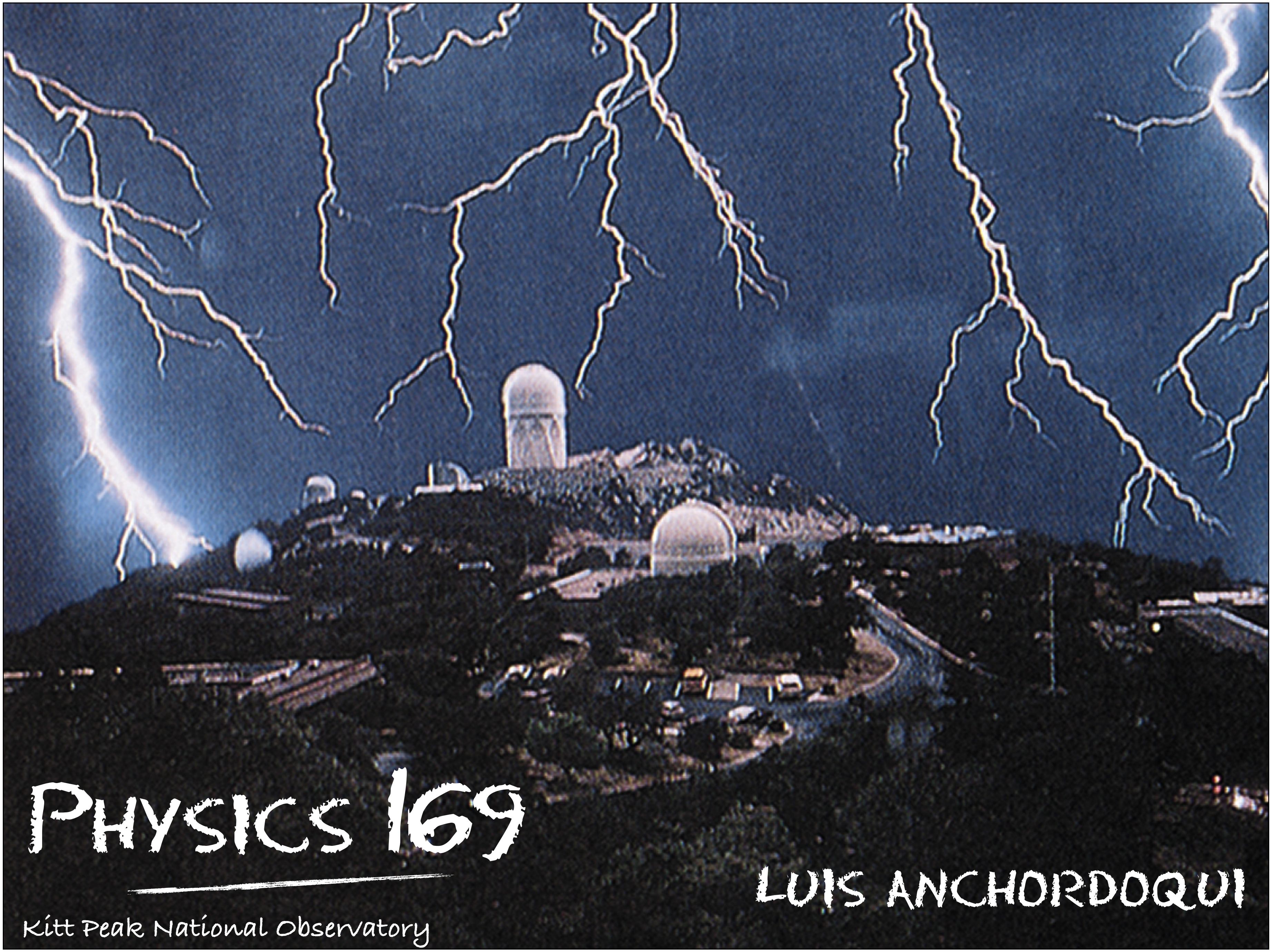
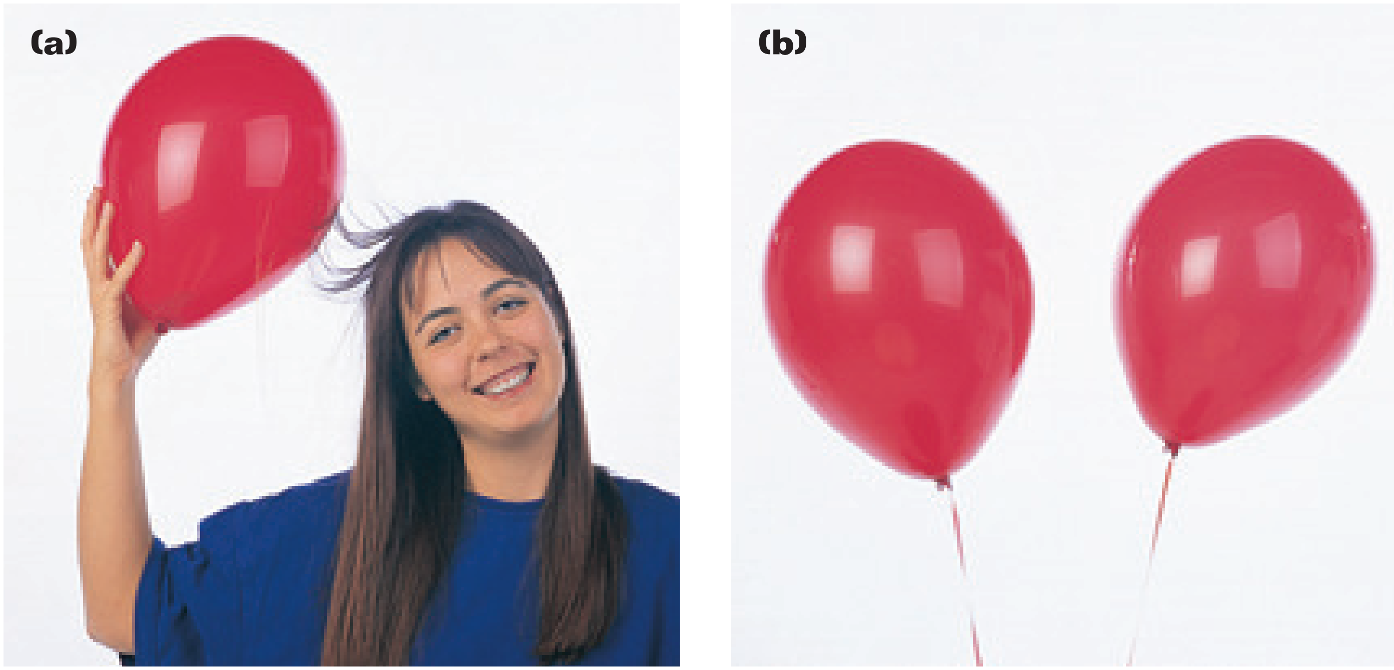


Physics 169
Luis Anchordoqui
Kitt Peak National Observatory

1.1 Electric Charge

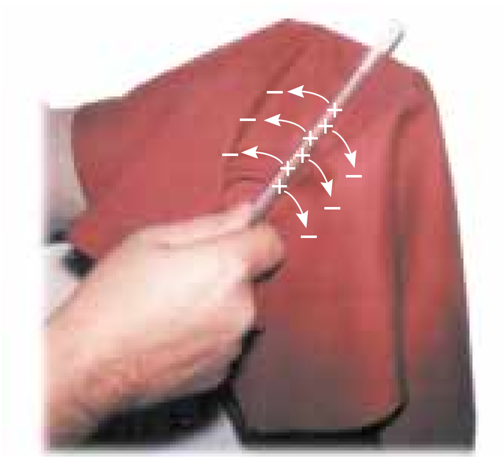


(a) If you rub a balloon across your hair on a dry day
the balloon and your hair become charged and attract each other
(b) Two charged balloons, on the other hand, repel each other.

The two balloons must have the same kind of charge
because each became charged in the same way
Because two charged balloons repel one another we see that like charges repel

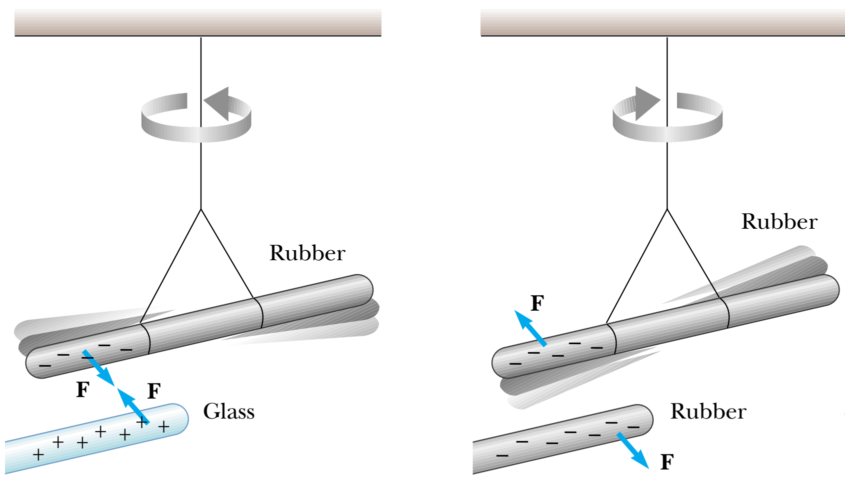
Conversely ☛ a rubbed balloon and your hair
which do not have the same kind of charge
are attracted to one another ☛ unlike charges attract

Charge is conserved and quantized



When a glass rod is rubbed with silk
electrons are transferred from the glass to the silk
Because of conservation of charge
each electron adds negative charge to the silk
and an equal positive charge is left behind on the rod

Also ☛ because charges are transferred in discrete bundles
charges on the two objects are $\pm e, \pm 2e, \pm 3e, \cdots$



left

A negatively charged rubber rod
suspended by a thread is attracted
to a positively charged glass rod

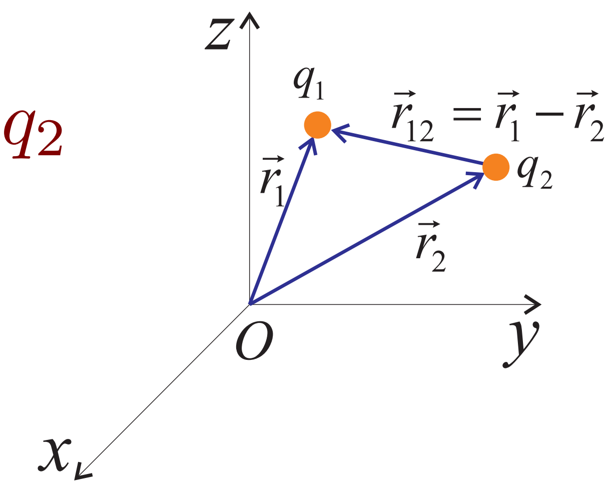
right

A negatively charged rubber rod
is repelled by another negatively charged
rubber rod

1.2 Electric Force

Electric force between two **charges** q_1 and q_2

described by ☛ **Coulomb's Law**



$\vec{F}_{12}$ = Force on q_1 exerted by q_2

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r_{12}^2} \cdot \hat{r}_{12}$$

$\hat{r}_{12} = \frac{\vec{r}_{12}}{|\vec{r}_{12}|}$ ☛ **unit vector** which locates particle 1 relative to particle 2

i.e. $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$

- q_1, q_2 are electrical charges in units of **Coulomb** (C)
- Charge is **quantized** ☛ electron carries 1.602×10^{-19} C
- **Permittivity of free space** $\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/Nm^2}$

COULOMB'S LAW:

(1) q_1, q_2 can be either positive or negative

(2) If q_1, q_2 **are of same sign**

force experienced by q_2 is in direction **away from** q_1 i.e. ☛ **repulsive**

(3) Force on q_2 exerted by q_1 :

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_2 q_1}{r_{21}^2} \cdot \hat{r}_{21}$$

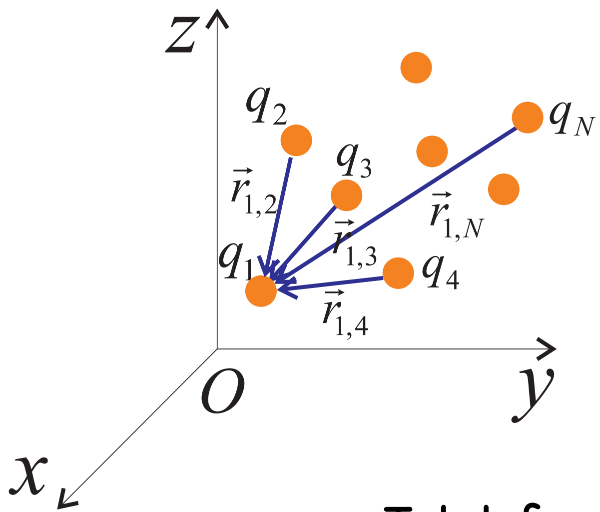
BUT

$r_{12} = r_{21} =$ **distance between** q_1, q_2

$$\hat{r}_{21} = \frac{\vec{r}_{21}}{r_{21}} = \frac{\vec{r}_2 - \vec{r}_1}{r_{21}} = \frac{-\vec{r}_{12}}{r_{12}} = -\hat{r}_{12}$$

$\vec{F}_{21} = -\vec{F}_{12}$ **Newton's 3rd Law**

SYSTEM WITH MANY CHARGES:



Total force experienced by charge q_1

vector sum of forces on q_1 exerted by other charges

$\vec{F}_1$ = **Force experienced by** q_1

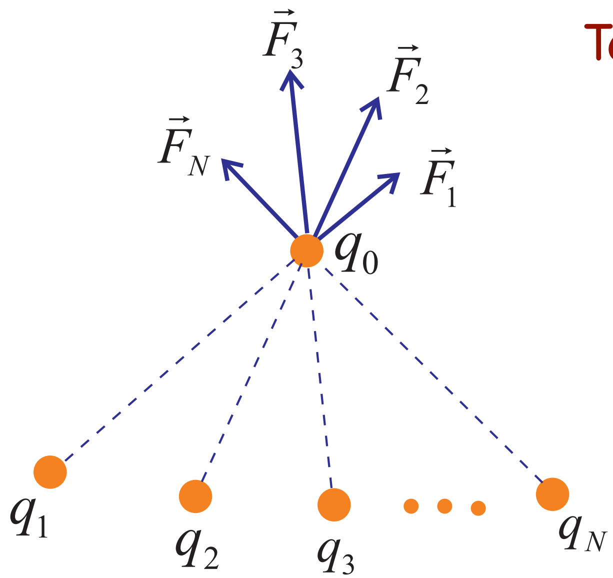
$$= \vec{F}_{1,2} + \vec{F}_{1,3} + \vec{F}_{1,4} + \cdots + \vec{F}_{1,N}$$

PRINCIPLE OF SUPERPOSITION

$$\vec{F}_1 = \sum_{j=2}^{N} \vec{F}_{1,j}$$

1.3 Electric Field

While we need two charges to quantify **electric force** we define **electric field** for any single charge distribution to describe its effect on other charges

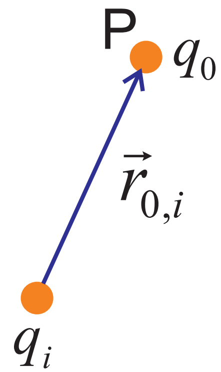


Total force $\vec{F} = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_N$

Electric field is defined as

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$$

(i) E-field due to a single charge q_i



From definitions of **Coulomb's Law**

force experienced at location of q_0 (point P)

$$\vec{F}_{0,i} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_0 q_i}{r_{0,i}^2} \cdot \hat{r}_{0,i}$$

$\hat{r}_{0,i}$ ☛ unit vector along direction from charge q_i to q_0

Recall $\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$ $\quad\therefore\quad$ $\vec{E}$-field due to q_i at point P

$$\vec{E}_i = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_i}{r_i^2} \cdot \hat{r}_i$$

$\vec{r}_i$ ☛ vector pointing from q_i to point P

$\hat{r}_i$ ☛ unit vector pointing from q_i to point P

Note:
(1) $\vec{E}$-field is a **vector**
(2) Direction of $\vec{E}$-field depends on **both** position of P and sign of q_i

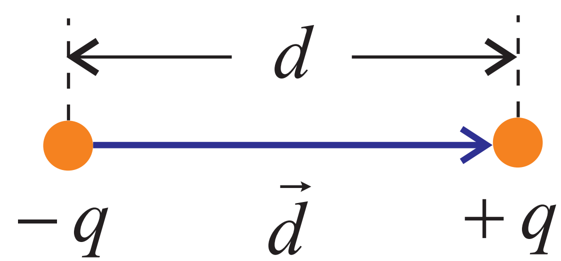
(ii) $\vec{E}$-field due to system of charges:

Principle of Superposition

In a system with N charges ☛ **total** $\vec{E}$-field due to all charges **vector sum** of $\vec{E}$-field due to individual charges

i.e. ☛ $$\vec{E} = \sum_i \vec{E}_i = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i^2}\hat{r}_i$$

(iii) Electric Dipole

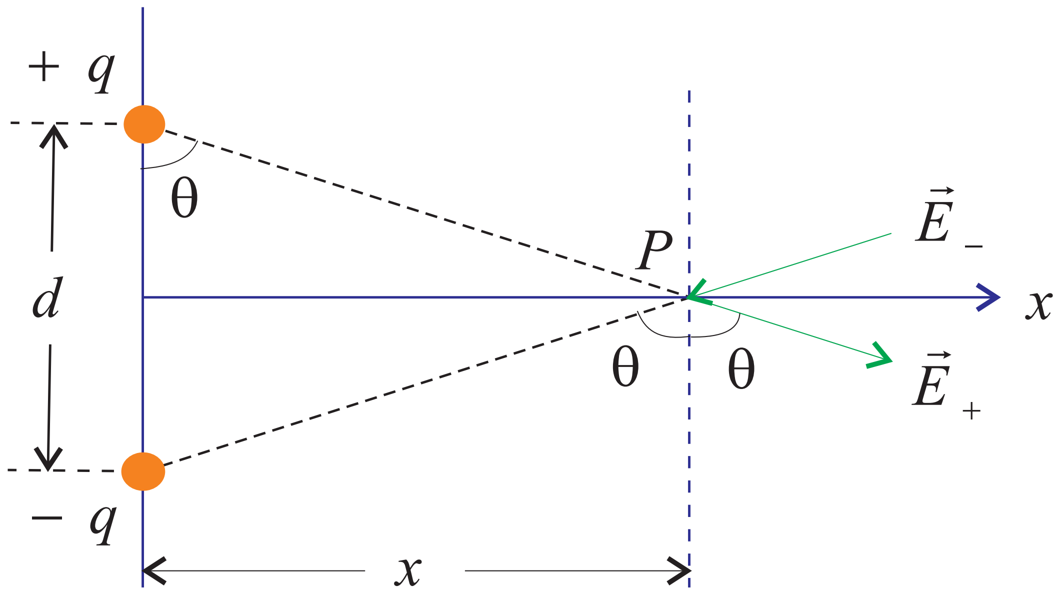


System of **equal** and **opposite** charges separated by a distance d

Electric Dipole Moment ☛ $\vec{p} = q\vec{d} = qd\hat{d}$

$$p = qd$$

Example: $\vec{E}$ due to dipole along x-axis

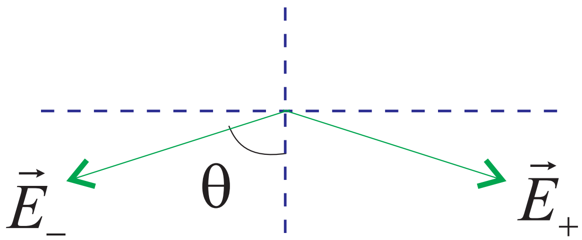


Consider point P at distance x along perpendicular axis of dipole $\vec{p}$

$$\vec{E} = \vec{E}_+ + \vec{E}_-$$

E-field due to $+q$ E-field due to $-q$

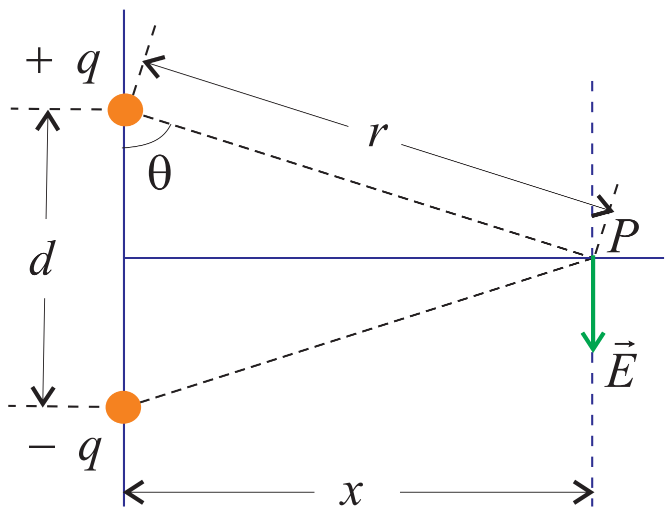
Notice: Horizontal $\vec{E}$-field components of $\vec{E}_+$ and $\vec{E}_-$ cancel out



$\therefore$ Net $\vec{E}$ points along axis parallel but opposite to dipole moment vector

Magnitude of $\vec{E}$-field $= 2E_+ \cos\theta$

$$\therefore E_- = 2\left(\underbrace{\frac{1}{4\pi\epsilon_0}\cdot\frac{q}{r^2}}_{E_+ \text{ or } E_- \text{ magnitude}}\right)\cos\theta$$



But

$$r = \sqrt{\left(\frac{d}{2}\right)^2 + x^2}$$

$$\cos\theta = \frac{d/2}{r}$$

$$\therefore E = \frac{1}{4\pi\epsilon_0}\cdot\frac{p}{\left[x^2 + \left(\frac{d}{2}\right)^2\right]^{\frac{3}{2}}}$$

$$(p = qd)$$

Special case ☛ When $x \gg d$

$$\left[x^2 + \left(\frac{d}{2}\right)^2\right]^{\frac{3}{2}} = x^3 \left[1 + \left(\frac{d}{2x}\right)^2\right]^{\frac{3}{2}}$$

- Binomial Approximation

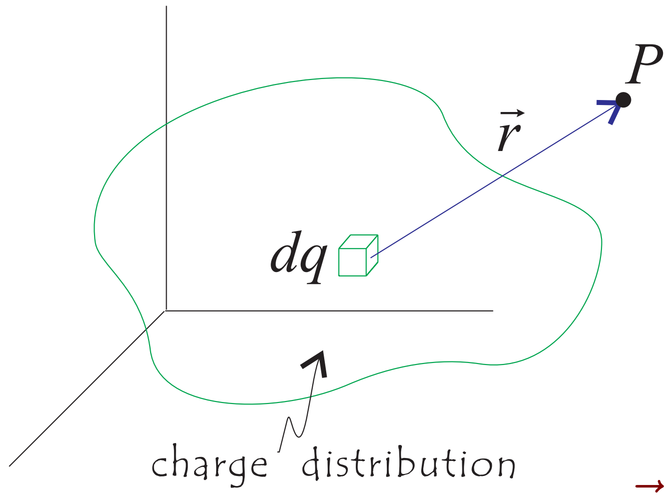
$$(1 + y)^n \approx 1 + ny \qquad \text{if} \qquad y \ll 1$$

$$\vec{E} - \text{field of dipole} \simeq \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{x^3} \propto \frac{1}{x^3}$$

- Compare with $\dfrac{1}{r^2}$ $\vec{E}$-field for single charge

- Result also valid for point P along any axis with respect to dipole

1.4 Continuous Charge Distribution



E-field at point P due to dq

$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{r^2} \cdot \hat{r}$$

$\therefore$ E-field due to charge distribution

$$\vec{E} = \int d\vec{E} = \int \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{r^2} \cdot \hat{r}$$

(1) Take advantage of symmetry of system to simplify integral

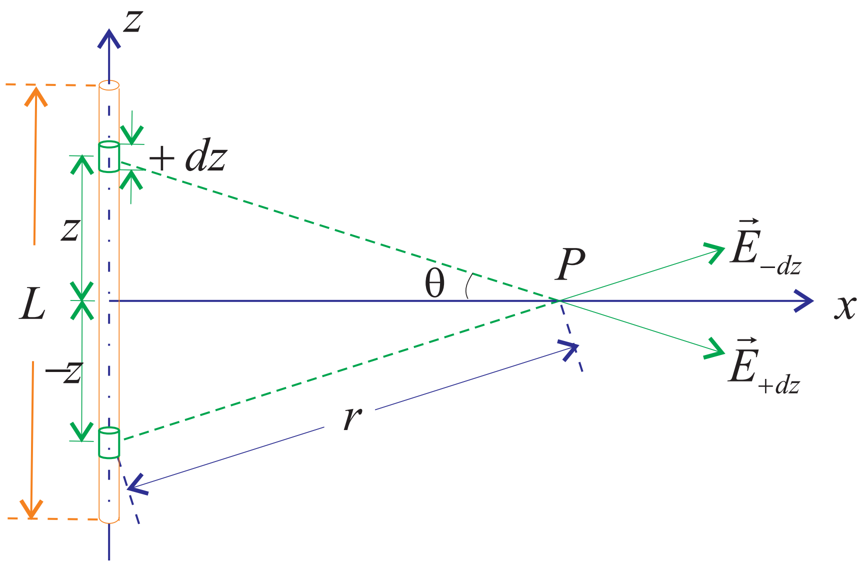
(2) To write down small charge element dq ☛

$1-\mathrm{D}$ $\quad dq = \lambda\, ds \quad \lambda = \text{linear charge density} \quad ds = \text{small length element}$

$2-\mathrm{D}$ $\quad dq = \sigma\, dA \quad \sigma = \text{surface charge density} \quad dA = \text{small area element}$

$3-\mathrm{D}$ $\quad dq = \rho\, dV \quad \rho = \text{volume charge density} \quad dV = \text{small volume element}$

Example 1 **Uniform line of charge**



charge per unit length $= \lambda$

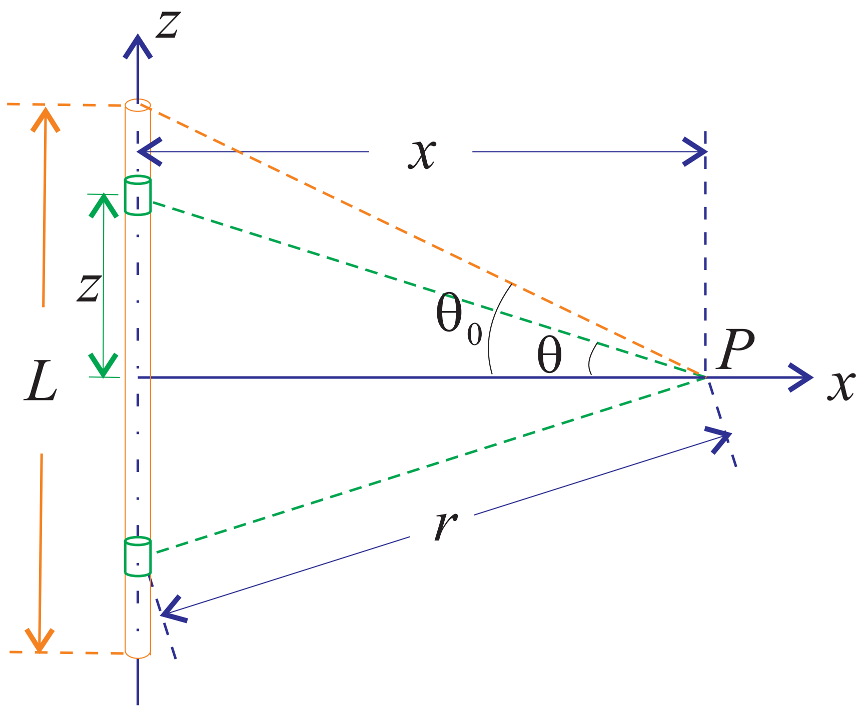
(1) Symmetry considered ☛ E-field from $+z$ and $-z$ directions cancel along z-direction, $\therefore$ Only horizontal E-field components need to be considered

(2) For each element of length dz, charge $dq = \lambda dz$

$\therefore$ Horizontal E-field at point P due to element dz is

$$|d\vec{E}| \cos\theta = \underbrace{\frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda dz}{r^2}}_{dE_{dz}} \cos\theta$$

$\therefore$ E-field due to entire line charge at point P



$$E = \int_{-L/2}^{L/2} \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda dz}{r^2} \cos\theta$$

$$= 2\int_{0}^{L/2} \frac{\lambda}{4\pi\epsilon_0} \cdot \frac{dz}{r^2} \cos\theta$$

To calculate this integral ☛

- First, notice that x is fixed, but z, r, θ all varies

- Change of variable (from z to θ)

(1)
$$z = x\tan\theta \quad \therefore dz = x\sec^2\theta\, d\theta$$
$$x = r\cos\theta \quad \therefore r^2 = x^2\sec^2\theta$$
$$z = 0 \quad \theta = 0^\circ$$

(2) When $z = L/2 \quad \theta = \theta_0$ where $\tan\theta_0 = \dfrac{L/2}{x}$

$$E = 2\cdot\frac{\lambda}{4\pi\epsilon_0}\int_0^{\theta_0}\frac{x\sec^2\theta\, d\theta}{x^2\sec^2\theta}\cdot\cos\theta$$
$$= 2\cdot\frac{\lambda}{4\pi\epsilon_0}\int_0^{\theta_0}\frac{1}{x}\cdot\cos\theta\, d\theta$$
$$= 2\cdot\frac{\lambda}{4\pi\epsilon_0}\cdot\frac{1}{x}\cdot(\sin\theta)\Big|_0^{\theta_0}$$
$$= 2\cdot\frac{\lambda}{4\pi\epsilon_0}\cdot\frac{1}{x}\cdot\sin\theta_0$$
$$= 2\cdot\frac{\lambda}{4\pi\epsilon_0}\cdot\frac{1}{x}\cdot\frac{L/2}{\sqrt{x^2+\left(\frac{L}{2}\right)^2}}$$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda L}{x\sqrt{x^2 + \left(\frac{L}{2}\right)^2}}$$ **along x-direction**

Important limiting cases

(1) $$x \gg L : \quad E \doteqdot \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda L}{x^2}$$

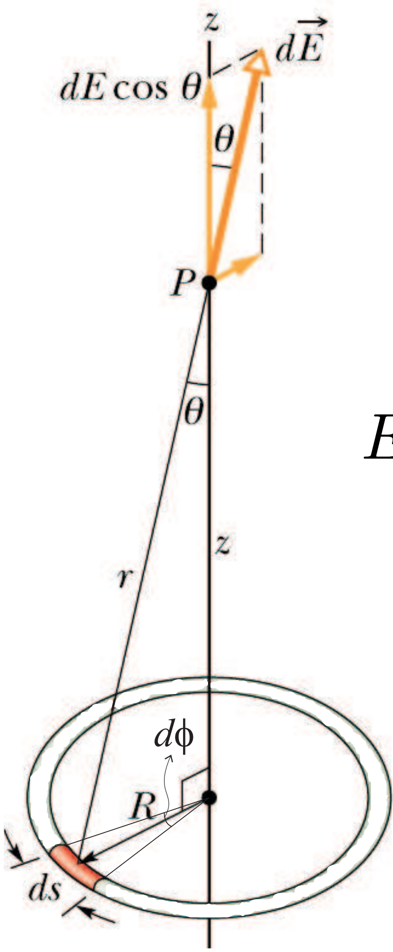
But $\lambda L =$ Total charge on rod $\therefore$ System behave like a point charge

(2) $$L \gg x : \quad E \doteqdot \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda L}{x \cdot \frac{L}{2}}$$

$$E_x = \frac{\lambda}{2\pi\epsilon_0 x}$$

ELECTRIC FIELD DUE TO INFINITELY LONG LINE OF CHARGE

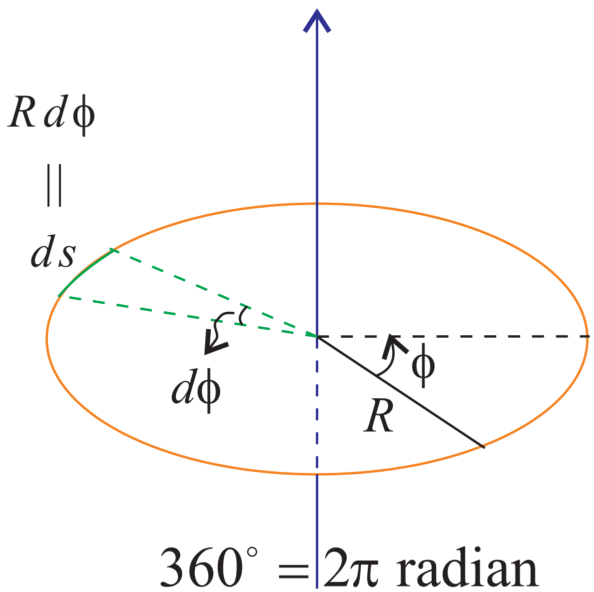
Example 2 **Ring of Charge**



E-field at a height z above a ring of charge of radius R

(1) Symmetry considered ☛ For every charge element dq considered, there exists dq' where horizontal $\vec{E}$ field components cancel

(2) For each element of length dz, charge



$$dq = \lambda \cdot ds$$

λ: Linear charge density

ds: Circular length element

$$dq = \lambda \cdot R\,d\phi$$

where ϕ is angle measured on ring plane

$360^\circ = 2\pi$ radian
$180^\circ = \pi$ radian

$\therefore$ Net E-field along z-axis due to dq

$$dE = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{r^2} \cdot \cos\theta$$

Total E-field $= \int dE$

$$= \int_0^{2\pi} \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda R \, d\phi}{r^2} \cdot \cos\theta \qquad (\cos\theta = \frac{z}{r})$$

Note: Here in this case, θ, R and r are fixed as ϕ varies!

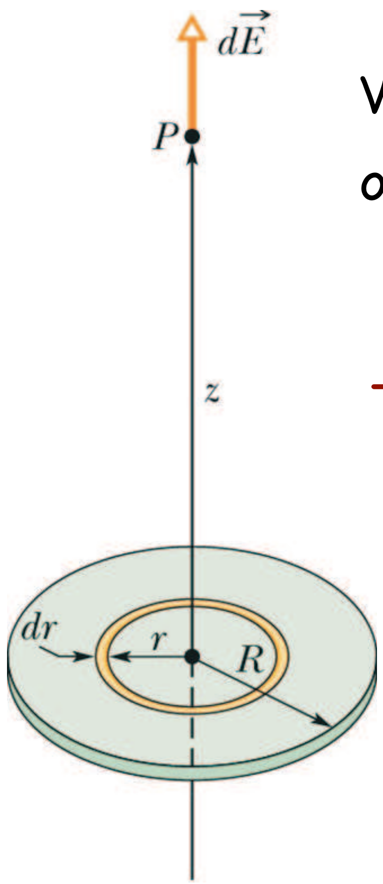
BUT we want to convert r, θ to R, z

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda R z}{r^3} \int_0^{2\pi} d\phi$$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda(2\pi R) z}{(z^2 + R^2)^{3/2}}$$ along z-axis

BUT ☛ $\lambda(2\pi R) =$ **total charge on ring**

Example 3 **E-field from a disk of surface charge density σ**

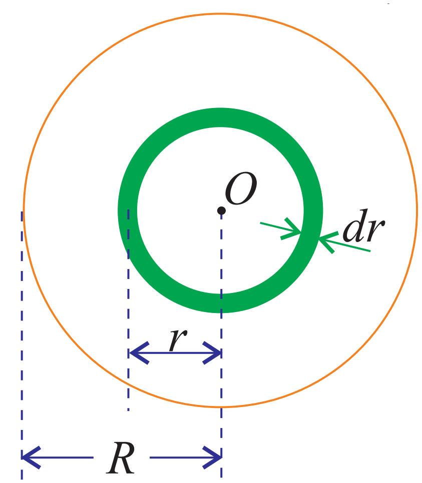


We find E-field of a disk by integrating concentric rings of charges

Total charge of ring

$$dq = \sigma \cdot \underbrace{(2\pi r\, dr)}_{\text{Area of ring}}$$

view from top



Recall from Example 2

E-field from ring ☛ $dE = \dfrac{1}{4\pi\epsilon_0} \cdot \dfrac{dq\, z}{(z^2 + r^2)^{3/2}}$

$$\therefore\ E = \frac{1}{4\pi\epsilon_0} \int_0^R \frac{2\pi\sigma r\, dr \cdot z}{(z^2 + r^2)^{3/2}}$$

$$= \frac{1}{4\pi\epsilon_0} \int_0^R 2\pi\sigma z \frac{r\, dr}{(z^2 + r^2)^{3/2}}$$

- Change of variable:

$$u = z^2 + r^2 \Rightarrow \quad (z^2 + r^2)^{3/2} = u^{3/2}$$

$$\Rightarrow du = 2r\, dr \Rightarrow \quad r\, dr = \frac{1}{2} du$$

- Change of integration limit:

$$\begin{cases} r = 0 & u = z^2 \\ r = R & u = z^2 + R^2 \end{cases}$$

$$\therefore \; E = \frac{1}{4\pi\epsilon_0} \cdot 2\pi\sigma z \int_{z^2}^{z^2+R^2} \frac{1}{2} u^{-3/2} du$$

BUT

$$\int u^{-3/2} du = \frac{u^{-1/2}}{-1/2} = -2u^{-1/2}$$

$$\therefore \; E = \frac{1}{2\epsilon_0} \sigma z \left(-u^{-1/2}\right)\Big|_{z^2}^{z^2+R^2}$$

$$= \frac{1}{2\epsilon_0} \sigma z \left(\frac{-1}{\sqrt{z^2+R^2}} + \frac{1}{z} \right)$$

$$E = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{z}{\sqrt{z^2+R^2}} \right]$$

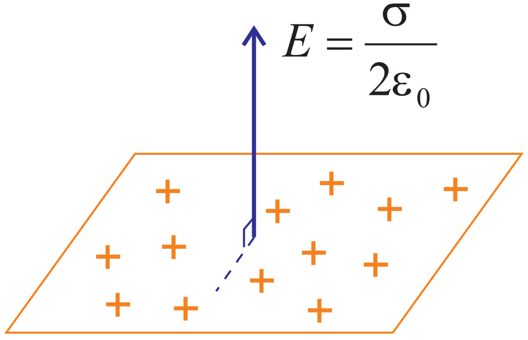
VERY IMPORTANT LIMITING CASE

If $R \gg z$, that is if we have an infinite sheet of charge with charge density σ

$$E = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right]$$

$$\simeq \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{R}\right]$$

$$E \approx \frac{\sigma}{2\epsilon_0}$$



E-field is normal to charged surface

1.5 Electric Field Lines

To visualize electric field

we can use a graphical tool called electric field lines

Conventions

1. Start on positive charges and end on negative charges

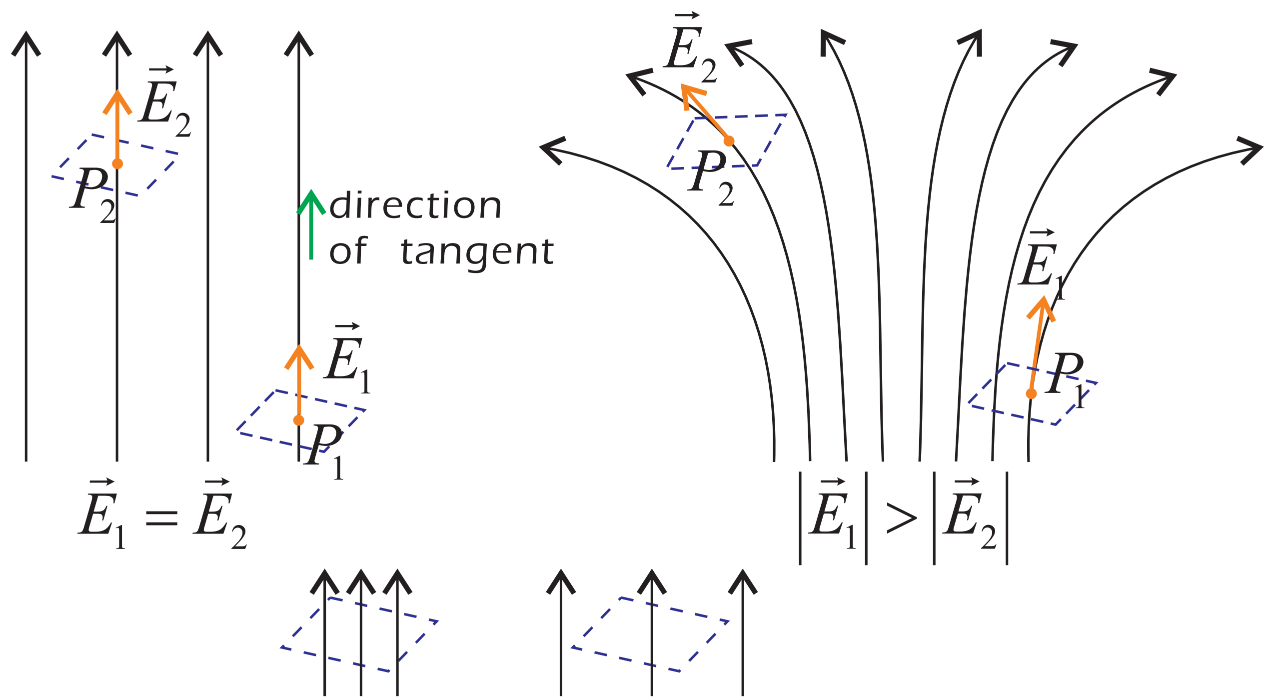
2. Direction of E-field at any point is given by tangent of E-field line

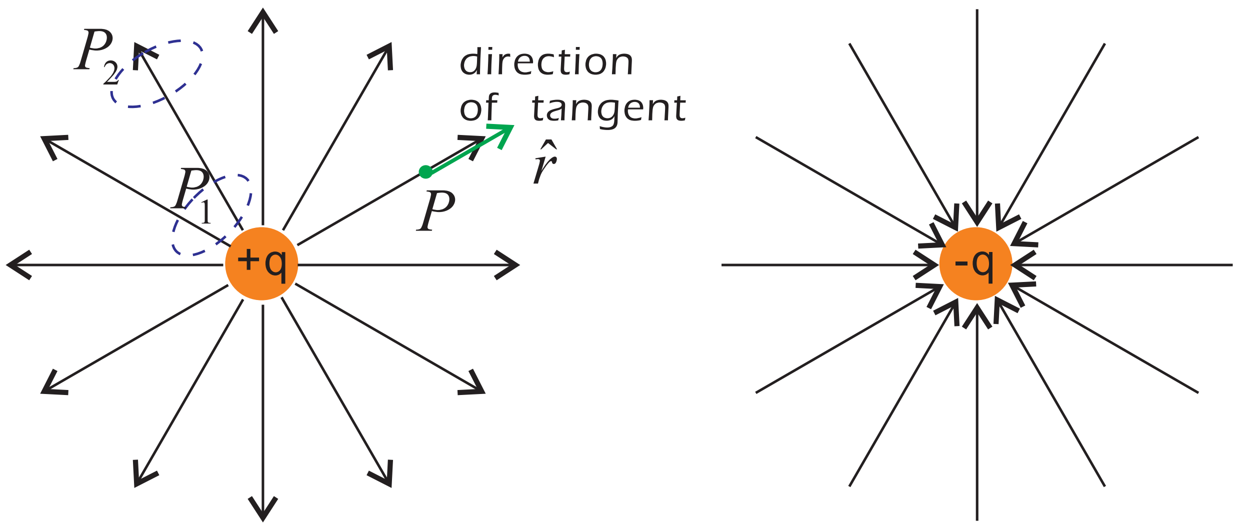
3. Magnitude of E-field at any point

proportional to number of E-field lines per unit area perpendicular to lines

Uniform E-field

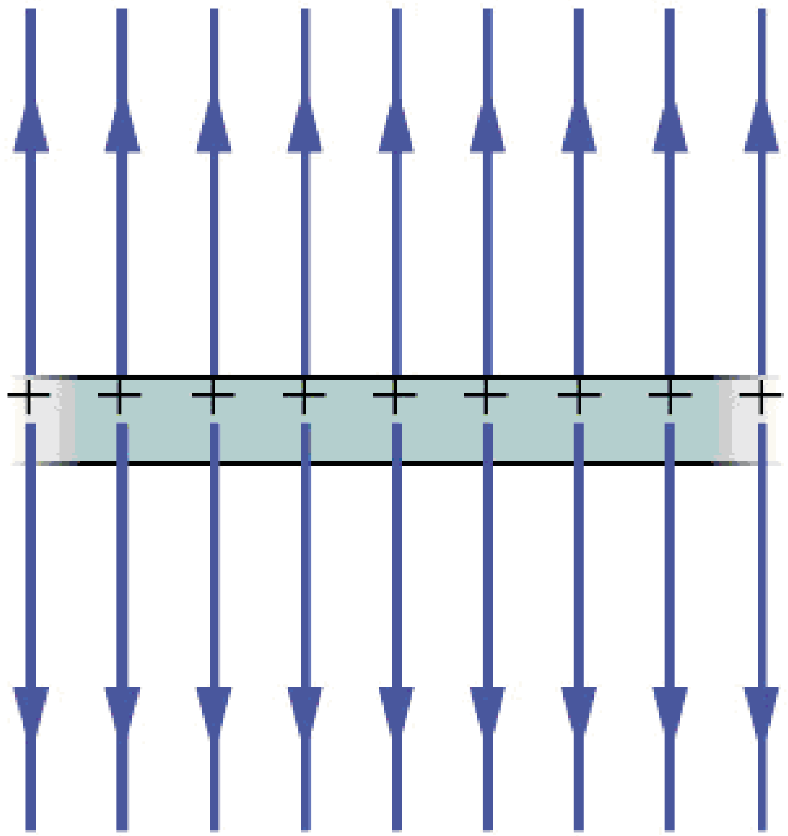
Non-uniform E-field



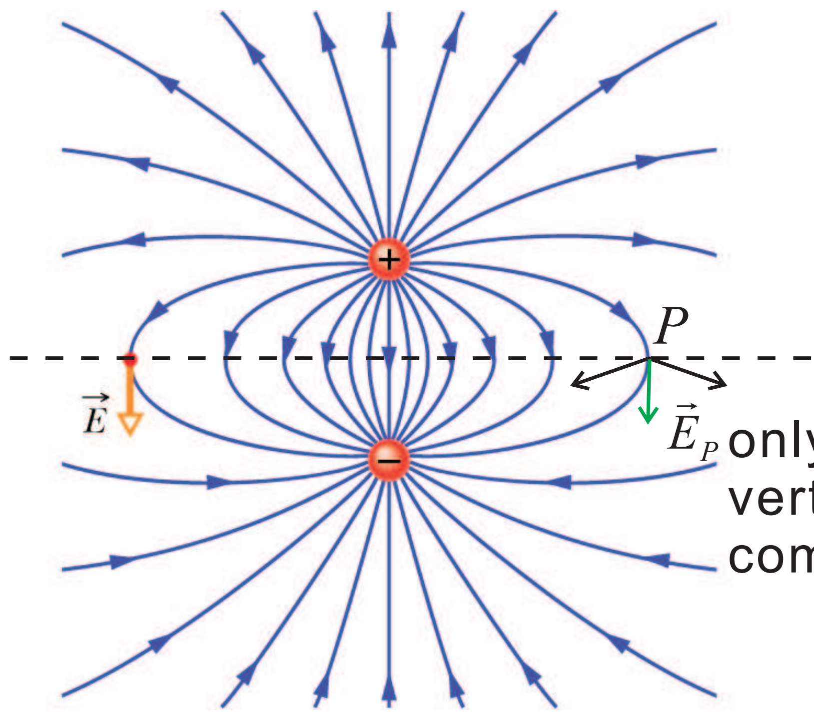


$$\left|\vec{E}_{P_1}\right| > \left|\vec{E}_{P_2}\right| \qquad \vec{E} = \frac{+q}{4\pi\epsilon_0 r^2}\hat{r}$$

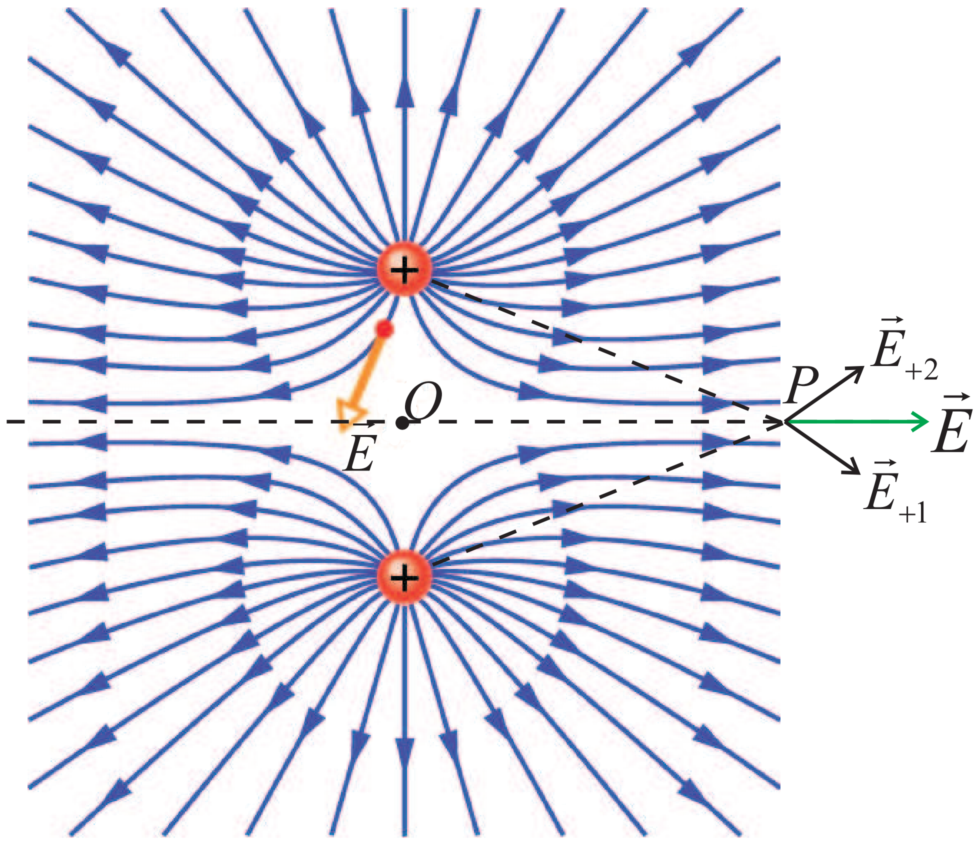
Infinite sheet of charge



$$E = \frac{\sigma}{2\epsilon_0}$$



only has vertical component.



$$\vec{E}_{\text{at point } O} = 0$$

1.6 Point Charge in E-field

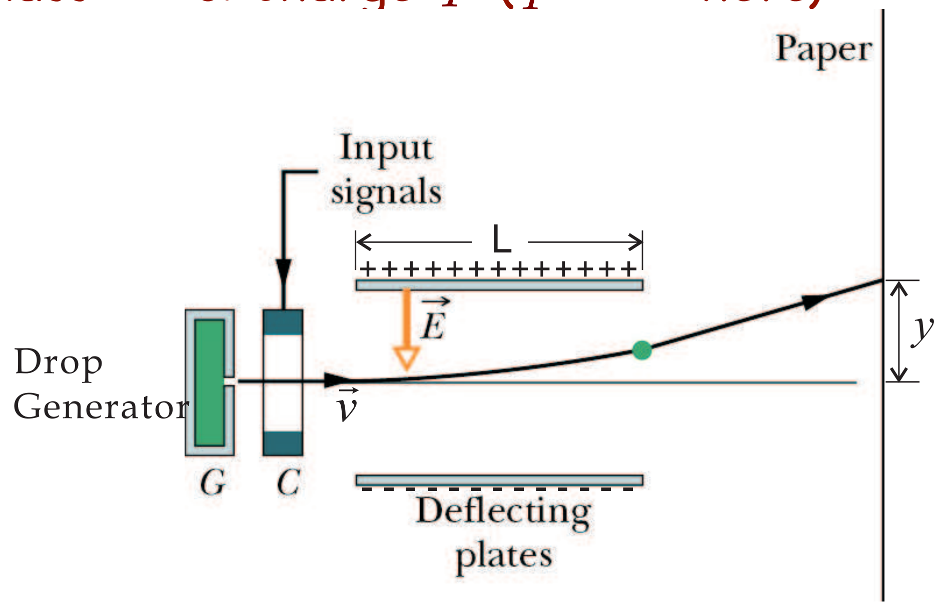
When we place a charge q in an E-field $\vec{E}$,force experienced by charge is

$$\vec{F} = q\vec{E} = m\vec{a}$$

Applications ☛ Ink-jet printer, TV cathode ray tube

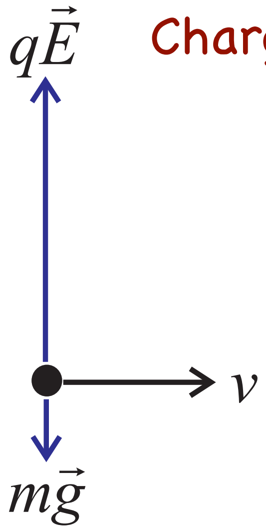
Example

Ink particle has mass m & charge q ($q < 0$ here)



Assume that mass of inkdrop is small, what's deflection of charge?

Solution ☛



Charge carried by inkdrop is **negative** ☛ $q < 0$

Note: $q\vec{E}$ points in opposite direction of $\vec{E}$

Horizontal motion ☛ Net force $= 0$

$$\therefore L = vt$$

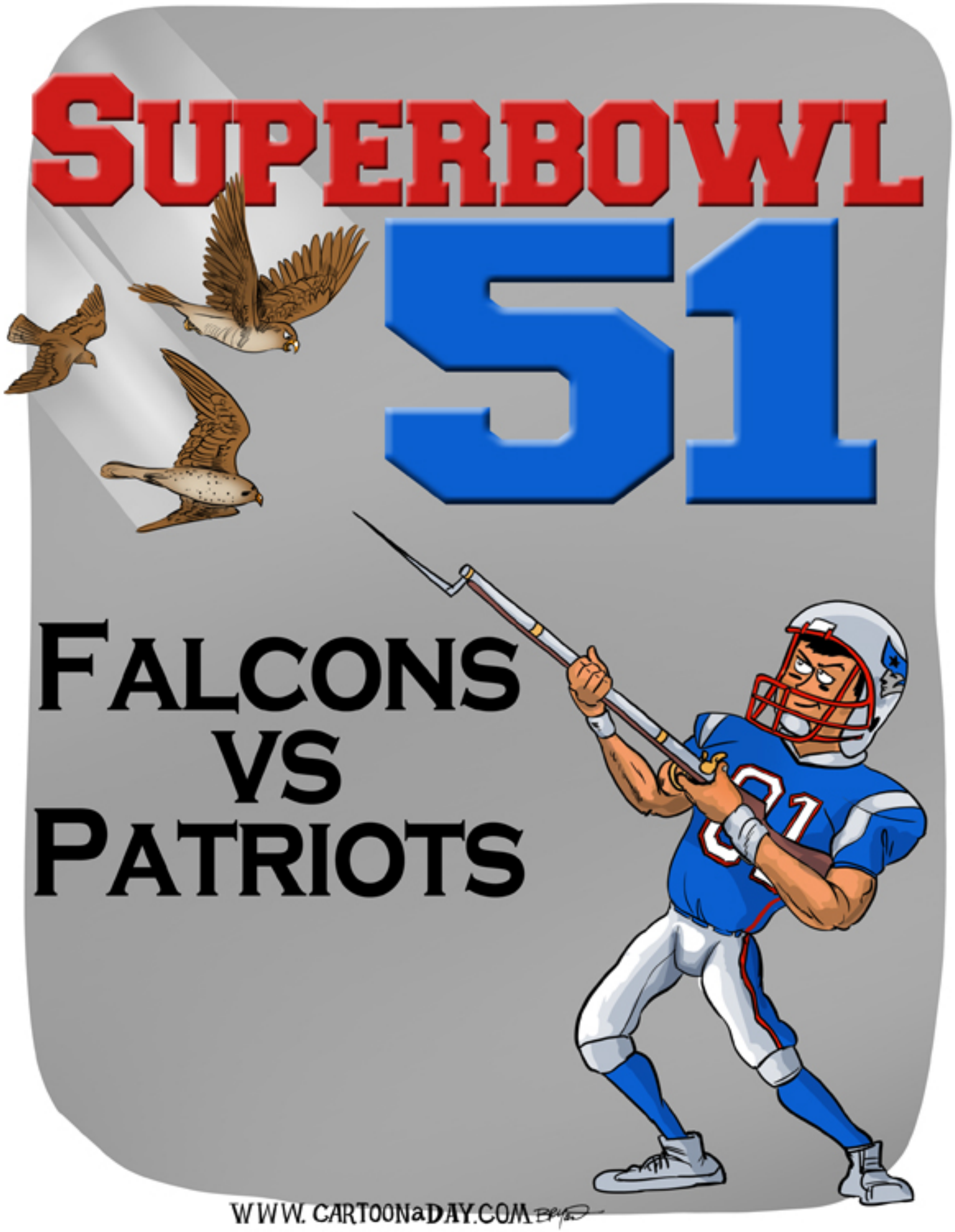
Vertical motion ☛ $|q\vec{E}| \gg |m\vec{g}|$ q is negative

$\therefore$ Net force $= -|q|E = ma$ ☛ **Newton's 2nd Law**

$$\therefore a = -\frac{|q|E}{m}$$

Vertical distance travelled ☛ $y = \dfrac{1}{2}at^2$

Review everything for next class BUT don't forget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LIII
SUPER BOWL

HOMEWORK

Review these slides B4 watching superbowl

1 Definitions

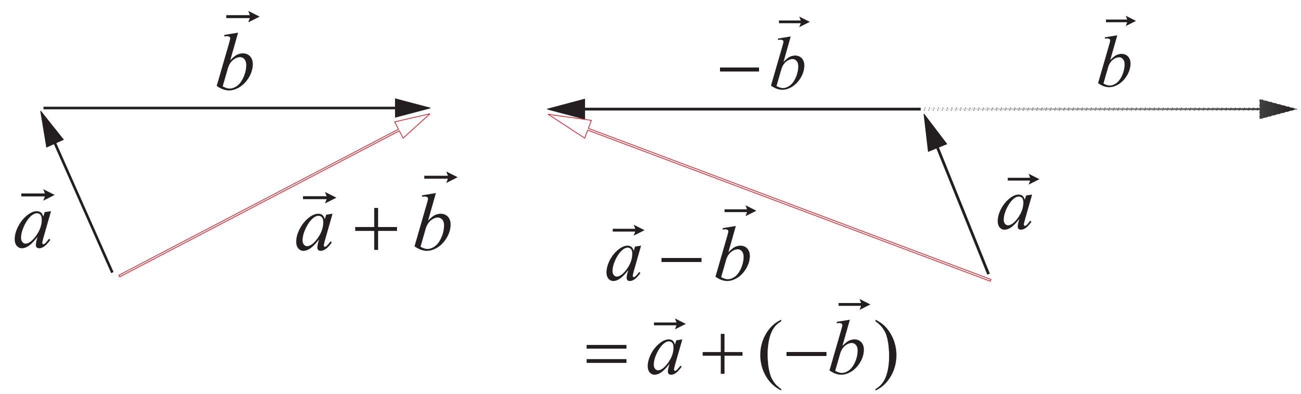
A **vector** consists of two components ☛ **magnitude** and **direction**
(e.g. force, velocity)

A **scalar** consists of **magnitude** only
(e.g. mass, charge, density)

Euclidean vector, a geometric entity endowed with magnitude and direction as well as a positive-definite inner product; an element of a Euclidean vector space

In physics, Euclidean vectors are used to represent physical quantities that have both magnitude and direction, such as force, in contrast to scalar quantities, which have no direction

.2 Vector Algebra



$$\vec{a}+\vec{b}=\vec{b}+\vec{a}$$

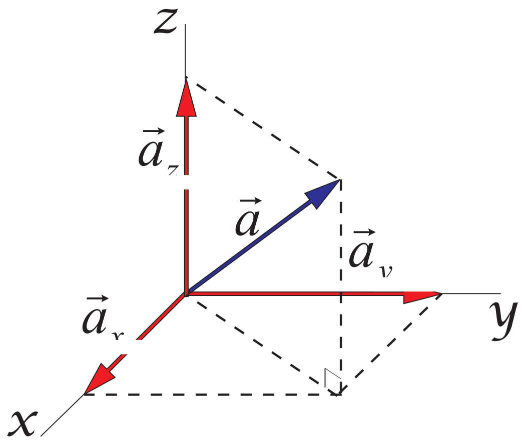
$$\vec{a}+(\vec{b}+\vec{c})=(\vec{a}+\vec{b})+\vec{c}$$

3 Components of Vectors

Usually vectors are expressed according to coordinate system

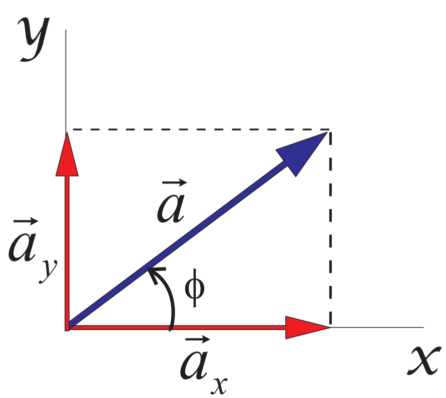
Each vector can be expressed in terms of components

The most common coordinate system ☛ Cartesian



Magnitude of $\vec{a} = |\vec{a}| = a$

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$



$$a = \sqrt{a_x^2 + a_y^2}$$

$$a_x = a\cos\phi$$

$$a_y = a\sin\phi$$

$$\tan\phi = a_y/a_x$$

Unit vectors have magnitude of 1

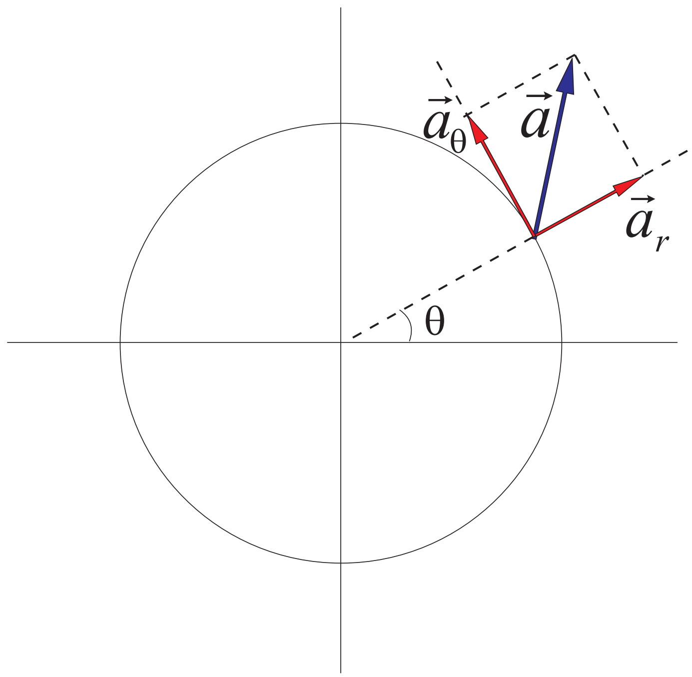
$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \text{unit vector along } \vec{a} \text{ direction}$$

$$\begin{array}{ccc} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \updownarrow & \updownarrow & \updownarrow \\ x & y & z \end{array}$$

$\hat{\imath} \quad \hat{\jmath} \quad \hat{k}$ are unit vectors along $x \quad y \quad z$ directions

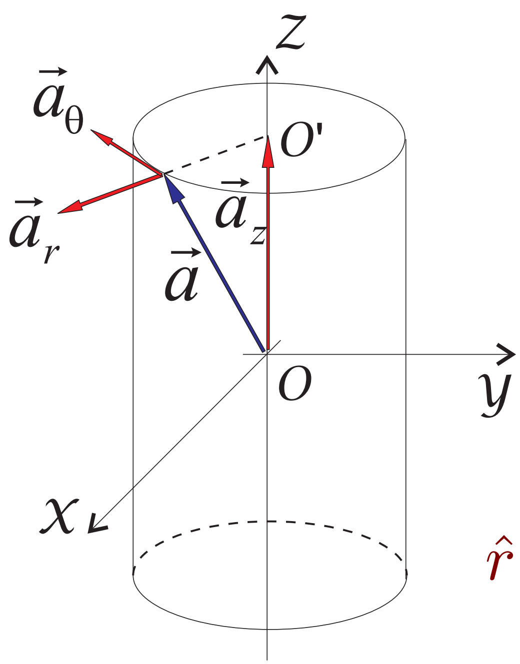
$$\vec{a} = a_x \,\hat{\imath} + a_y \,\hat{\jmath} + a_z \,\hat{k}$$

1. Polar Coordinates



$$\vec{a} = a_r \, \hat{r} + a_\theta \, \hat{\theta}$$

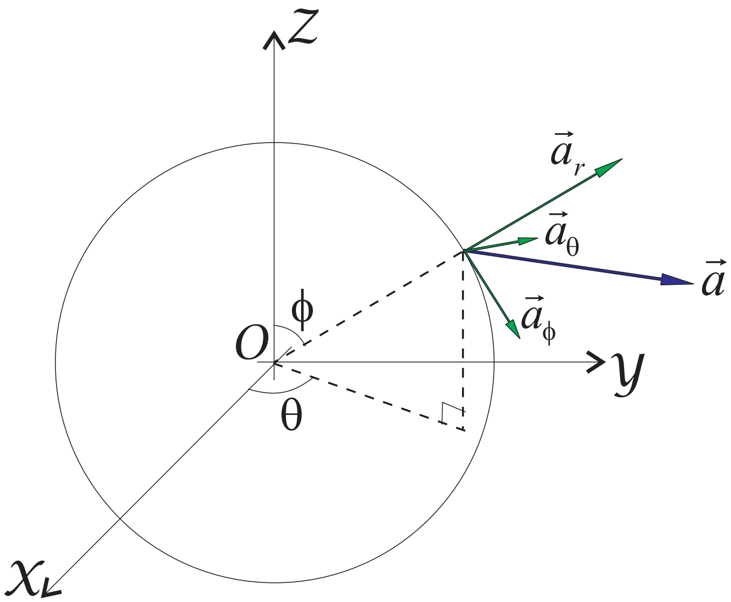
2. Cylindrical Coordinates



$$\vec{a} = a_r\,\hat{r} + a_\theta\,\hat{\theta} + a_z\,\hat{z}$$

$\hat{r}$ originated from nearest point on z-axis (Point O')

3. Spherical Coordinates



$$\vec{a} = a_r \, \hat{r} + a_\theta \, \hat{\theta} + a_\phi \, \hat{\phi}$$

$\hat{r}$ originated from Origin O

4 Multiplication of Vectors

1. Scalar multiplication

If $\vec{b} = m\,\vec{a}$ $\vec{b}, \vec{a}$ are vectors; m is a scalar

then $b = m\,a$ (Relation between magnitude)

$$\left.\begin{aligned} b_x &= m\,a_x \\ b_y &= m\,a_y \end{aligned}\right\}$$ Components also follow relation

i.e.

$$\vec{a} = a_x \; \hat{\imath} + \; a_y \; \hat{\jmath} + \; a_z \; \hat{k}$$

$$m\vec{a} = m a_x \hat{\imath} + m a_y \; \hat{\jmath} + m a_z \; \hat{k}$$

2. Dot Product (Scalar Product) Cont'd

$$\hat{\imath} \cdot \hat{\imath} = |\hat{\imath}|\,|\hat{\imath}| \cos 0^\circ = 1 \cdot 1 \cdot 1 = 1$$

$$\hat{\imath} \cdot \hat{\jmath} = |\hat{\imath}|\,|\hat{\jmath}| \cos 90^\circ = 1 \cdot 1 \cdot 0 = 0$$

$$\hat{\imath} \cdot \hat{\imath} = \hat{\jmath} \cdot \hat{\jmath} = \hat{k} \cdot \hat{k} = 1$$

$$\hat{\imath} \cdot \hat{\jmath} = \hat{\jmath} \cdot \hat{k} = \hat{k} \cdot \hat{\imath} = 0$$

If $\vec{a} = a_x\,\hat{\imath} + a_y\,\hat{\jmath} + a_z\,\hat{k}$

$\vec{b} = b_x\,\hat{\imath} + b_y\,\hat{\jmath} + b_z\,\hat{k}$

then $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$

$$\vec{a} \cdot \vec{a} = |\vec{a}| \cdot |\vec{a}| \cos 0^\circ = a \cdot a = a^2$$

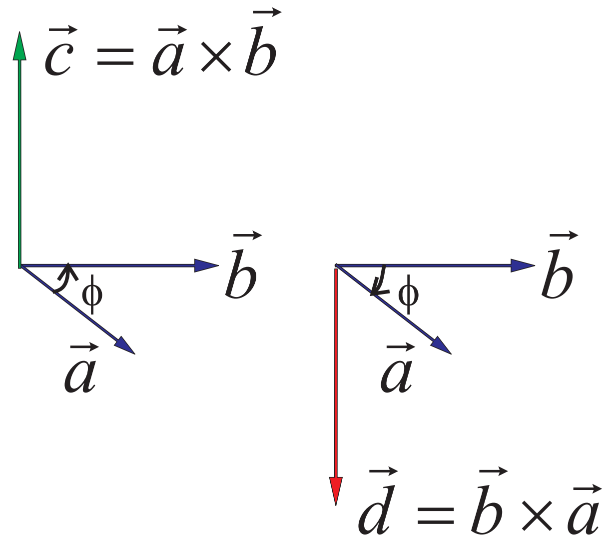
2. Cross Product (Vector Product)

If $\vec{c} = \vec{a} \times \vec{b}$

then $c = |\vec{c}| = a\, b \sin\phi$

$$\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a} \quad !!!$$

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$



- Direction of cross product determined from **right hand rule**
- Also, $\vec{a} \times \vec{b}$ is $\perp$ to $\vec{a}$ and $\vec{b}$

i.e.
$$\vec{a} \cdot (\vec{a} \times \vec{b}) = 0$$
$$\vec{b} \cdot (\vec{a} \times \vec{b}) = 0$$

- IMPORTANT

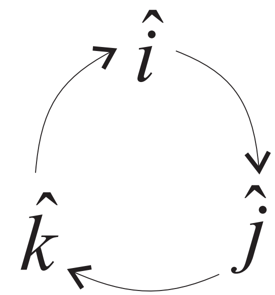
$$\vec{a} \times \vec{a} = a \cdot a \sin 0^\circ = 0$$

$$|\hat{\imath} \times \hat{\imath}| = |\hat{\imath}|\,|\hat{\imath}| \sin 0^\circ = 1 \cdot 1 \cdot 0 = 0$$

$$|\hat{\imath} \times \hat{\jmath}| = |\hat{\imath}|\,|\hat{\jmath}| \sin 90^\circ = 1 \cdot 1 \cdot 1 = 1$$

$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$$

$$\hat{\imath} \times \hat{\jmath} = \hat{k};\ \hat{\jmath} \times \hat{k} = \hat{\imath};\ \hat{k} \times \hat{\imath} = \hat{\jmath}$$



$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = (a_y b_z - a_z b_y)\,\hat{\imath} + (a_z b_x - a_x b_z)\,\hat{\jmath} + (a_x b_y - a_y b_x)\,\hat{k}$$

4. Vector identities

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{c} \times \vec{a}) = \vec{c} \cdot (\vec{a} \times \vec{b})$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$$

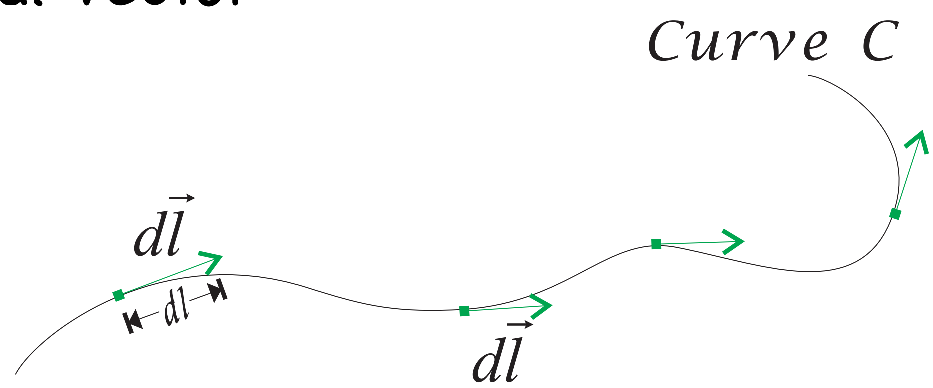
5 Vector Field (Physics Point of View)

A **vector field** $\vec{\mathcal{F}}(x, y, z)$ is a mathematical function which has a **vector** output for a **position** input

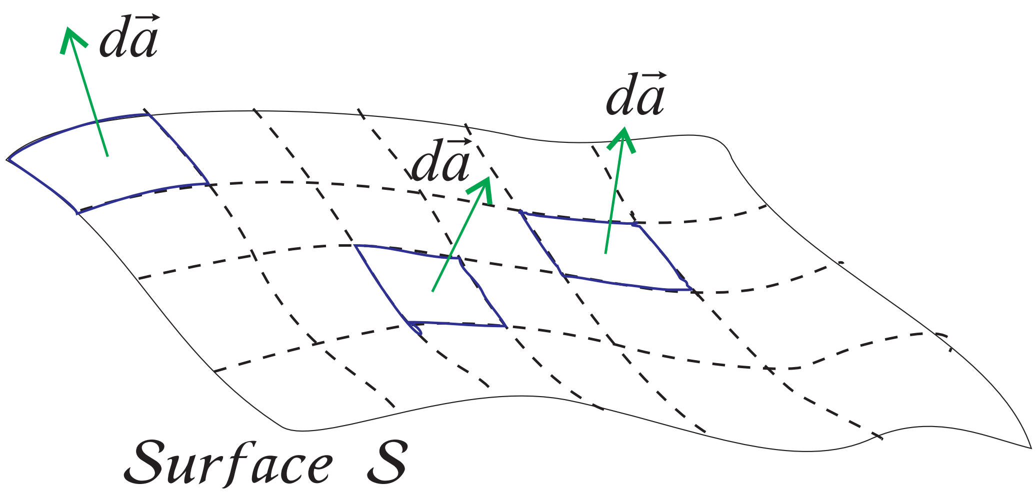
(Scalar field $\mathcal{U}(x, y, z)$)

6 Analytic Geometry

Tangential Vector



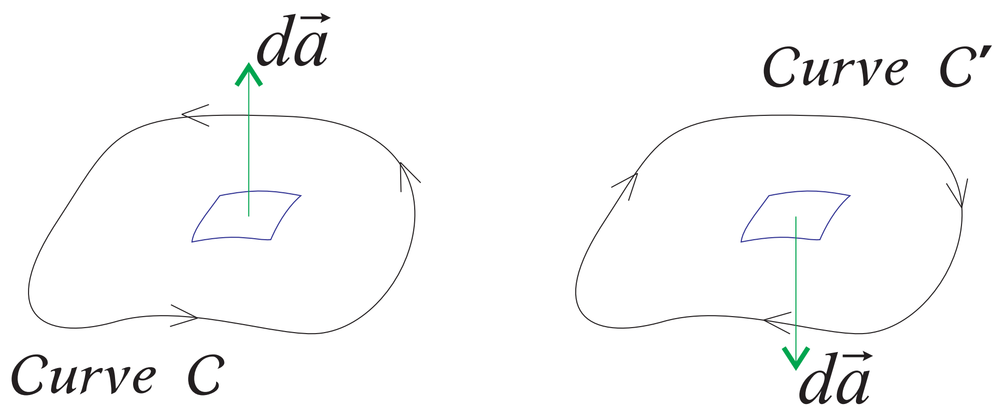
Surface Vector



Some uncertainty! ($d\vec{a}$ versus $-d\vec{a}$)

Two conventions:

- Area formed from a closed curve



- Closed surface enclosing a volume