Prof. Anchordoqui

Problems set # 12 **Physics 169** May 12, 2015

1. Two concentric conducting spheres of inner and outer radii a and b, respectively, carry charges $\pm Q$. The empty space between the spheres is half-filled by a hemispherical shell of dielectric (of dielectric constant ϵ/ϵ_0), as shown in Fig. 1. *(i)* Find the electric field everywhere between the spheres. *(ii)* Calculate the surface-charge distribution on the inner sphere. *(iii)* Calculate the polarization-charge density induced on the surface of the dielectric at $r = a$.

2. A coaxial capacitor of length $l = 6$ cm uses an insulating dielectric material with $\epsilon/\epsilon_0 = 9$, see Fig. 2. The radii of the cylindrical conductors are 0.5 cm and 1 cm. If the voltage applied across the capacitor is $V(t) = 50\sin(120\pi t)$ what is the displacement current?

3. The parallel-plate capacitor shown in Fig. 3 is filled with a lossy dielectric material of relative permittivity κ and conductivity σ. The separation between the plates is d and each plate is of area A. The capacitor is connected to a time-varying voltage source $V(t)$. *(i)* Obtain an expression for I_c, the conduction current flowing between the plates inside the capacitor, in terms of the given quantities. *(ii)* Obtain an expression for I_d, the displacement current flowing inside the capacitor. *(iii)* Based on your expressions for parts (i) and (ii), give an equivalent-circuit representation for the capacitor. *(iv)* Evaluate the values of the circuit elements for $A = 4$ cm^2, $d = 0.5$ cm, $\kappa = 4$, $\sigma = 2.5$ (S/m), and $V(t) = 10\cos(3\pi 10^3 t)$ V. [*Hint*: 1 $S = 1\Omega^{-1}$, (S stands for siemens)]

4. Figure 4 shows a plane electromagnetic sinusoidal wave propagating in the x-direction. Suppose that the wavelength is 50 m, and the electric field vibrates in the xy plane with an amplitude of 22 V/m. Calculate *(i)* the frequency of the wave and *(ii)* the magnitude and direction of the magnetic field when the electric field has its maximum value in the negative y-direction. *(iii)* Write an expression for the magnetic field with the correct unit vector, with numerical values for $B_{\rm max}$, k, and ω, and its magnitude in the form $B = B_{\rm max}\cos(kx - \omega t)$.

5. Some science fiction writers have described solar sails that could propel interstellar spaceships. Imagine a giant sail on a spacecraft subjected to radiation pressure from our Sun. *(i)* Explain why this arrangement works better if the sail is highly reflective rather than highly absorptive. *(ii)* If the sail is assumed highly reflective, show that the force exerted by the sunlight on the spacecraft's sail is given by $F_{\rm rad} = \frac{P_\odot A}{2\pi r^2 c}$, where $P_\odot$ is the power output of the Sun (3.8×10^{26} W), A is the surface area of the sail, r is the distance from the Sun, and c is the speed of light. (Assume that the area of the sail is much larger than the area of the spacecraft so that all the force is due to radiation pressure on the sail, only. *(iii)* Using a reasonable value for A, compute the force on the spacecraft due to the radiation pressure and the force on the spacecraft due to the gravitational force of the Sun on the spacecraft. Does this result imply that such a system will work? Explain your answer.

6. A pulsed laser fires a 1000 MW pulse that has a 200 ns duration at a small object that has a mass equal to 10.0 mg and is suspended by a fine fiber that is 4.00 cm long. If the radiation is completely absorbed by the object, what is the maximum angle of deflection of this pendulum? [*Hint*: Think of the system as a ballistic pendulum and assume the small object was hanging vertically before the radiation hit it.]

7. An electromagnetic wave has a frequency of 100 MHz and is traveling in a vacuum. The

magnetic field is given by $B(z,t) = 1.00 \times 10^{-8}\cos(kz-\omega t)\hat{\imath}$. *(i)* Find the wavelength and the direction of propagation of this wave. *(ii)* Find the electric field vector, $\vec{E}(z,t)$. *(iii)* Determine the Poynting vector, and use it to find the intensity of the wave.

8. A dish antenna having a diameter of 20 m receives (at normal incidence) a radio signal from a distant source as shown in Fig. 5. The radio signal is a continuous sinusoidal wave with amplitude $E_{\rm m} = 0.2\mu\text{V/m}$. Assume the antenna absorbs all the radiation that falls on the dish. *(i)* What is the amplitude of the magnetic field in this wave? *(ii)* What is the intensity of the radiation received by the antenna? *(iii)* What is the power received by the antenna? *(iv)* What force is exerted by the radio waves on the antenna?

9. Show that any function of the form $y(x,t) = f(x-ct)+g(x+ct)$ satisfies the one-dimensional wave equation for light, $\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = 0$.

10. Suppose that we have a cylindrical capacitor, as seen in the Fig. 6. Suppose further that we put an AC current across the plates, starting at a low frequency, ω. As the voltage alternates, the positive charge on the top plate is take off and negative charge is put on. While that is happening, the electric field disappears and then builds up in the opposite direction. As the charge sloshes back and forth slowly, the electric field follows. At each instant the electric field is uniform, as shown in the figure, except for some edge effects which we are going to disregard. We can write the electric field as $E = E_0\cos(\omega t)$, where $E_0 = Q_0/\epsilon_0 A$ is constant, and $A = \pi a^2$ is the area of the plate. Now will this continue to be right as the frequency goes up? No, because as the electric field is going up and down, there is a flux of electric field through any circular loop, say Γ, of radius r inside the capacitor. And, as you know, a changing electric field acts to produce a magnetic field. From Maxwell's equations, the magnetic field is given by a

$$c^2 \oint \vec{B}\cdot d\vec{\ell} = \frac{d}{dt}\int \vec{E}\cdot d\vec{A} \Rightarrow c^2 B 2\pi r = \frac{d}{dt}(E\pi r^2),$$

or

$$B = -\frac{\omega r}{2c^2}E_0\sin(\omega t).$$

So, the changing electric field has produced a magnetic field circulating around inside the capacitor, and oscillating at the same frequency as the electric field. Now, are we done? No! This magnetic field also oscillates, which produces a new electric field! The uniform field, $E_1 = E_0\cos(\omega t)$, is only the first term! The changing magnetic field produces a new electric field, E_2, such that the total field is $E = E_1 + E_2$. Now, in general, E_2 is also oscillating! This means that there will be a new magnetic field from E_2, which will be oscillating, which will create a new electric field, E_3, which will create a new magnetic field.... Your task is to calculate the first four terms of the series, enough to get the pattern, and write the total electric field, taking the field at the center of the capacitor to be exactly $E_0\cos(\omega t)$, (i.e., there is no correction at the center). Then, compare your result with Bessel functions, and see if you can write the full electric field you find in terms of one of the Bessel functions, in closed form. Can you find an exact expression in terms of one of the Bessel functions for the magnetic field?

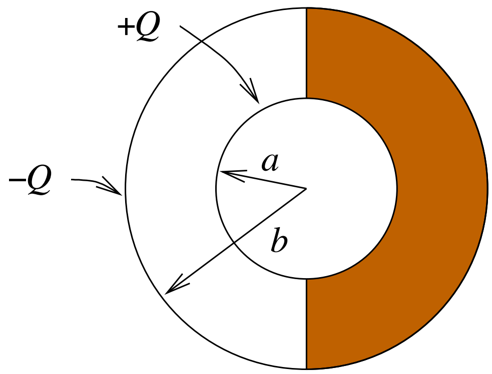

Figure 1: Problem 1.

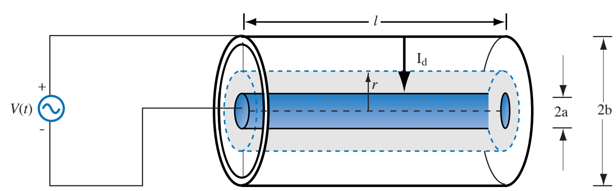

Figure 2: Problem 2.

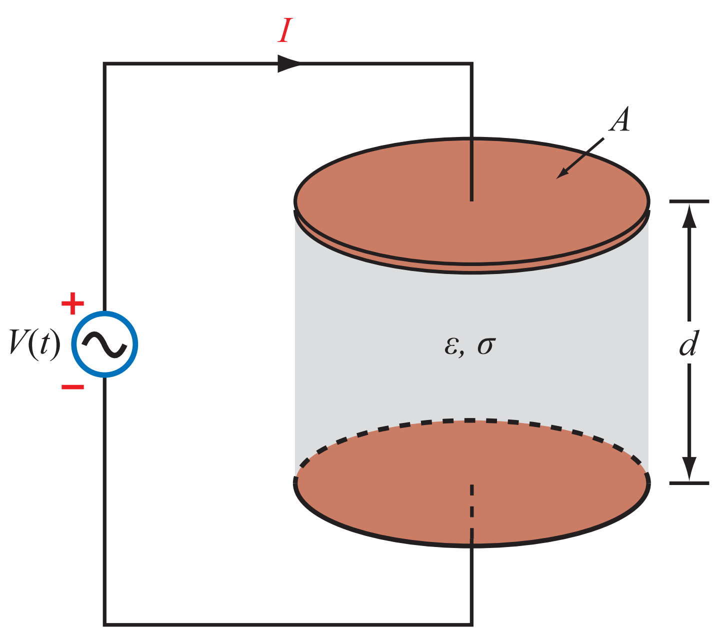

Figure 3: Problem 3.

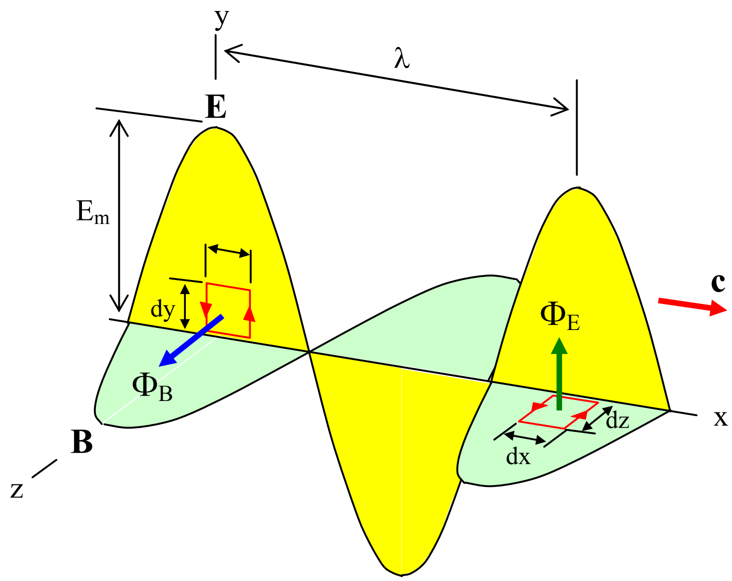

Figure 4: Problem 4.

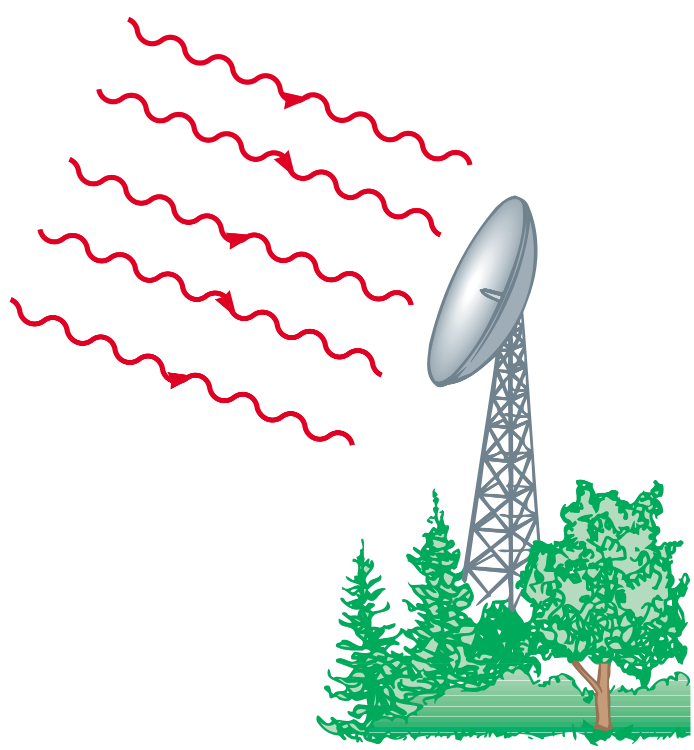

Figure 5: Problem 8.

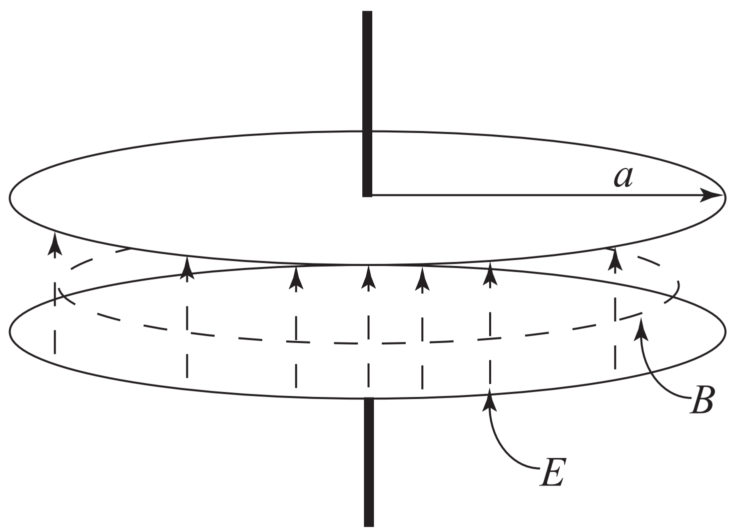

Figure 6: Problem 10.