Prof. Anchordoqui

**Problems set # 4** **Physics 169** March 3, 2015

1. *(i)* Eight equal charges $q$ are located at the corners of a cube of side $s$, as shown in Fig. 1. Find the electric potential at one corner, taking zero potential to be infinitely far away. *(ii)* Four point charges are fixed at the corners of a square centered at the origin, as shown in Fig. 1. The length of each side of the square is $2a$. The charges are located as follows: $+q$ is at $(-a, +a)$, $+2q$ is at $(+a, +a)$, $-3q$ is at $(+a, -a)$, and $+6q$ is at $(-a, -a)$. A fifth particle that has a mass $m$ and a charge $+q$ is placed at the origin and released from rest. Find its speed when it is a very far from the origin.

2. Five identical point charges $+q$ are arranged in two different manners as shown in Fig. 2: in once case as a face-centered square, in the other as a regular pentagon. Find the potential energy of each system of charges, taking the zero of potential energy to be infinitely far away. Express your answer in terms of a constant times the energy of two charges $+q$ separated by a distance $a$.

3. Consider a system of two charges shown in Fig. 3. Find the electric potential at an arbitrary point on the $x$ axis and make a plot of the electric potential as a function of $x/a$.

4. A point particle that has a charge of +11.1 nC is at the origin. *(i)* What is (are) the shapes of the equipotential surfaces in the region around this charge? *(ii)* Assuming the potential to be zero at $r = \infty$, calculate the radii of the five surfaces that have potentials equal to 20.0 V, 40.0 V, 60.0 V, 80.0 V and 100.0 V, and sketch them to scale centered on the charge. *(iii)* Are these surfaces equally spaced? Explain your answer. *(iv)* Estimate the electric field strength between the 40.0-V and 60.0-V equipotential surfaces by dividing the difference between the two potentials by the difference between the two radii. Compare this estimate to the exact value at the location midway between these two surfaces.

5. Two coaxial conducting cylindrical shells have equal and opposite charges. The inner shell has charge $+q$ and an outer radius $a$, and the outer shell has charge $-q$ and an inner radius $b$. The length of each cylindrical shell is $L$, and L is very long compared with $b$. Find the potential difference, $V_a - V_b$ between the shells.

6. An electric potential $V(z)$ is described by the function

$$V(z) = \begin{cases} -2\ \mathrm{V \cdot m^{-1}}\, z + 4\ \mathrm{V}, & z > 2.0\ \mathrm{m} \\ 0, & 1.0\ \mathrm{m} < z < 2.0\ \mathrm{m} \\ \frac{2}{3}\ \mathrm{V} - \frac{2}{3}\mathrm{V \cdot m^{-3}}\, z^3, & 0\ \mathrm{m} < z < 1.0\ \mathrm{m} \\ \frac{2}{3}\ \mathrm{V} + \frac{2}{3}\mathrm{V \cdot m^{-3}}\, z^3, & -1.0\ \mathrm{m} < z < 0\ \mathrm{m} \\ 0, & -2.0\ \mathrm{m} < z < -1.0\ \mathrm{m} \\ 2\ \mathrm{V \cdot m^{-1}} z + 4\ \mathrm{V}, & z < -2.0\ \mathrm{m} \end{cases}$$

The graph in Fig. 4 shows the variation of an electric potential $V(z)$ as a function of $z$. *(i)* Give the electric field vector $\vec{E}$ for each of the six regions. *(ii)* Make a plot of the $z$-component of the electric field, $E_z$, as a function of $z$. Make sure you label the axes to indicate the numeric magnitude of the field.

7. Two conducting, concentric spheres have radii $a$ and $b$. The outer sphere is given a charge $Q$. What is the charge on inner sphere if it is earthed?.

8. Consider two nested, spherical conducting shells. The first has inner radius $a$ and outer radius $b$. The second has inner radius $c$ and outer radius $d$. The system is shown in Fig. 5. In the following four situations, determine the total charge on each of the faces of the conducting spheres (inner and outer for each), as well as the electric field and potential everywhere in space (as a function of distance $r$ from the center of the spherical shells). In all cases the shells begin uncharged, and a charge is then instantly introduced somewhere. *(i)* Both shells are not connected to any other conductors (floating) – that is, their net charge will remain fixed. A positive charge $+Q$ is introduced into the center of the inner spherical shell. Take the zero of potential to be at infinity. *(ii)* The inner shell is not connected to ground (floating) but the outer shell is grounded – that is, it is fixed at $V = 0$ and has whatever charge is necessary on it to maintain this potential. A negative charge $-Q$ is introduced into the center of the inner spherical shell. *(iii)* The inner shell is grounded but the outer shell is floating. A positive charge $+Q$ is introduced into the center of the inner spherical shell. *(iv)* Finally, the outer shell is grounded and the inner shell is floating. This time the positive charge $+Q$ is introduced into the region in between the two shells. In this case the question "What are $\vec{E}(r)$ and $V(r)$?" cannot be answered analytically in some regions of space. In the regions where these questions can be answered analytically, give answers. In the regions where they cannot be answered analytically, explain why, but try to draw what you think the electric field should look like and give as much information about the potential as possible.

9. The hydrogen atom in its ground state can be modeled as a positive point charge of magnitude $+e$ (the proton) surrounded by a negative charge distribution that has a charge density (the electron) that varies with the distance from the center of the proton $r$ as: $\rho(r) = -\rho_0 e^{-2r/a}$ (a result obtained from quantum mechanics), where $a = 0.523$ nm is the most probable distance of the electron from the proton. *(i)* Calculate the value of $\rho_0$ needed for the hydrogen atom to be neutral. *(ii)* Calculate the electrostatic potential (relative to infinity) of this system as a function of the distance $r$ from the proton.

10. A particle that has a mass $m$ and a positive charge $q$ is constrained to move along the $x$-axis. At $x = -L$ and $x = L$ are two ring charges of radius $L$. Each ring is centered on the $x$-axis and lies in a plane perpendicular to it. Each ring has a total positive charge $Q$ uniformly distributed on it. *(i)* Obtain an expression for the potential $V(x)$ on the $x$ axis due to the charge on the rings. *(ii)* Show that $V(x)$ has a minimum at $x = 0$. *(iii)* Show that for $x << L$, the potential approaches the form $V(x) = V(0) + \alpha x^2$. *(iv)* Use the result of Part *(iii)* to derive an expression for the angular frequency of oscillation of the mass $m$ if it is displaced slightly from the origin and released. (Assume the potential equals zero at points far from the rings.)

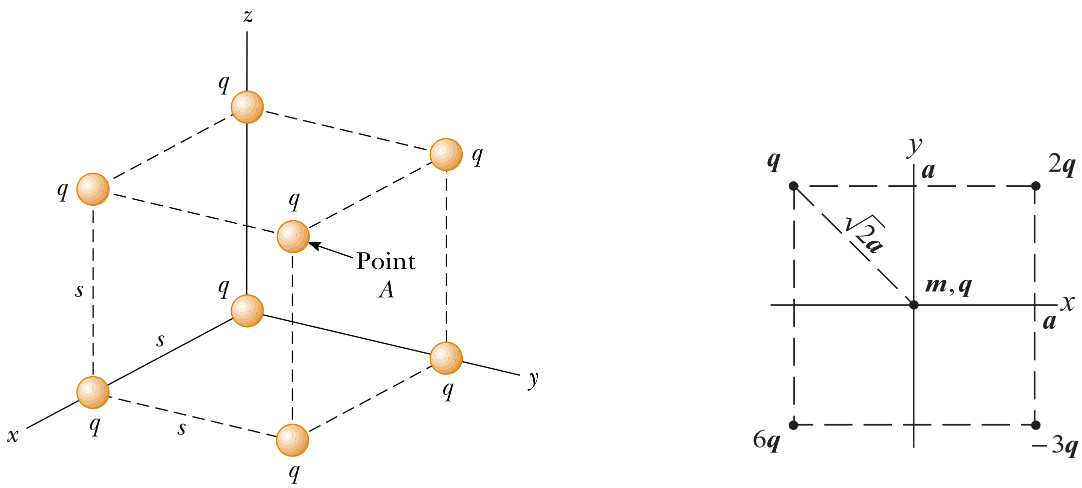


Figure 1: Problem 1.

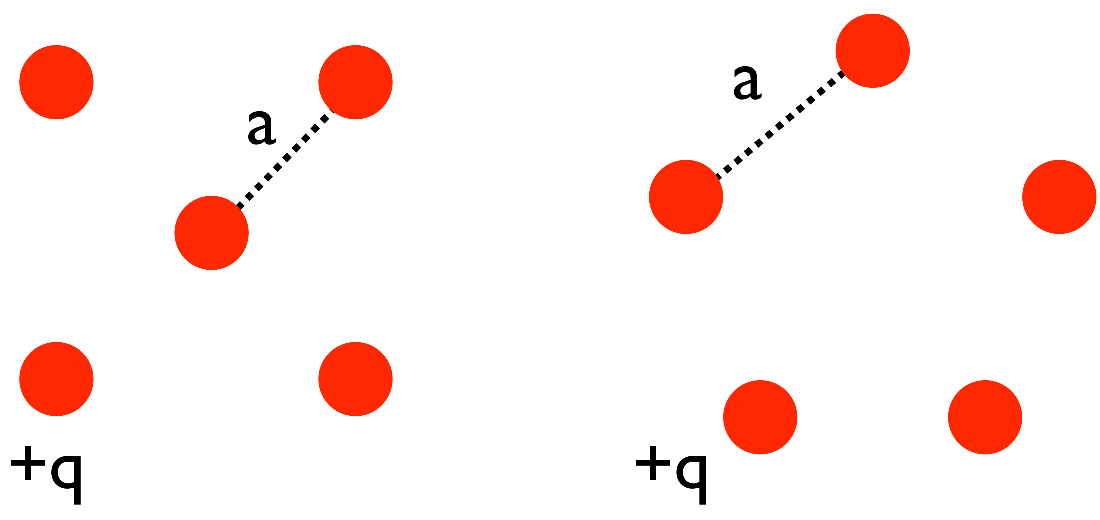


Figure 2: Problem 2.

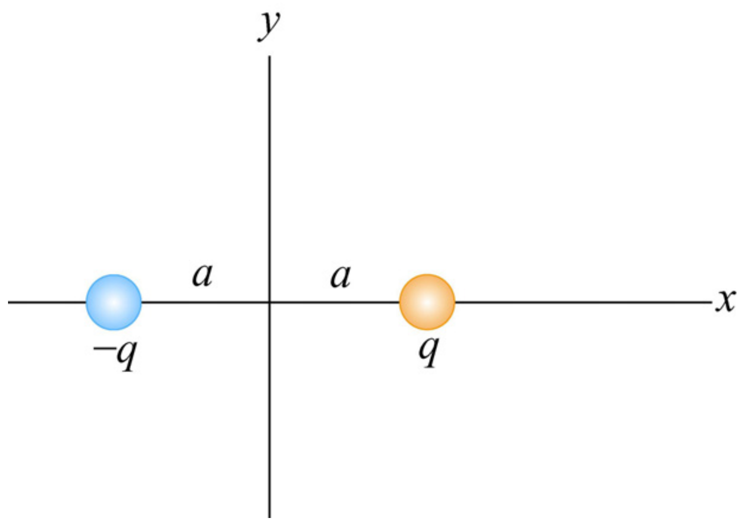


Figure 3: The electric dipole of problem 3.

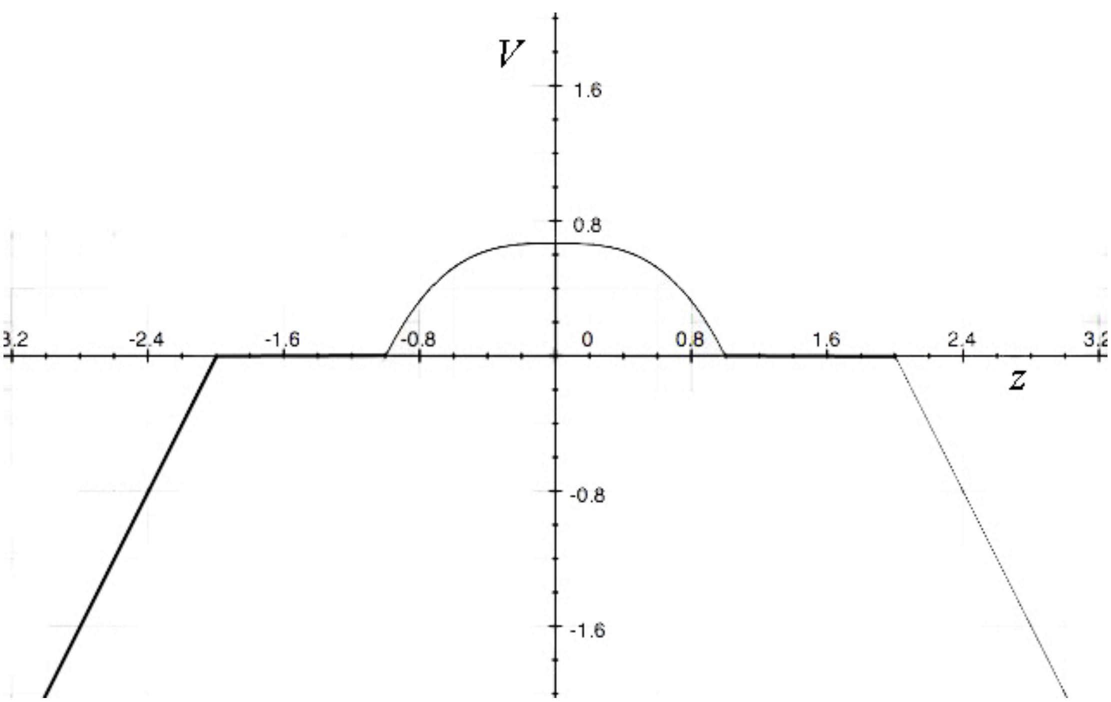


Figure 4: Problem 6.

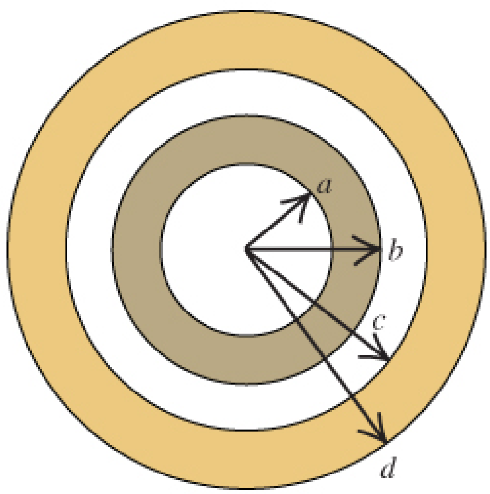


Figure 5: The Farady cage of problem 8.