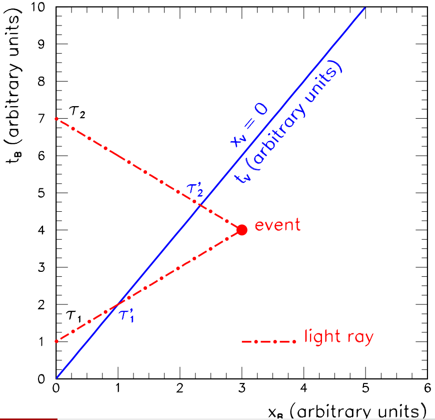
- Einstein postulates retain Galilean invariance
  there is no experiment that can detect a uniform state of motion
- However ☞ Galilean transformation rule *must be changed*
- Since light travels driven by Maxwell's equations
  transformation law between inertial observers
  must preserve Maxwell's equations
- Actually ☞ it is even more general than that:
  we will have a set of transformations that leave c *invariant*

Recall that...

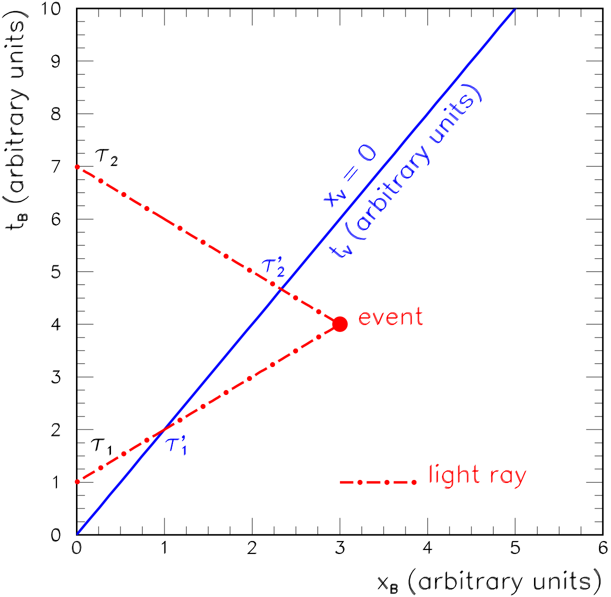
- When relatively moving observers label event
  all observers must use same two light rays
- Any event is characterized uniquely
  by two light rays that pass through it
- All observers finding labels of particular event
  MUST use same transmitted and received rays
- This apparent coincidence implies that all observers agree:
  1. on speed of light
  2. that intersection of two light rays defines event
  3. indeed ☞ unique label of event

Example

- Vinnie and Brittany share same origin to coordinatize same event
- Transverse coordinates are same ☞ used to construct clocks
- Red event is coordinatized by: Brittany as $(x_{\rm B}, t_{\rm B})$
  Vinnie as $(x_{\rm V}, t_{\rm V})$



Lorentz transformations are relationship between $(x_{\rm B}, t_{\rm B})$ and $(x_{\rm V}, t_{\rm V})$



- Start by finding coordinates of Vinnie's proper times τ_1' and τ_2' in terms of coordinates of red event in Brittany's reference frame

- Event τ_1' has form (vt_1, t_1) in Brittany's coordinates since it is on Vinnie's time axis
  and he is moving at a speed v with respect to her

- This event is also on light ray with red event

- The equation of that light ray is

$$x - x_{\rm B} = c(t - t_{\rm B}) \tag{12}$$

- Putting in coordinates of event τ_1'

$$vt_1 - x_{\rm B} = c(t_1 - t_{\rm B}) \tag{13}$$

- Solving for t_1

$$t_1 = \frac{ct_{\rm B} - x_{\rm B}}{c - v} \tag{14}$$

- Because of time dilation

$$\tau_1' = t_1 \sqrt{1 - v^2/c^2} \tag{15}$$

- Combining these

$$\tau_1' = \frac{ct_{\rm B} - x_{\rm B}}{c - v} \sqrt{1 - \frac{v^2}{c^2}} \tag{16}$$

- Similarly for event τ_2'

$$\tau_2' = t_2 \sqrt{1 - \frac{v^2}{c^2}} \tag{17}$$

and

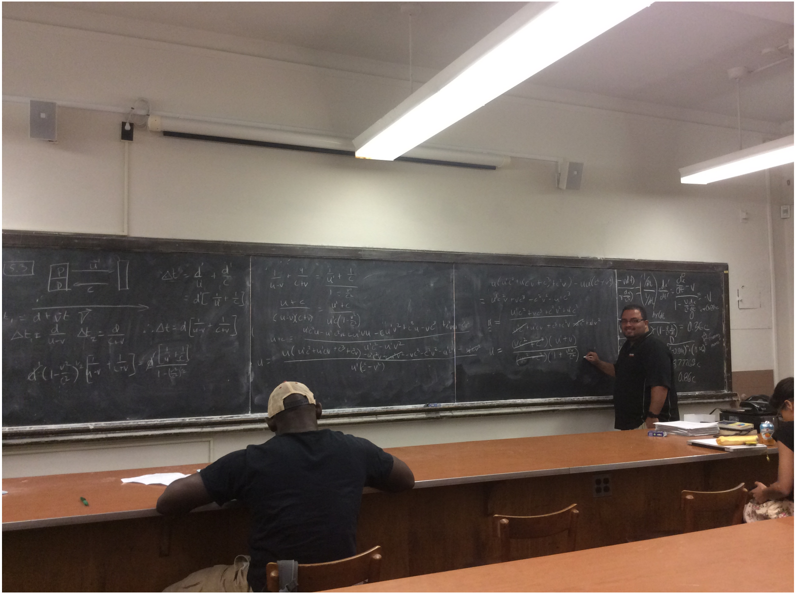
$$\tau_2' = \frac{ct_{\rm B} + x_{\rm B}}{c + v} \sqrt{1 - \frac{v^2}{c^2}} \tag{18}$$

- Substituting (16) and (18) into the definitions (1) and (2)

$$\begin{aligned} x_V &= \frac{x_B - v t_B}{\sqrt{1 - v^2/c^2}} = \gamma(x_B - v t_B) \\ y_V &= y_B \\ z_V &= z_B \\ t_V &= \frac{t_B - v x_B/c^2}{\sqrt{1 - v^2/c^2}} = \gamma(t_B - v x_B/c^2) \end{aligned} \tag{19}$$

- Lorentz transformations
  if transverse directions are unaffected by velocity transformation

Homework hint: exercise 5.3



Homework hint: exercise 5.5