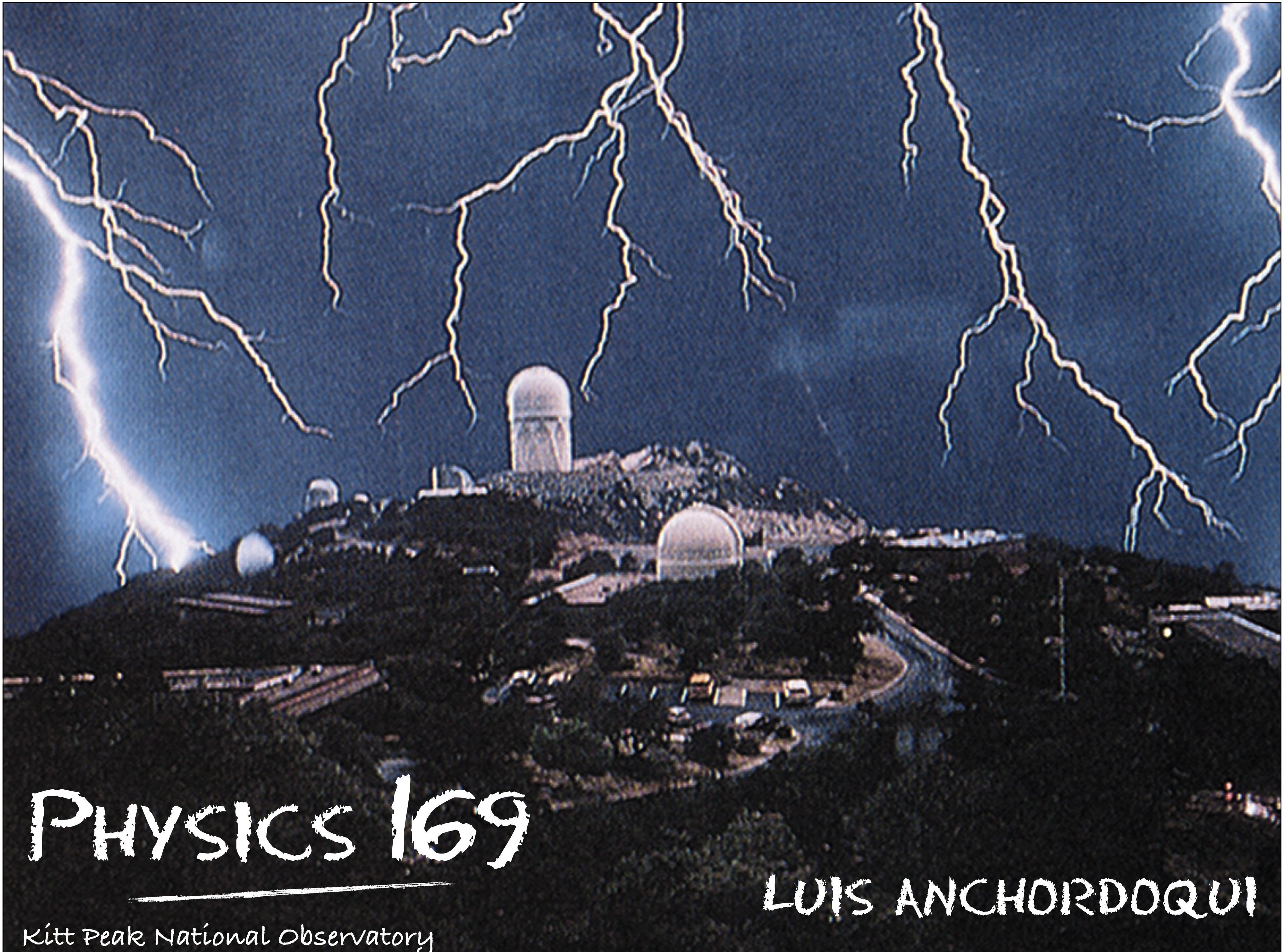


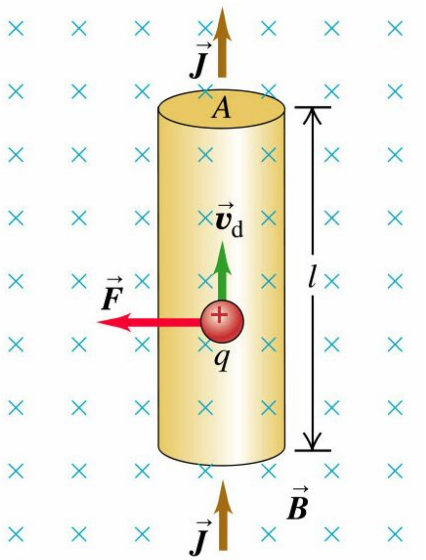
Physics 169
Luis Anchordoqui
Kitt Peak National Observatory

7.1 Magnetic Force on Currents (cont'd)

We know that a single moving charge experiences a force when it moves in a magnetic field

What is the net effect if we have multiple charges moving together, as a current in a wire?

We start with a wire of length *l* and cross section area *A* in a magnetic field of strength B with the charges having a drift velocity of v_d



The total number of charges in this section is then nAl where *n* is the charge density

The force on a single charge moving with drift velocity v_d is given by $F = qv_d B$

So the total force on this segment is

$$F = nqv_d AlB$$

We have so far that $F = nqv_d\,A\,l\,B$

But we also have that $J = nqv_d$ **and** $I = J\,A$

Combining these, we then have that $F = I\,l\,B$

The force on the wire is related to the current in the wire and the length of the wire in the magnetic field

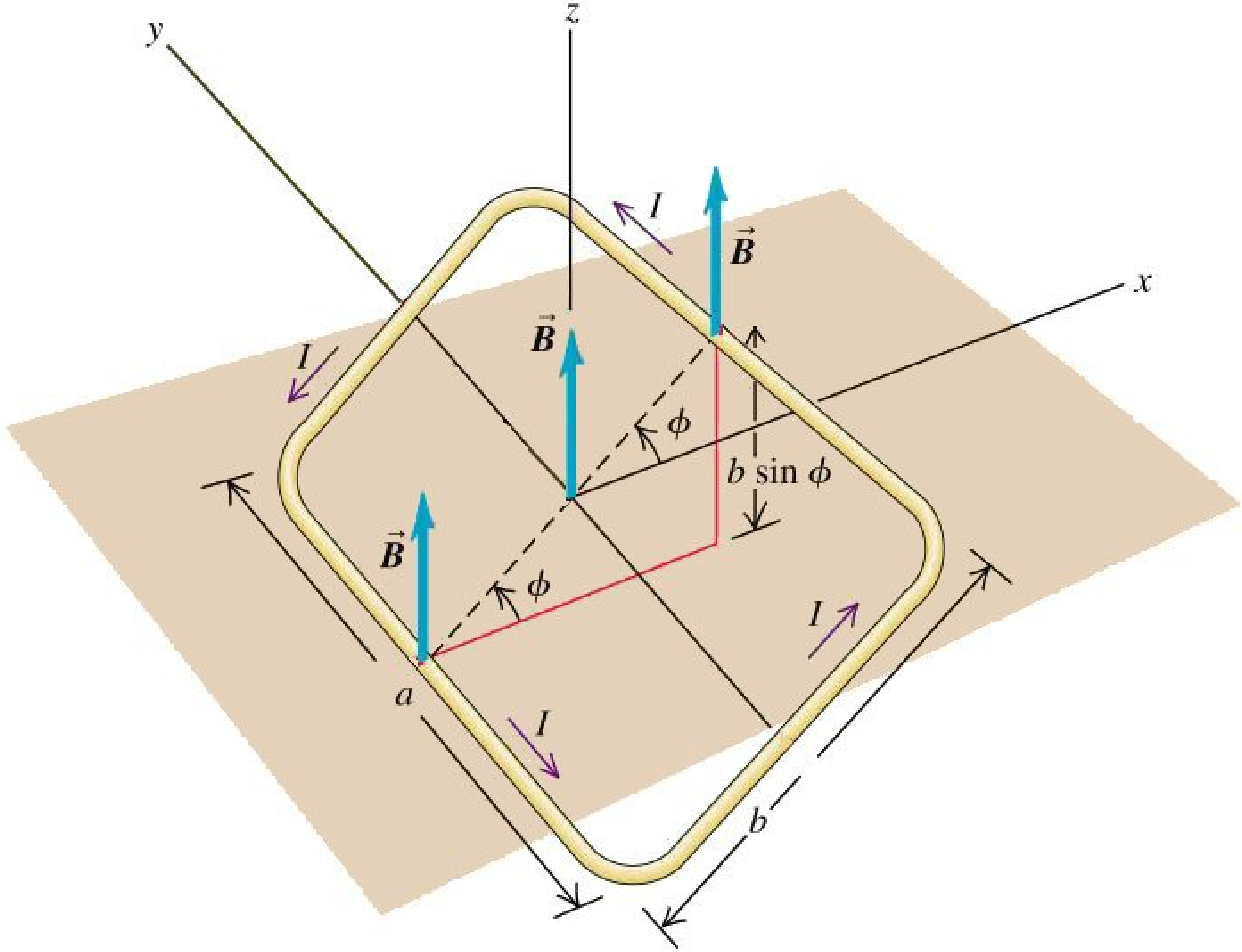
If the field and the wire are not perpendicular to each the full relationship is

$$\vec{F} = I\,\vec{l} \times \vec{B}$$

The direction of *l* is the direction of the current

Example 3: Current loop in a magnetic field

Suppose that instead of a current element, we have a closed loop in a magnetic field



We ask what happens to this loop

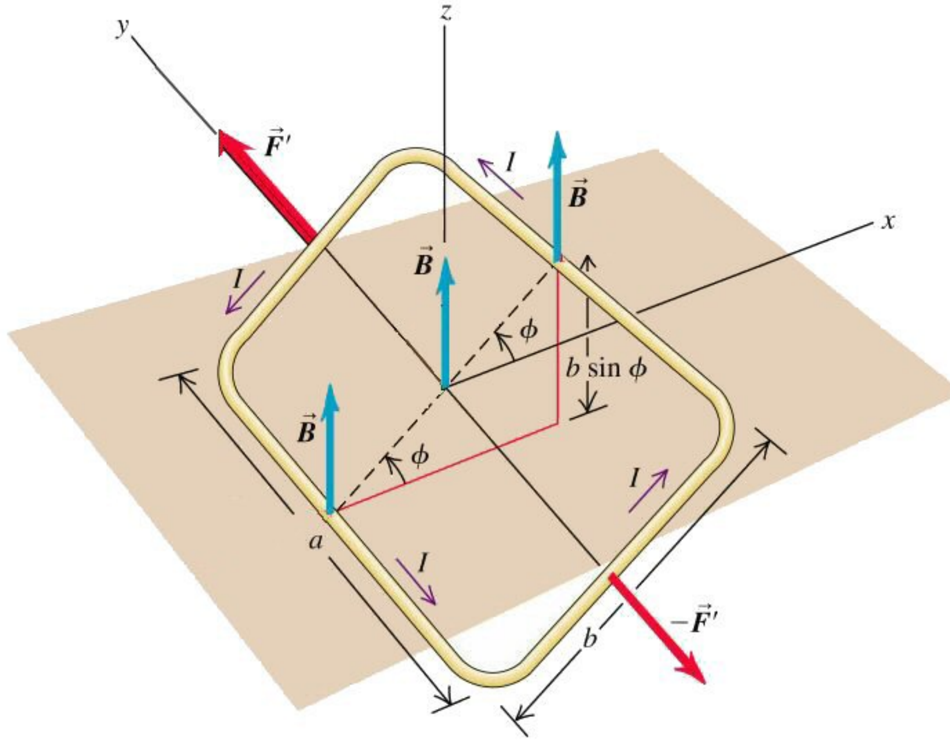
Each segment experiences a magnetic force since there is a current in each segment

As with the velocity, it is only the component of the wire that is perpendicular to B that matters

Each of the two shorter sides experiences a force given by

$$F' = I\,b\,B\cos\phi$$

in the directions shown



Since the magnitudes are the same, the net force in the y-direction is $\sum F_y = 0$

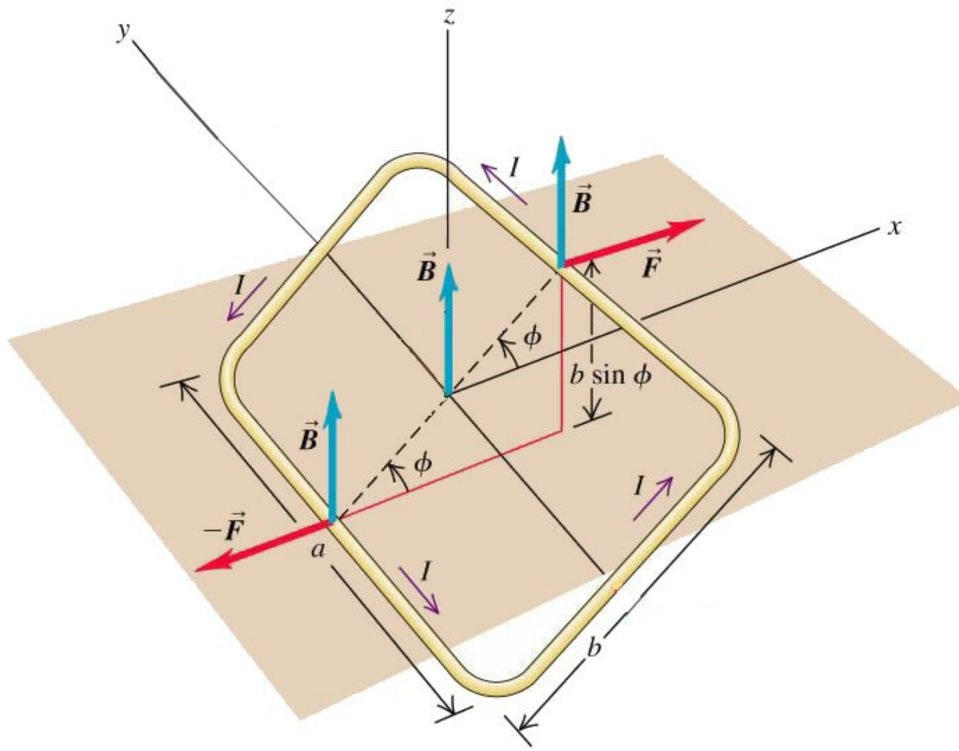
No translational motion in the y-direction

Now for the two longer sides of length *a*

Each of these two sides experiences a force given by

$$F = I\,a\,B$$

in the directions shown



But since the forces are of the same magnitude but in opposite directions we have

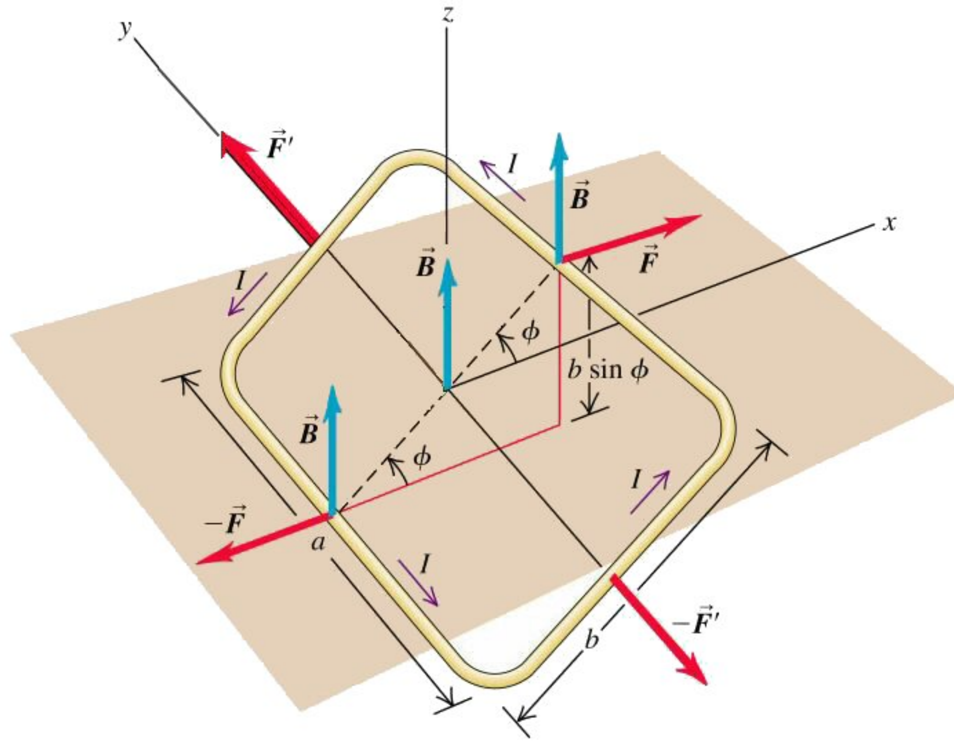
$$\sum F_x = 0$$

No translational motion in the x-direction

There is no translational motion in either the x- or y-directions

While the two forces in the y-direction are colinear, the two forces in the x-direction are not

Therefore there is a *torque* about the y-axis



The lever arm for each force is

$$\frac{\boldsymbol{b}}{2}\sin\phi$$

The net torque about the y-axis is

$$\tau = 2\boldsymbol{F}\left(\frac{\boldsymbol{b}}{2}\right)\sin\phi = \boldsymbol{I}\,\boldsymbol{B}\,\boldsymbol{a}\,\boldsymbol{b}\sin\phi$$

This torque is along the positive y-axis and is given by

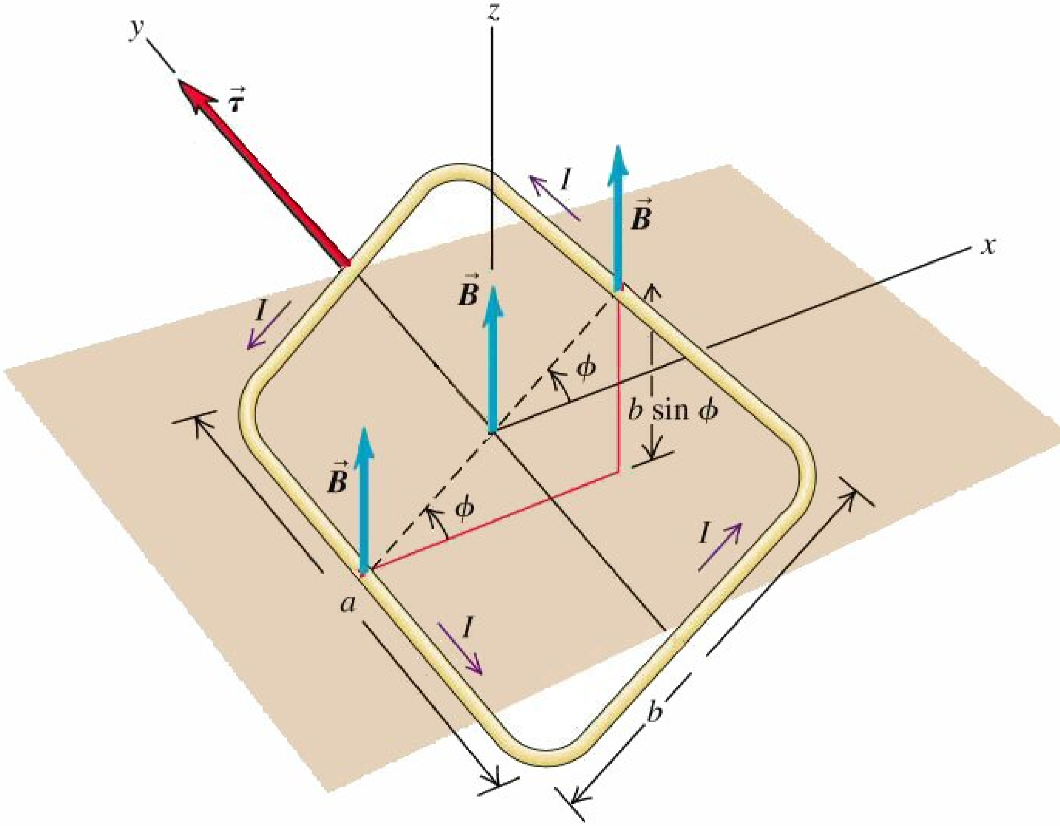
$$\tau = \boldsymbol{I}\,\boldsymbol{B}\,\boldsymbol{A}\sin\phi$$

The product *IA* is referred to as the ***magnetic moment***

$$\mu = \boldsymbol{I}\,\boldsymbol{A}$$

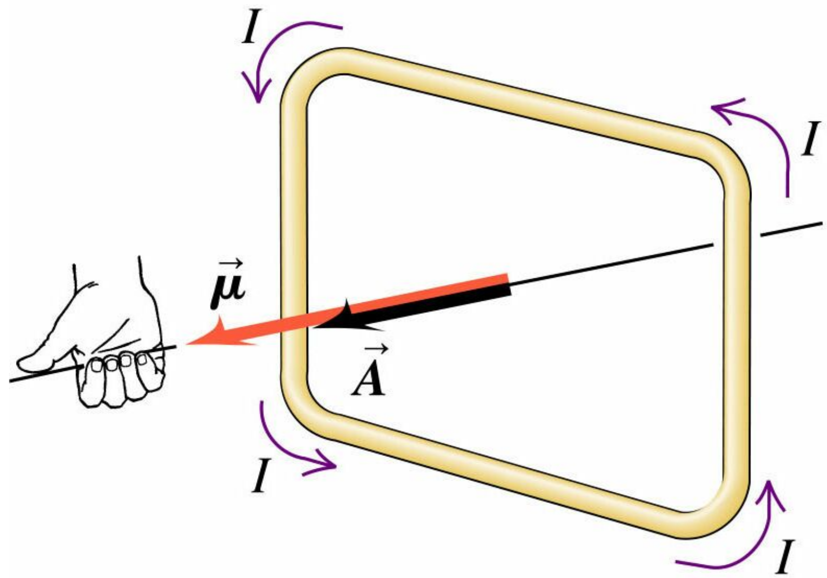
We rewrite the torque as

$$\tau = \mu\,\boldsymbol{B}\sin\phi$$



We defined the magnetic moment to be $\mu = I\,A$

It also is a *vector* whose direction is given by the direction of the area of the loop



The direction of the area is defined by the sense of the current

We can now write the torque as $\vec{\tau} = \vec{\mu} \times \vec{B}$

7.2 Ampere's Law

In our study of electricity

we noticed that **inverse square force** law leads to **Gauss' law**

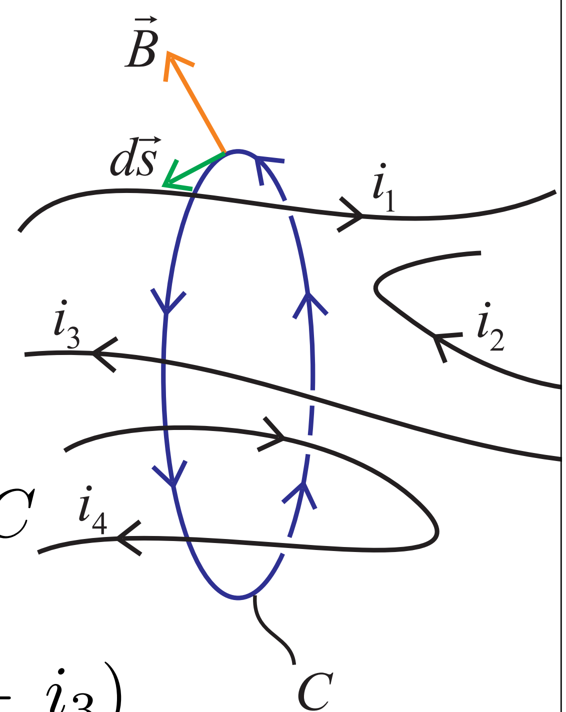
useful for **finding $\vec{E}$-field for systems with high level of symmetry**

For magnetism ➡ Gauss' law is simple

$$\oiint_S \;☛\; \oint_S \vec{B}\cdot d\vec{A} = 0 \;☛\; \because \text{ there are no magnetic monopoles}$$

For calculating $\vec{B}$-field for highly symmetric situations ☛ **Ampere's Law**

$$\oint_C \vec{B}\cdot d\vec{s} = \mu_0 i$$



$\oint_C$ ☛ line integral evaluated around a closed loop C

Amperian curve

i ☛ net current penetrating area bounded by curve C

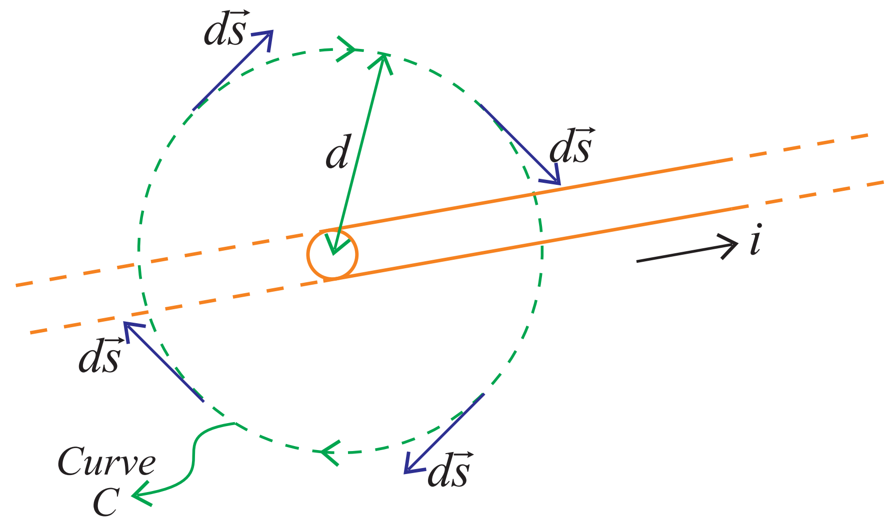
topological property

$$\oint_C \vec{B}\cdot d\vec{s} = \mu_0(i_1 - i_3 + i_4 - i_4) = \mu_0(i_1 - i_3)$$

Applications of Ampere's Law

① Long-straight wire

Construct Amperian curve of radius d



By symmetry argument $\vec{B}$-field only has **tangential component**

$$\therefore \quad \oint_C \vec{B} \cdot d\vec{s} = \mu_0 i$$

Take $d\vec{s}$ to be tangential vector around circular path

$$\therefore \quad \vec{B} \cdot d\vec{s} = B\, ds$$

$$B \underbrace{\oint_C ds}_{\text{Circumference of circle } = 2\pi d} = \mu_0 i$$

$$\therefore \quad B(2\pi d) = \mu_0 i$$

$\vec{B}$-field due to long straight current ☛ $B = \dfrac{\mu_0 i}{2\pi d}$

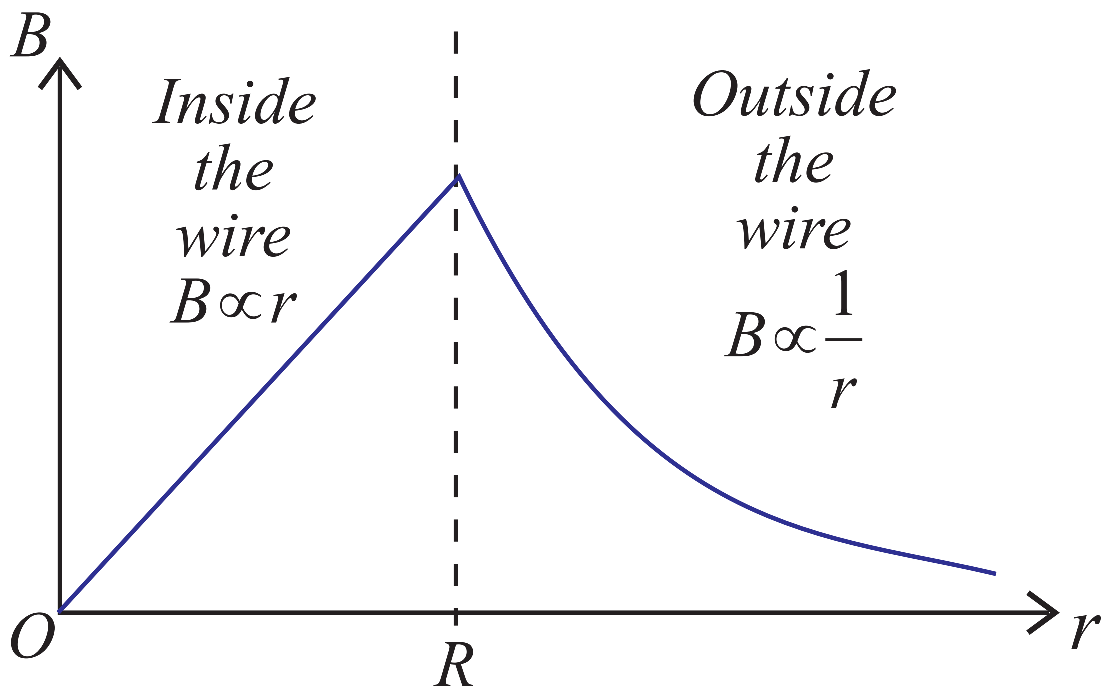
② Inside a current-carrying wire

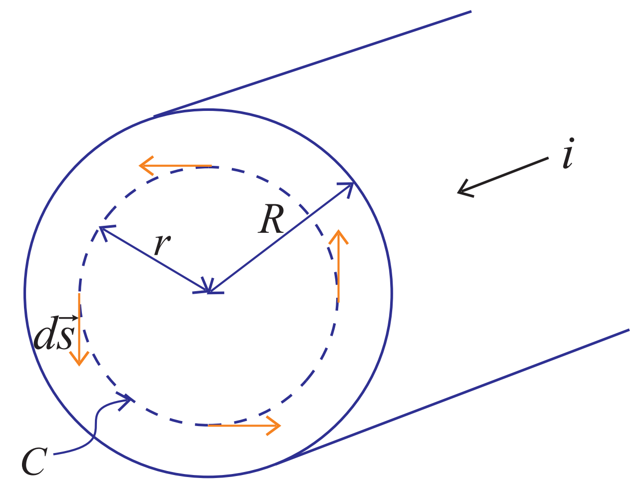
Again ☛ symmetry argument

implies that $\vec{B}$ is tangential to Amperian curve and $\vec{B} \to B(r)\hat{\theta}$

Consider Amperian curve of radius $r(< R)$

$i_{\text{included}} \propto$ cross-sectional area of C





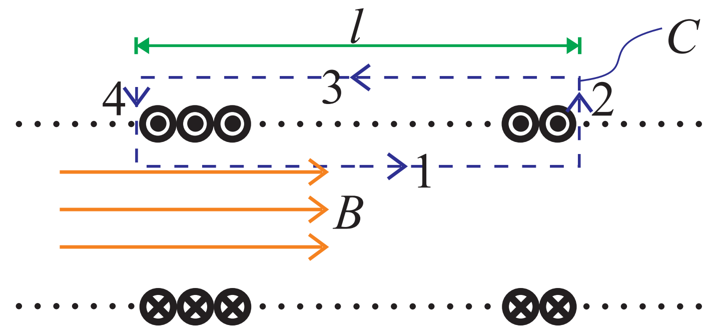
$$\therefore \quad \frac{i_{\text{included}}}{i} = \frac{\pi r^2}{\pi R^2}$$

$$\therefore \quad i_{\text{included}} = \frac{r^2}{R^2}\, i$$

$$\therefore \quad B = \frac{\mu_0 i}{2\pi R^2} \cdot r \propto r$$

③ Solenoid (Ideal)

Consider rectangular Amperian curve 1234 ➢



$$\oint_C \vec{B}\cdot d\vec{s} = \int_1 \vec{B}\cdot d\vec{s} + \cancel{\int_2 \vec{B}\cdot d\vec{s}} + \cancel{\int_3 \vec{B}\cdot d\vec{s}} + \cancel{\int_4 \vec{B}\cdot d\vec{s}}$$

$$\int_2 = \int_4 = 0 \quad \because$$ field perpendicular to path

$$\int_3 = 0 \quad \because \quad \vec{B} = 0$$ outside solenoid

$$\therefore \quad \oint_C \vec{B}\cdot d\vec{s} = \int_1 \vec{B}\cdot d\vec{s} = Bl = \mu_0 i_{tot}$$

But $i_{tot} = \underbrace{nl}_{\text{numbers of coils included}} \cdot i$

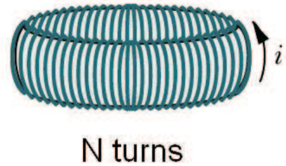
$$\therefore \quad B = \mu_0 n i$$

Note

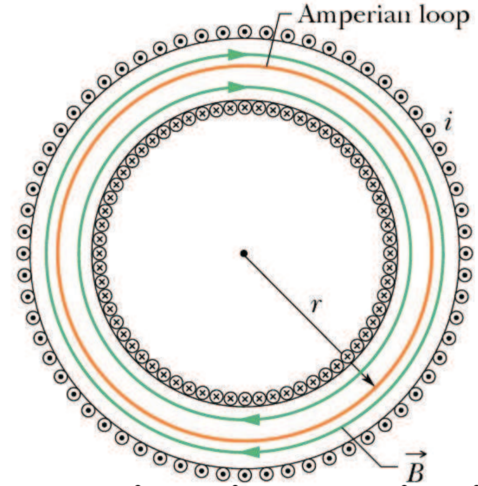
(i) Assumption that $\vec{B} = 0$ outside ideal solenoid is only **approximate**

(ii) $\vec{B}$-field everywhere inside solenoid is a **constant** (for ideal solenoid)

④ Toroid (circular solenoid)



approximated as



By symmetry argument ☛ field lines form **concentric circles inside toroid**

Take Amperian curve C to be a circle of radius r inside toroid

$$\oint_C \vec{B}\cdot d\vec{s} = B\oint_C ds = B\cdot 2\pi r = \mu_0(Ni)$$

$$\therefore \quad B = \frac{\mu_0 N i}{2\pi r} \quad \text{inside toroid}$$

Note

(i) $B \neq$ constant inside toroid

(ii) Outside toroid ☛ take Amperian curve to be circle of radius $r > R$

$$\oint_C \vec{B}\cdot d\vec{s} = B\oint_C ds = B\cdot 2\pi r = \mu_0 \cdot i_{\text{incl}} = 0$$

$$\therefore \quad B = 0$$

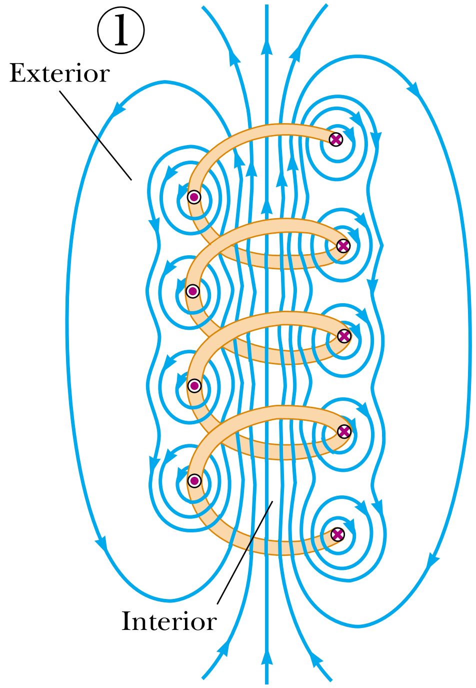
Similarly ☛ in central cavity $B = 0$

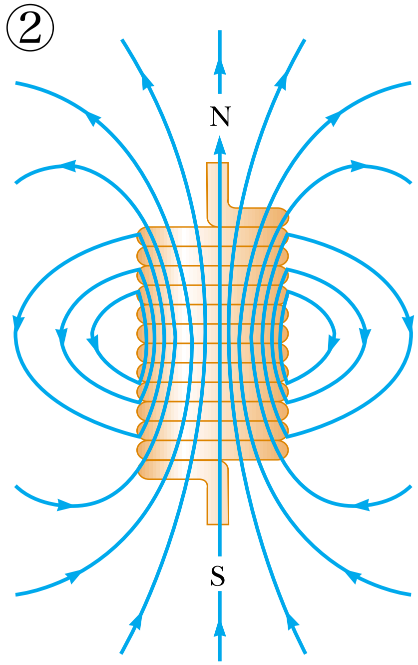
① Magnetic field lines for a loosely wound solenoid

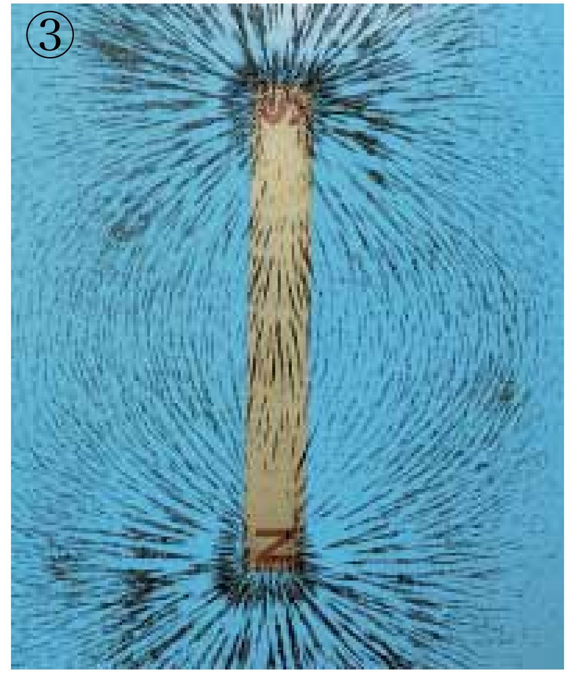
② Magnetic field lines for tightly wound solenoid of finite length carrying steady current

③ Magnetic field pattern of a bar magnet

(displayed with small iron filings on sheet of paper)

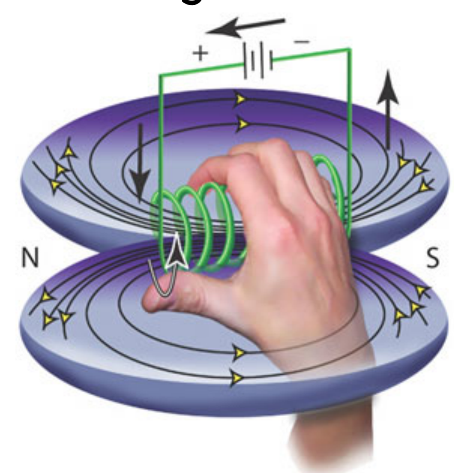






Field lines in interior of tightly wound solenoid resemble those of a bar magnet

meaning that solenoid effectively has north and south poles



7.3 Magnetic Dipole

In §7.1 we defined **magnetic dipole moment** of rectangular current loop

$$\vec{\mu} = Ni\,A\hat{n}$$

$\hat{n} =$ area unit vector with direction determined by right-hand rule

$N =$ number of turns in current loop

$A =$ area of current loop

This is actually a general definition of a magnetic dipole

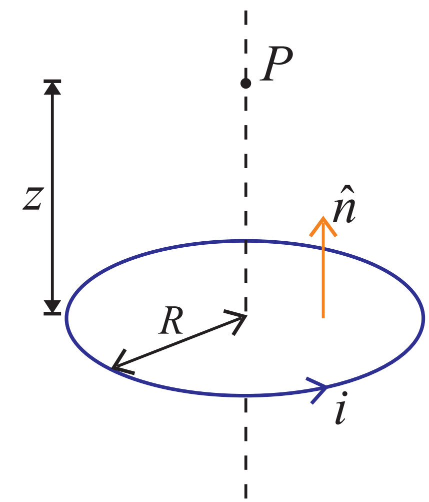
i.e. we use it for current loops of **all** shapes

A common and symmetric example ☛ circular current

Recall from §6.1 (Example 2)

magnetic field at point P (height z above ring)

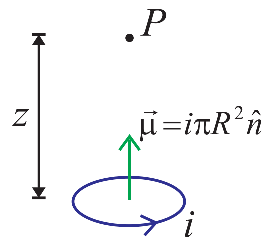
$$\vec{B} = \frac{\mu_0 i R^2 \hat{n}}{2(R^2+z^2)^{3/2}} = \frac{\mu_0 \vec{\mu}}{2\pi(R^2+z^2)^{3/2}}$$



At distance $z \gg R$

$$\vec{B} = \frac{\mu_0 \vec{\mu}}{2\pi z^3}$$

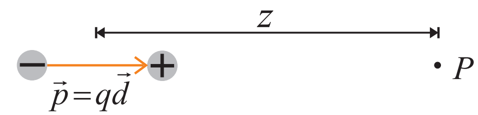
due to **magnetic dipole**
(for $z \gg R$)

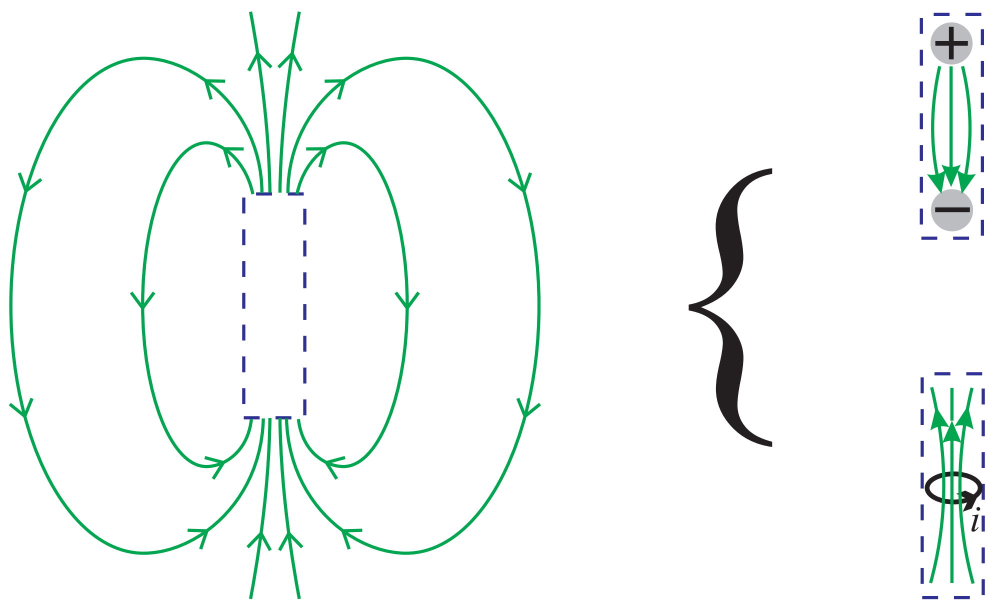


Recall

$$\vec{E} = \frac{\vec{p}}{4\pi\epsilon_0 z^3}$$

due to **electric dipole**
(for $z \gg d$)





Also note that

$\vec{\mu}$ = magnetic dipole moment [Unit: Am^2 / J/T]

μ_0 = Permeability of free space

$= 4\pi \times 10^{-7} \mathrm{Tm/A}$

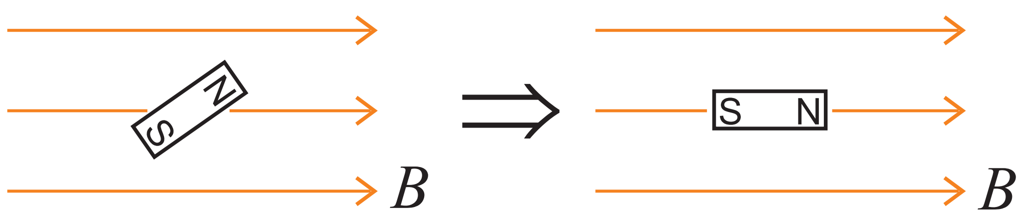
7.4 Magnetic Dipole in A Constant B-field

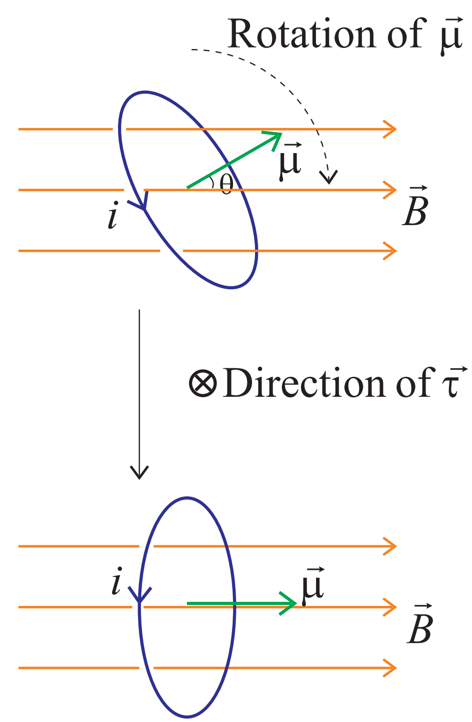
In presence of a constant magnetic field ☛ we have shown that

rectangular current loop experiences a **torque** $\vec{\tau} = \vec{\mu} \times \vec{B}$

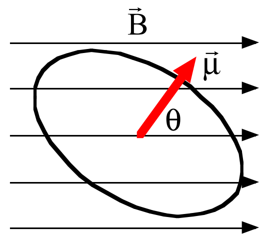
This applies to any magnetic dipole in general

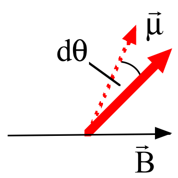
external magnetic field aligns magnetic dipoles





When a torque is applied on an object that is free to rotate, work is done. The incremental work done by the field when a magnetic dipole is rotated through an angle $d\theta$ is:





$dW = -\tau.d\theta$

$= -\mu B \sin\theta.d\theta,$

where θ is the angle between $\vec{\mu}$ and $\vec{B}$. The work done is equal to the decrease in potential energy of the system, i.e.,

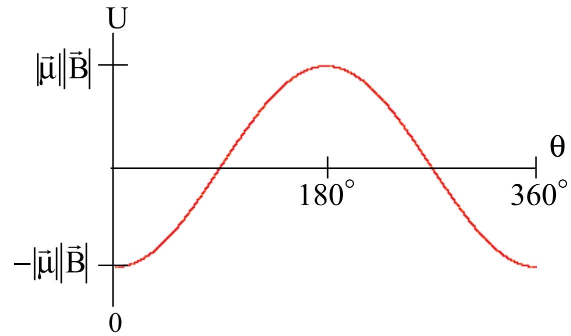
$$dU = -dW = \mu B \sin\theta.d\theta.$$

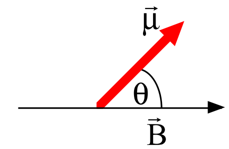
$$\therefore U = \int \mu B \sin\theta.d\theta = -\mu B \cos\theta + U_{\circ},$$

where $U_{\circ}$ is an integration constant. Choosing $U = 0$ when $\theta = 90^{\circ}$, then $U_{\circ} = 0$.

$$\therefore U = -\mu B \cos\theta = -\vec{\mu} \bullet \vec{B},$$

the potential energy of a magnetic dipole at angle θ to the direction of a magnetic field.





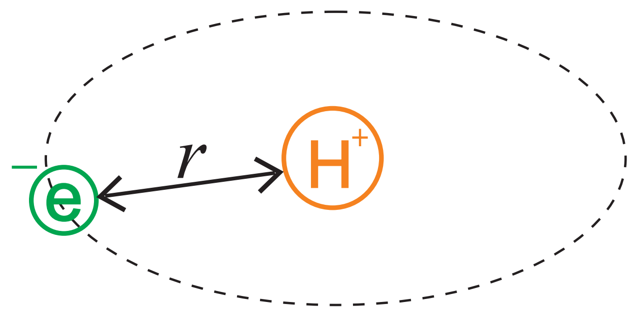
- When $\theta = 0$, U has its ***minimum*** value (*stable* equilibrium).
- When $\theta = 180^{\circ}$, U has its ***maximum*** value (*unstable* equilibrium).

Note that the torque acts to align the dipole with $\vec{\mu}$ parallel to $\vec{B}$.

7.5 Magnetic Properties of Materials

Intrinsic electric dipole moment of molecules

Intrinsic magnetic dipole moment of atoms



In our classical model of atoms electrons revolve around positive nucleus

Current $i = \dfrac{e}{P}$

P is period of one orbit around nucleus

$$P = \frac{2\pi r}{v}$$

v is velocity of electron

Magnetic dipole moment of atom

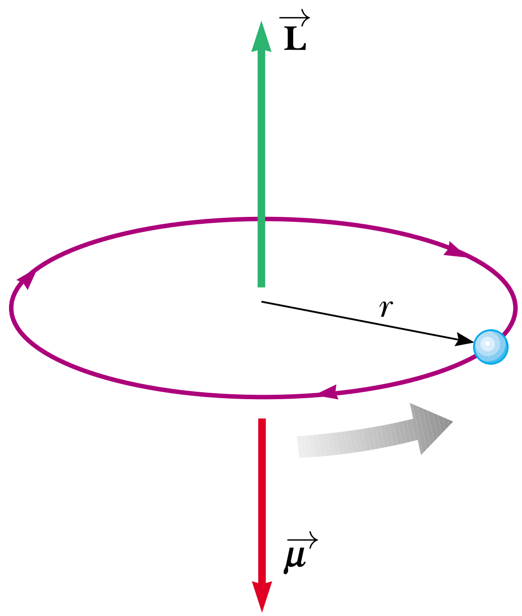
$$\mu = iA = \left(\frac{ev}{2\pi r}\right)(\pi r^2) = \frac{erv}{2}$$

Recall ☛ angular momentum of rotation ➢ $L = mrv$

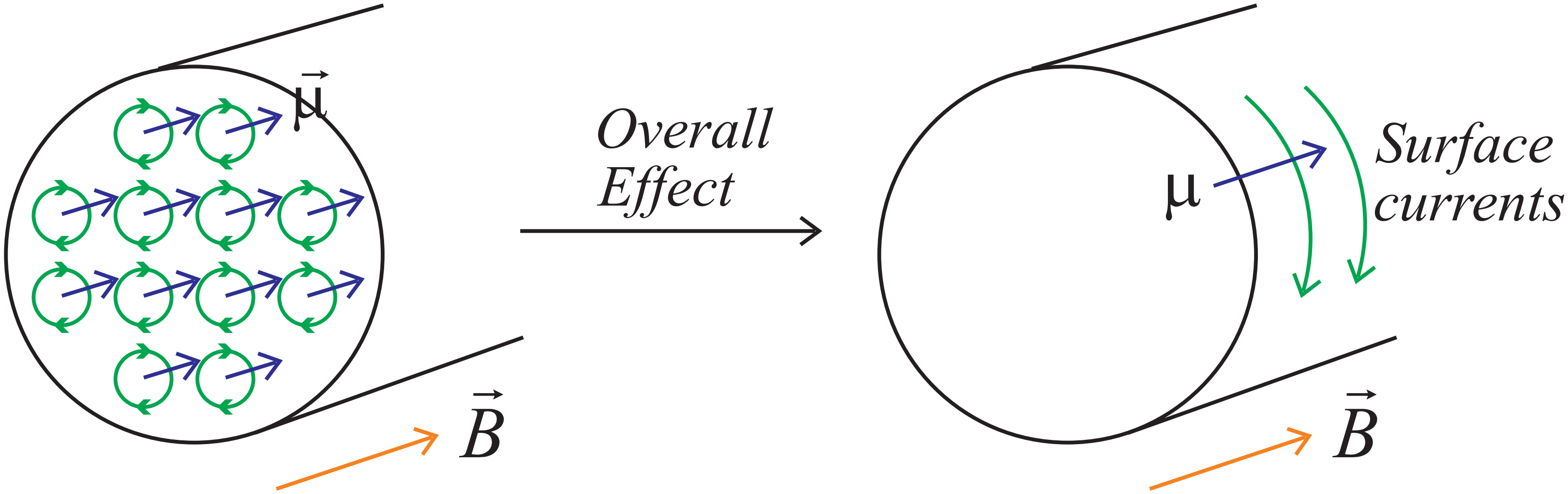
$$\mu = \frac{e}{2m} L$$

magnetic moment of e^- is proportional to its orbital angular momentum
Because electron is negatively charged

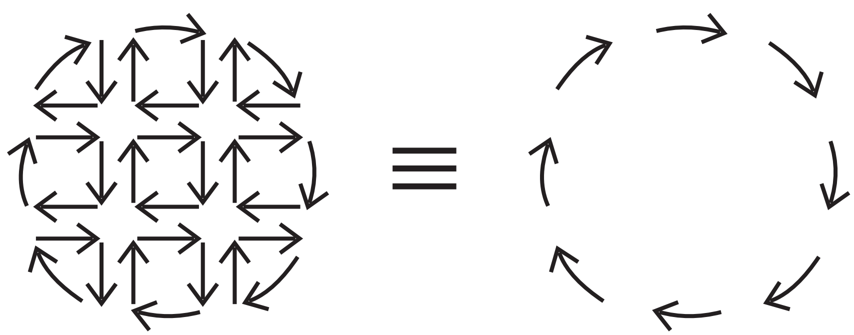
vectors $\vec{\mu}$ and $\vec{L}$ point in opposite directions



Total magnetic moment of atom is vector sum of magnetic moments



Induced $\vec{\mu}$ aligned with B-field



Magnetic Moments of Some Atoms and Ions

Atom or Ion	**Magnetic Moment (10^{-24} J/T)**
H	9.27
He	0
Ne	0
Ce^{3+}	19.8
Yb^{3+}	37.1

Magnetization and Magnetic Field Strength

Magnetic state of a substance is described magnetization vector
magnitude of $\vec{M}$ ☛ magnetic moment per unit volume of substance

$$|\vec{M}| = \mu/V$$

When a substance is placed in a magnetic field
total magnetic field in the region is expressed as

$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

$\vec{H}$ ☛ magnetic field strength

To better understand these definitions
consider the torus region of a toroid that carries a current I

If this region is a vacuum $\vec{M} = 0$ (because no magnetic material is present)
total magnetic field is that arising from current alone

$$\vec{B} = \mu_0 \vec{H}$$

Because $|\vec{B}| = \mu_0 n I$ in the torus region ➡ $H = B/\mu_0 = nI$
number of turns per unit length of the toroid

In general ☛ part of $\vec{B}$-field arises from term $\mu_0\vec{H}$ associated with current in toroid
and part arises from term $\mu_0\vec{M}$ due to magnetization of substance
of which the torus is made

Classification of Magnetic Substances

Substances can be classified as belonging to one of three categories depending on their magnetic properties

Paramagnetic and **Ferromagnetic** materials are those made of atoms that have permanent magnetic moments
(atoms have net magnetic moment due to unpaired electrons in partially filled orbitals)

Diamagnetic materials are those made of atoms that do not have permanent magnetic moments
(non-cooperative behavior of orbiting electrons)

However when exposed to external field a negative magnetization is induced

For paramagnetic and diamagnetic substances magnetization vector is proportional to magnetic field strength

$$\vec{M} = \chi \vec{H}$$

magnetic susceptibility

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) = \mu_0(\vec{H} + \chi\vec{H}) = \mu_0(1+\chi)\vec{H} = \mu_{\rm m}\vec{H}$$

magnetic permeability

paramagnetic materials ☛ $\mu_{\rm m} > \mu_0$

diamagnetic materials ☛ $\mu_{\rm m} < \mu_0$

Some crystalline substances exhibit strong magnetic effects called ferromagnetism

(e.g. iron, cobalt, nickel, gadolinium, and dysprosium)

These substances contain permanent atomic magnetic moments

that tend to align parallel to each other even in a weak external magnetic field

Once moments are aligned

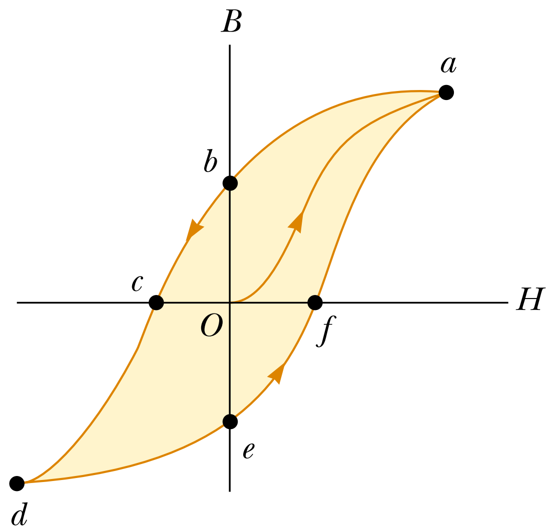
substance remains magnetized after external field is removed

This permanent alignment is due to strong coupling between neighboring moments
(coupling that can be understood only in quantum-mechanical terms)

Magnetic Susceptibilities of Some Paramagnetic and Diamagnetic Substances at 300 K

Paramagnetic Substance	χ	Diamagnetic Substance	χ
Aluminum	2.3×10^{-5}	Bismuth	-1.66×10^{-5}
Calcium	1.9×10^{-5}	Copper	-9.8×10^{-6}
Chromium	2.7×10^{-4}	Diamond	-2.2×10^{-5}
Lithium	2.1×10^{-5}	Gold	-3.6×10^{-5}
Magnesium	1.2×10^{-5}	Lead	-1.7×10^{-5}
Niobium	2.6×10^{-4}	Mercury	-2.9×10^{-5}
Oxygen	2.1×10^{-6}	Nitrogen	-5.0×10^{-9}
Platinum	2.9×10^{-4}	Silver	-2.6×10^{-5}
Tungsten	6.8×10^{-5}	Silicon	-4.2×10^{-6}

Magnetization curve for ferromagnetic material



Note that at point b

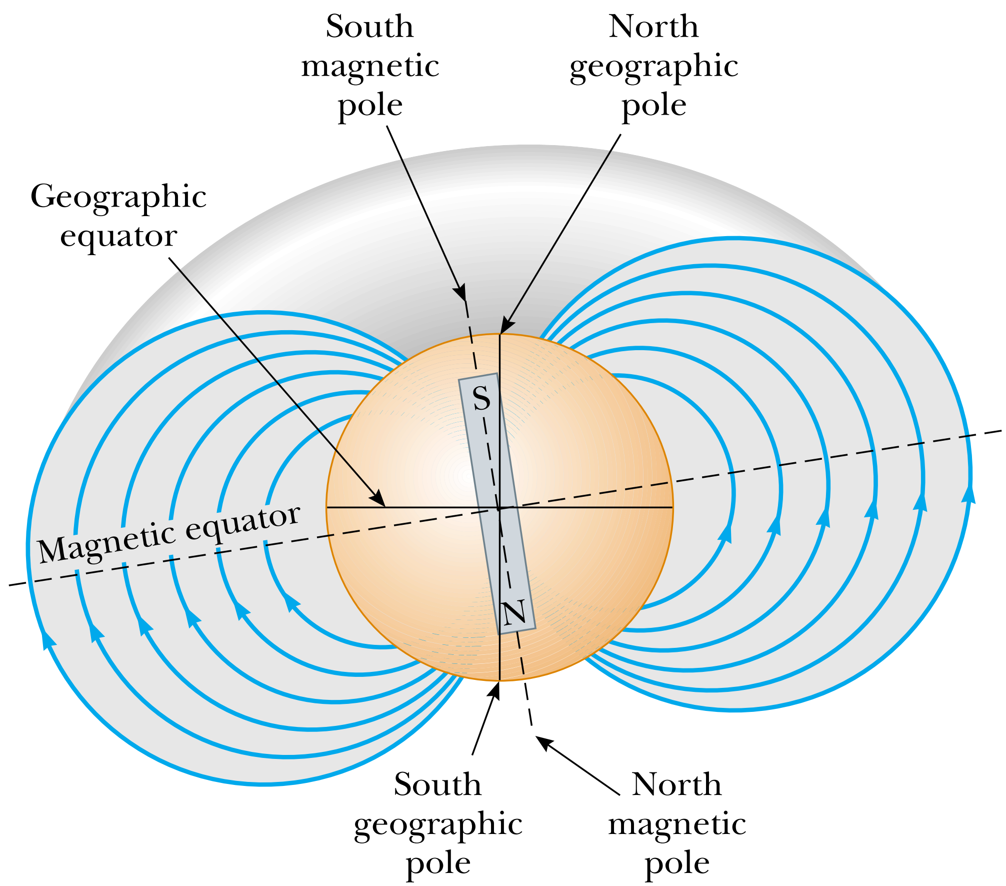
$\vec{B}$ is not zero

even though external field $\mu_0 \vec{H}$ is zero

7.6 Earth's Magnetic Field

When we speak of compass magnet having north pole and south pole
we should say more properly ☛ it has "north-seeking" pole and "south-seeking" pole
This mean that one pole of the magnet seeks for north geographic pole of Earth

Because north pole of magnet is attracted toward north geographic pole of Earth
we conclude that Earth's south magnetic pole is located near north geographic pole
and the Earth's north magnetic pole is located near south geographic pole



Configuration of the Earth's magnetic field resembles the one that would be achieved
by burying a gigantic bar magnet deep in Earth's interior


SEATTLE SEAHAWKS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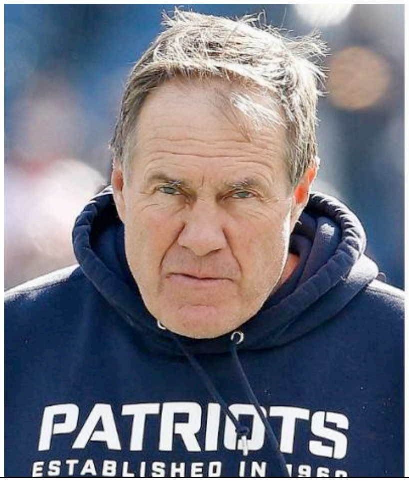
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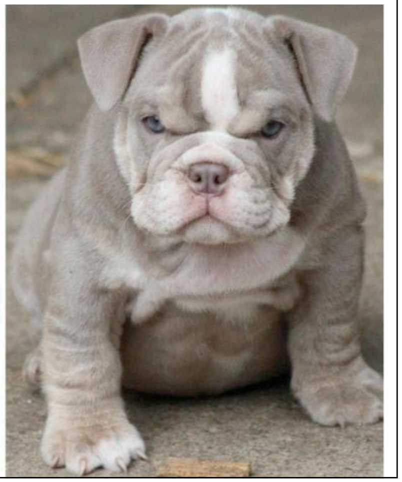

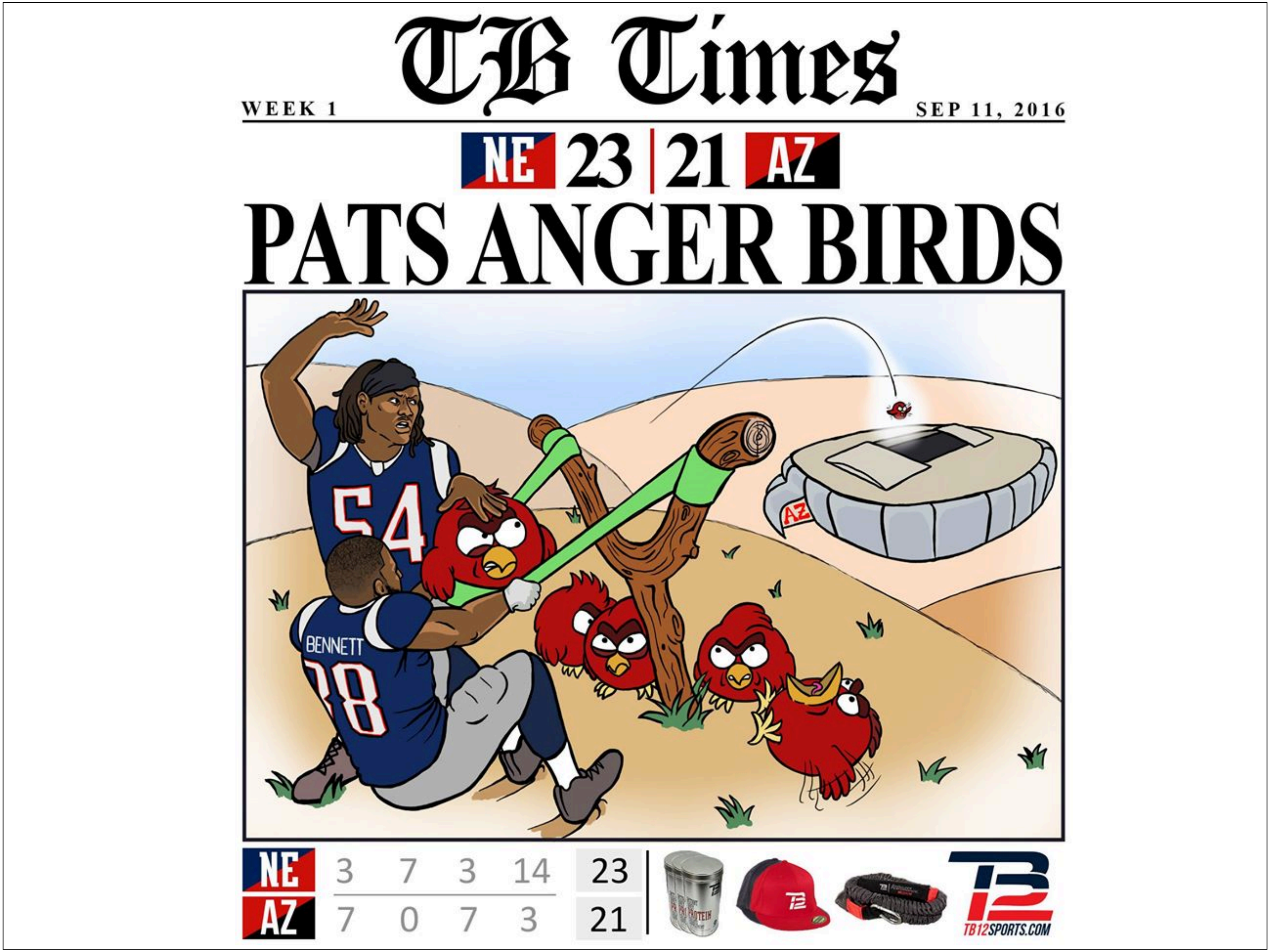
TB Times
WEEK 1
SEP 11, 2016
NE 23 | 21 AZ
PATS ANGER BIRDS
54
BENNETT
28
AZ
NE 3 7 3 14 23
AZ 7 0 7 3 21
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