Modern Physics

Luis A. Anchordoqui

Department of Physics and Astronomy
Lehman College, City University of New York

Lesson I
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Galilean Law

- Simplest statement of the law ☞ there is no experiment which can be implemented to measure a uniform velocity
- Since we can only know what can be measured ☞ we can never know how fast we are moving

Galileo Galilei
(1564-1642)

There is no speedometer on the starship Enterprise

Relative velocity

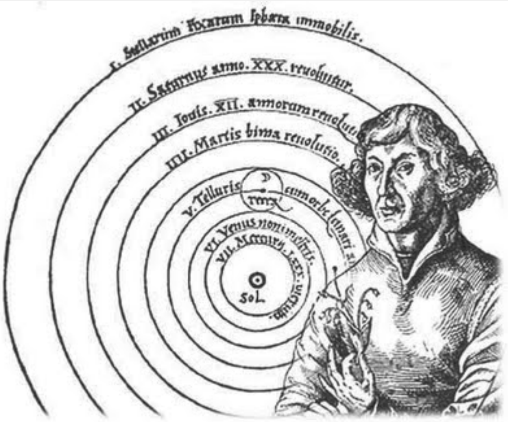
- Idea very counter to our experience because what we generally measure is not velocity in space but our velocity relative to Earth
- Relative velocities are detectable
  e.g. we note the amount of street that passes below our car
- We do know that the Earth moves around the Sun ☞ we can find our velocity relative to the Sun
- We know that the Sun is moving in our Galaxy and our Galaxy is moving relative to other nearby galaxies ☞ we can determine our velocity relative to the local cluster of galaxies
- We are also able to infer our velocity relative to the place that we occupied in the early universe ☞ our motion relative CMB
  but again we cannot know whether that place had a velocity
- Inability to detect uniform velocity is one of the most mysterious and counter intuitive concepts that has ever been layed down

Space

- Consider a remote and empty part of the universe
- Without stars or galaxies around there are no discernible forces
  A released particle moves in straight line with constant velocity
- This is one of Newton's laws:
  It was his way of enunciating Galilean Invariance
- From now on we will assume this empty region is "space"
- We envision this space as the stable background structure
  introduced by Newton against which motion takes place

Copernican Principle

Space is not centered on some special place like the Earth



General Copernican Principle

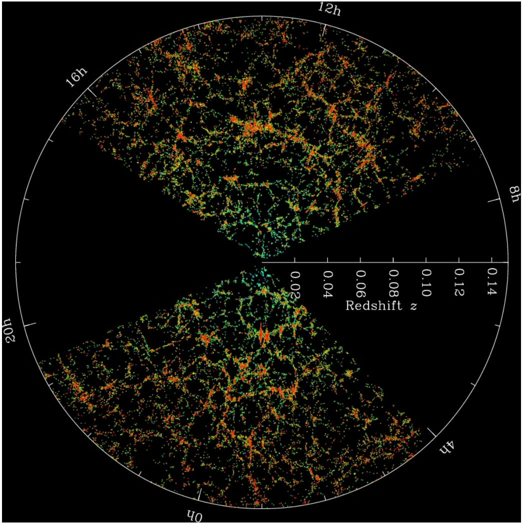
In empty universe there is no special place that could be called center

1. **Space is homogeneous**
   - There is no experiment that can be performed in space to distinguish one place from another
   - If you cannot distinguish between places ☞ of course you cannot have a center or a boundary
   - These are special places contrary to the viewpoint that all places are the same

2. **Space is isotropic**
   - Space is the same in all directions

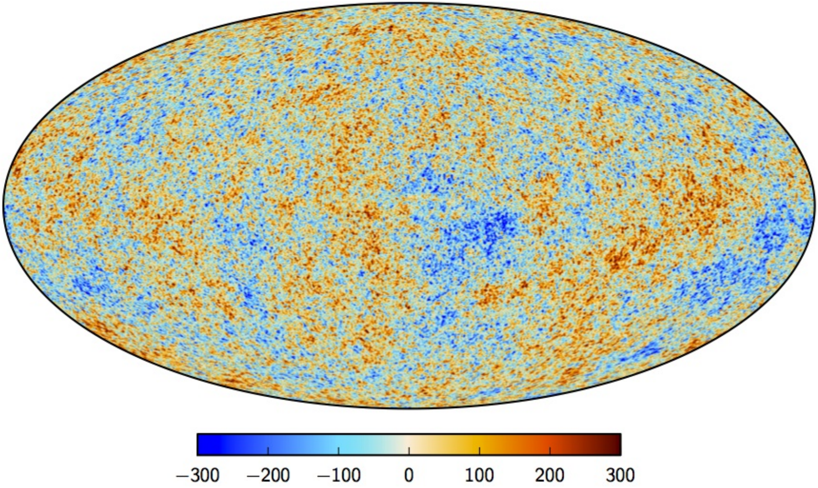
3. How do we test these hypothesis?
   - We cannot be anywhere other than where we are

Stars in remote galaxies evolve in the same way as nearby stars



There is no indication that the universe is not homogeneous

Cosmic Microwave Background



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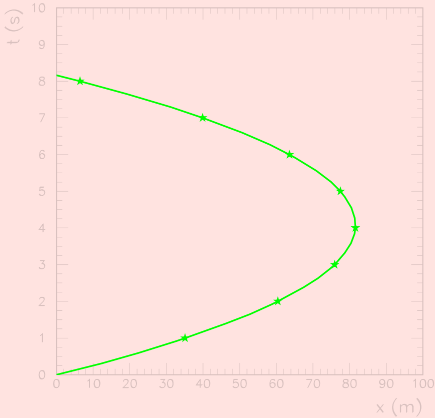
In the classical world...

- we describe state of a system by motion of its particles
- as point particle moves around use Cartesian coordinates (x, y, z) to describe where particle is at any given time t



Spacetime diagram: freely falling object lanunched upward at 40 m/s

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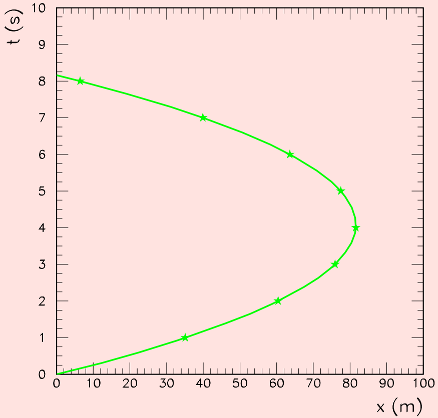

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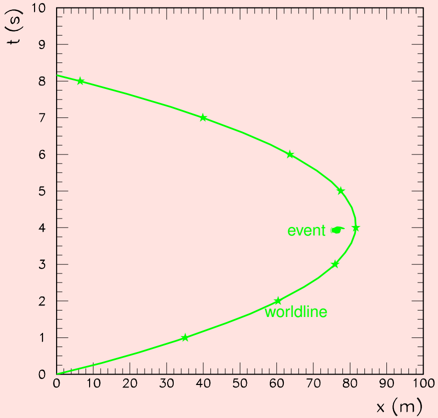

Spacetime diagram: freely falling object lanunched upward at 40 m/s

- We made tacit assumption about observer making measurements
- Simplistically ☞ take observer at rest
- We have seen that concept of absolute rest is pretty meaningless
- It is more useful to specify whether observer is accelerating or not
- We make measurements to ascertain if object is accelerating
  since acceleration of massive bodies originates in forces
- Any observer can make experiment
  to see whether or not he is accelerating
- A non-accelerating observer is called an inertial observer
- Throughout this course all our observers will be inertial

Galilean transformations

- Consider case where observer
  is moving at constant velocity with respect to another
- If we now impose Galilean Invariance
  ☛ each must have the same rules of physics
- Each observer
  – one measuring with (x, y, z, t) and the other with (x', y', z', t') – will conclude that the universe is homogeneous and isotropic
- If watches are synchronized ☞ how do coordinates of same event written down by two different observers differ ?

$$\begin{aligned} t' &= t \\ x' &= x - vt \\ y' &= y \\ z' &= z \end{aligned} \quad (1)$$

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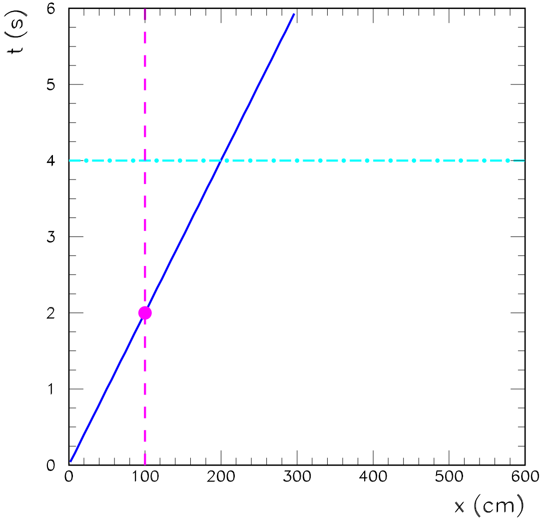
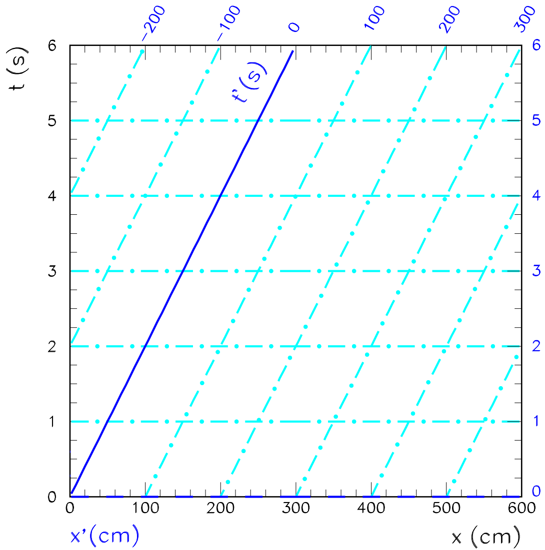
Consider two observers ☞ the gondolieri and the tourist on the bridge

What is the world line of the gondola ($v = 50~\mathrm{cm/s}$) for the two observers?



Gondola worldline in coordinate system of unprimed observer on bridge



Space-time diagram for moving gondola in bridge coordinate system with lines of constant t' and lines of constant x' overlaid

Light travels between two points
over the path that is the least travel time of all the possible paths

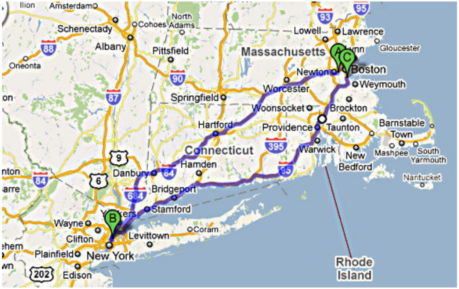
Pierre de Fermat
(1601-1665)

What is the best way to go between Boston and New York?

1. You take a map with all possible roads
2. You classify all routes dividing trip into segments
3. You estimate your speed in each segment.
4. Use speed and length of given segment to calculate time it takes and then you add up time for each segment to get a total

$$T(\text{route}) = \sum_{\text{segments}} \Delta t_i = \sum_{\text{segments}} \frac{\Delta s_i}{v_i} \tag{2}$$

5. Make ordered list of routes and repeat process for all routes
6. Once you have $T(\text{route}) \forall$ routes ☞ select the one with least time
7. This is route you take if you want the least time

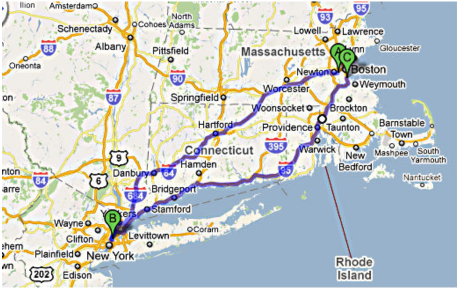


Likewise

If you know the speed of light at every place when light goes between two points you can apply this procedure to find the routes in space through which the light travels

Is it really this simple?

Note that in contrast to our highway problem there is an infinite number of paths



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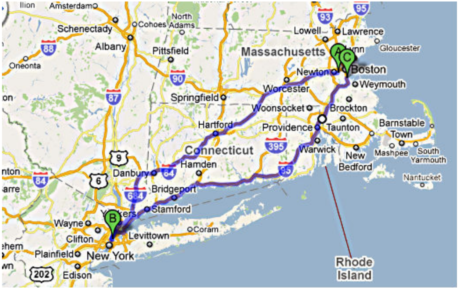

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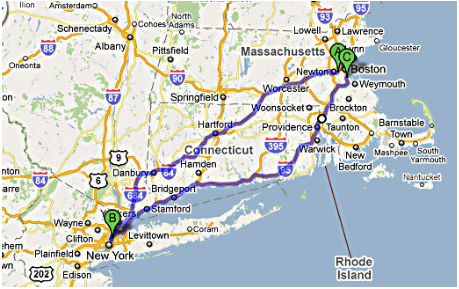

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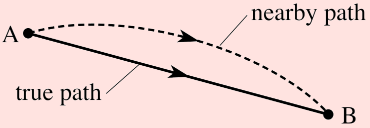
What are paths of light in homogeneous medium?

1. Homogeneous medium ☞ every point is the same
2. Speed of light must be the same at every point

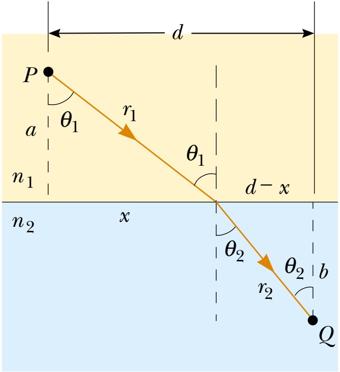
$$T(\text{path}) = \sum_{\text{segments}}^{\text{path}} \frac{\Delta s_i}{v_i} = \frac{1}{v} \sum_{\text{segments}}^{\text{path}} \Delta s_i \quad (3)$$

3. Since $\sum_{\text{segments}}^{\text{path}} \Delta s_i$ is definition of path length
   time for any path is proportional to length of the path
4. Least time path is the shortest-length path
   which (of course) is the straight line path



Refraction

- Refraction occurs when light passes through a medium
  that has varying speed for light and rays bend
- Simplest case
  ☞ system of two media that are themselves homogeneous
- How do we order paths that connect the two points
  ☞ there are infinity of them
- Least time paths in homogeneous medium must be straight lines
- Path with the least time overall must be among paths
  that are straight within either of two media and kinked at interface
- Curve path in one media would clearly be longer time path
  than one with same start point hitting other media at same point
- Label paths with distance of kink position
  from place at which path would meet interface
- Path space simplification to kinked straight line segments
  important reduction in nature of problem



- Consider light ray traveling from P to Q
- Speed of light is c/n_1 in medium 1 and c/n_2 in medium 2
- Time at which the ray arrives at Q is

$$T(x) = \frac{r_1}{v_1} + \frac{r_2}{v_2} = \frac{\sqrt{a^2 + x^2}}{c/n_1} + \frac{\sqrt{b^2 + (d-x)^2}}{c/n_2} \qquad (4)$$

Least time path

$$\frac{dT}{dx} = \frac{n_1 x}{c(a^2 + x^2)^{1/2}} - \frac{n_2(d - x)}{c[b^2 + (d - x)^2]^{1/2}} = 0 \tag{5}$$

yielding

$$\frac{n_1 x}{(a^2 + x^2)^{1/2}} = \frac{n_2(d - x)}{[b^2 + (d - x)^2]^{1/2}} \tag{6}$$

Since

$$\sin\theta_1 = \frac{x}{(a^2 + x^2)^{1/2}} \tag{7}$$

and

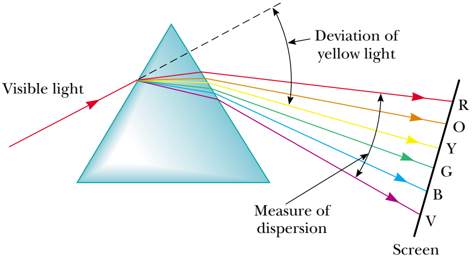
$$\sin\theta_2 = \frac{d - x}{[b^2 + (d - x)^2]^{1/2}} \tag{8}$$

Snell's law of refraction

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{9}$$

Prisms

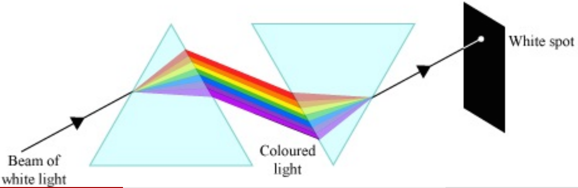
- Triangular cut of glass produce rainbow of color from sun light
- Narrow beam of white light
  incident at non-normal angle on one surface of glass is refracted
- Different colors of light have different speeds in glass
- Separated rays emerge from other interface
  spread in familiar rainbow pattern



Light as composite of colors

- Newton placed prism in path of narrow beam of sunlight.
- As expected ☞ beam was spread over band of angles
- He inserted second prism and allowed spread beam to enter it
- When arranged carefully
  second prism reconstituted original beam in original direction
- He labeled the different colors
  with continuously varying parameter that had the units of time
- λ and T characterizing given color connected by speed of light

$$\lambda/T = c/n$$



Mathematical Digression

- How do you determine the number of paths?
- Do you count them, or do you order them?
- Counting is a process of matching elements of two sets
- Smallest set of choice are integers numbers $\mathbb{Z}$
- Sets of this size are said to be in the class $\aleph_0$
- Functions are mappings of the real line onto the real line
- Real line example of the next larger infinite set ☞ $\aleph_1$
- Number of paths is larger than number of points on real line
- our selection of the point of intersection
  allowed a reduction in size of path space
- x is not just distance along the interface
  its real role is as a label in path space