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Problems set # 1	**Physics 169**	February 4, 2015

1. *(i)* Compare the electric force holding the electron in orbit ($r = 0.53 \times 10^{-10}$ m) around the proton nucleus of the hydrogen atom, with the gravitational force between the same electron and proton. What is the ratio of these two forces. *(ii)* Would life be different if the electron were positively charged and the proton were negatively charged? Does the choice of signs have any bearing on physical and chemical interactions? Explain.

Solution *(i)* Take the ratio of the electric force divided by the gravitational force, that is $\frac{F_E}{F_G} = \left(\frac{q_1 q_2}{4\pi\epsilon_0 \; r^2}\right) / \left(\frac{G\, m_1 m_2}{r^2}\right) = \frac{8.99\times10^9 \;\; \text{N}\cdot\text{m}^2/\text{C}^2 \;\; (1.602\times10^{-19} \;\; \text{C})^2}{6.67\times10^{-11} \;\; \text{N}\cdot\text{m}^2/\text{kg}^2 \; 9.11\times10^{-31} \;\; \text{kg} \; 1.67\times10^{-27} \;\; \text{kg}} \simeq 2.3 \times 10^{39}$. The electric force is about 2.3×10^{39} times stronger than the gravitational force for the given scenario. *(ii)* No. Life would be no different if electrons were positively charged and protons were negatively charged. Opposite charges would still attract, and like charges would still repel. The designation of charges as positive and negative is merely a definition.

2. The nucleus of ^{8}Be, which consists of 4 protons and 4 neutrons, is very unstable and spontaneously breaks into two alpha particles (helium nuclei, each consisting of 2 protons and 2 neutrons). *(i)* What is the force between the two alpha particles when they are 5.00×10^{-15} m apart, and *(ii)* what will be the magnitude of the acceleration of the alpha particles due to this force? Note that the mass of an alpha particle is 4.0026 u.

Solution *(i)* Since the charges have opposite signs, the force is repulsive. The magnitude of the force is given by Coulombs law, $F = \frac{1}{4\pi\epsilon_0}\frac{4e^2}{r^2} = 8.99 \times 10^9 \;\; \frac{\text{Nm}^2}{\text{C}^2}\frac{4(1.6\times10^{-19} \;\; \text{C})^2}{(5.00\times10^{-15} \;\; \text{m})^2} = 36.8$ N. *(ii)* The mass of an alpha particle is $m = 4.0026 \; u$, where $u = 1.66 \times 10^{-27}$ kg is the unified mass unit. Applying Newton's 2nd law, the acceleration of either alpha particle is then $a = \frac{F}{m} = \frac{36.8 \;\; \text{N}}{4.0026\cdot1.66\times10^{-27} \;\; \text{kg}} = 5.54 \times 10^{27}$ m/s^2. Of course from Newton's 3rd law, both alpha particles experience the same force, and hence undergo the same acceleration.

3. Suppose that 1.00 g of hydrogen is separated into electrons and protons. Suppose also that the protons are placed at Earth's North Pole and the electrons are placed at the South Pole. What is the resulting compressional force on Earth?

Solution 1.00 g of hydrogen contains Avogadro's number of atoms, each containing one proton and one electron. Thus, each charge has magnitude $|q| = N_A e$. The distance separating these charges is $r = 2R_\oplus$, where $R_\oplus$ is the Earths radius. Thus applying Coulombs law, $F = \frac{1}{4\pi\epsilon_0}\left(\frac{N_A e}{2R_\oplus}\right)^2 = 8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\frac{1}{4}\left(\frac{6.022\times10^{23}\cdot1.60\times10^{-19} \;\; \text{C}}{6.38\times10^6 \;\; \text{m}}\right)^2 = 5.12 \times 10^5$ N.

4. Three charge particles are placed at the corners of an equilateral triangle of side 0.500 m. The charges are $+7.00$ μC, $+2.00$ μC, and -4.00 μC. Calculate the magnitude and direction of the net force on the 7.00 μC charge.

Solution The forces on the 7.00 μC charge are shown in Fig. 1. Applying Coulombs law to calculate each force, we get $\vec{F_1} = 8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2} \frac{7.00\times10^{-6}\ \text{C}\cdot 2.00\times10^{-6}\ \text{C}}{(0.500\ \text{m})^2} = 0.503$ N and $\vec{F_2} = 8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2} \frac{7.00\times10^{-6}\ \text{C}\cdot 4.00\times10^{-6}\ \text{C}}{(0.500\ \text{m})^2} = 1.01$ N. From the superposition principle, we known $\sum F_x = (\vec{F_1}+\vec{F_2})\cos 60° = 0.755$ N and $\sum F_y = (\vec{F_1}+\vec{F_2})\sin 60° = -0.436$ N. So the resultant force on the 7.00 μC charge is $F_R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2} = 0.872$ N at $\theta = \tan^{-1}(\sum F_y / \sum F_x) = -30°$.

5. A charge of 6.00×10^{-9} C and a charge of -3.00×10^{-9} are at a distance of 60.0 cm. Find the position at which a third charge of 12.0×10^{-9} C can be placed so that the net electrostatic force on it is zero.

Solution The required position is shown in Fig. 2. Note that this places q closer to the smaller charge, which will allow the two forces to cancel. Applying Coulmobs law, and requiring that $F_6 = F_3$ gives $\frac{1}{4\pi\epsilon_0} \frac{6.00\ \text{nC}\ q}{(x+0.600\ \text{m})^2} = \frac{1}{4\pi\epsilon_0} \frac{3\ \text{nC}\, q}{x^2}$, or $2x^2 = (x + 0.600\ \text{m})^2$. Solving for x gives the equilibrium position as $x = \frac{0.600\ \text{m}}{\sqrt{2}-1} = 1.45$ m beyond the -3.00 nC charge.

6. An airplane is flying through a thundercloud at a height of $2,000$ m. (This is a very dangerous thing to do because of updrafts, turbulence, and the possibility of electric discharge.) If there are charge concentrations of +40.0 C at height $3,000$ m within the cloud and -40.0 C at height $1,000$ m, what is the electric field $\vec{E}$ at the aircraft?

Solution We shall treat the concentrations as point charges. Then, the resultant field consists of two contributions, one due to each concentration. The contribution due to the positive charge at 3000 m altitude is $E_+ = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2} = 8.99 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2} \frac{40.0\ \text{C}}{(1000\ \text{m})^2} = 3.6 \times 10^5$ N/C (downward). The contribution due to the negative charge at 1000 m altitude is $E_- = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2} = 8.99 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2} \frac{40.0\ \text{C}}{(1000\ \text{m})^2} = 3.6 \times 10^5$ N/C (downward). From the Superposition Principle, the resultant field is then $E = E_+ + E_- = 7.20 \times 10^5$ N/C. (downward).

7. Two positive point charges are a fixed distance apart. The sum of their charges is Q_T. What charge must each have in order to *(i)* maximize the electric force between them, and *(b)* minimize it?

Solution *(i)* Let one of the charges be q and then the other charge is $Q_T - q$. The force between the charges is $F = \frac{1}{4\pi\epsilon_0}\frac{q(Q_T-q)}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{(qQ_T-q^2)}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{Q_T^2}{r^2}\left[\frac{q}{Q_T} - \left(\frac{q}{Q_T}\right)^2\right]$. Define $x = q/Q_T$ such that $F = \frac{1}{4\pi\epsilon_0}\frac{Q_T^2}{r^2}(x - x^2)$, where $0 \le x \le 1$. The function $f(x) = x - x^2$, with $x \in [0,1]$, has a maximum at $x = 0.5$, and so the force is maximum if $q = 0.5Q_T$; i.e., if both charges are half of the total, the force is maximized and given by $F = \frac{Q_T^2}{16\pi\epsilon_0\ r^2}$. *(ii)* If one of the charges has all of the charge, and the other has no charge, then the force between them will be zero, which is the minimum possible force.

8. An electron is released a short distance above Earth's surface. A second electron (directly below it) exerts an electrostatic force on the first electron just great enough to cancel the gravita-

tional force on it. How far below the first electron is the second?

Solution The magnitude of the repulsive force between electrons must equal the weight of an electron. Thus by using Coulomb's law and Newton's 2nd law (applied to gravity), we have $\frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} = m_e g$, so rearranging this expression, and then making the substitutions, you obtain $r = \sqrt{\frac{1}{4\pi\epsilon_0}\frac{e^2}{m_e g}} = \sqrt{\frac{8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\,(1.60\times10^{-19}\text{ C})^2}{9.11\times10^{-31}\text{ kg } 9.80 \text{ m/s}^2}} = 5.08$ m.

9. *(i)* How much negative charge and how much positive charge are there on the electrons and the protons in a cup of water (0.25 kg)? Note Avogadro's number is $N_A = 6.022 \times 10^{23}$, and each oxygen atom has 8 electrons. *(ii)* What is the magnitude of the attractive force exerted by the electrons in a cup of water on the protons in a second cup of water at a distance of 10 m?

Solution The "molecular mass" of water is 18 g/mol, so 250 g of water amounts to 250/18 mol. For each mole of water, there are 6.02×10^{23} molecules. Each molecule has two hydrogen atoms (with one electron) and one oxygen atom (with eight electrons), with a total of ten electrons per molecule. Thus, we can calculate the total amount of negative charge in the cup of water readily: $[\text{neg. chg.}] = 0.25 \text{ kg}\left(\frac{1{,}000 \text{ g}}{1 \text{ kg}}\right)\left(\frac{1 \text{ mol}}{18 \text{ g}}\right)\left(\frac{6.022\times10^{23} \text{ molecules}}{\text{mol}}\right)\left(\frac{10 \text{ electrons}}{\text{molecules}}\right)\left(\frac{-1.6\times10^{-19}\text{ C}}{\text{electron}}\right) = 1.3 \times 10^7$ C. Clearly, since the cup of water is overall electrically neutral, the positive charge on the protons is just the opposite of this. If we treat the total charge in each glass of water as point charges, then in the first cup of water we have one point charge of -1.3×10^7 C and another of 1.3×10^7 C, separated by 10 m. The force is then: $|\vec{F}| = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}^2} = -1.5 \times 10^{22}$ N. This is an enormous force, equivalent to a weight of a billion billion tons! Thankfully, this attractive force on the protons is precisely canceled by an equally large repulsive force exerted by the protons in one cup on the protons in the other.

10. Eight point charges, each of magnitude q, are located on the corners of a cube of edge s, as shown in Fig. 3 *(i)* Determine the x, y, and z components of the resultant force exerted by the other charges on the charge located at point A. *(ii)* What are the magnitude and direction of this resultant force? *(iii)* Show that the magnitude of the electric field at the center of any face of the cube has a value of $2.18\frac{1}{4\pi\epsilon_0}\frac{q}{s^2}$. *(iv)* What is the direction of the electric field at the center of the top face of the cube?

Solution *(i)* There are 7 terms that contribute. There are 3 charges a distance s away (along sides), 3 a distance $\sqrt{2}s$ away (face diagonals), and one charge a distance $\sqrt{3}s$ away (body diagonal). By symmetry, the x, y, and z components of the electric force must be equal. Thus, we only need to calculate one component of the total force on the charge of interest. We will choose the coordinate system as indicated in Fig. 3, and calculate the y component of the force. We can already see that several charges will not give a y component of the force at all, just from symmetry - charges 3, 4 and 7. This leaves only charges 1, 2, 5, and 6 to deal with. Charge 6 will give a force purely in the y direction: $F_{6,y} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{s^2}$. Charges 5 and 2 are both a distance $s\sqrt{2}$ away, and a line connecting these charges with the charge of interest makes an angle $\theta = 45°$ with the y-axis in both cases. Hence, noting that $\cos\theta = 1/\sqrt{2}$, we obtain $F_{2,y} = F_{5,y} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{(s\sqrt{2})^2}\cos\theta = \frac{1}{4\pi\epsilon_0}\frac{q^2}{2s^2}\frac{1}{\sqrt{2}}$.

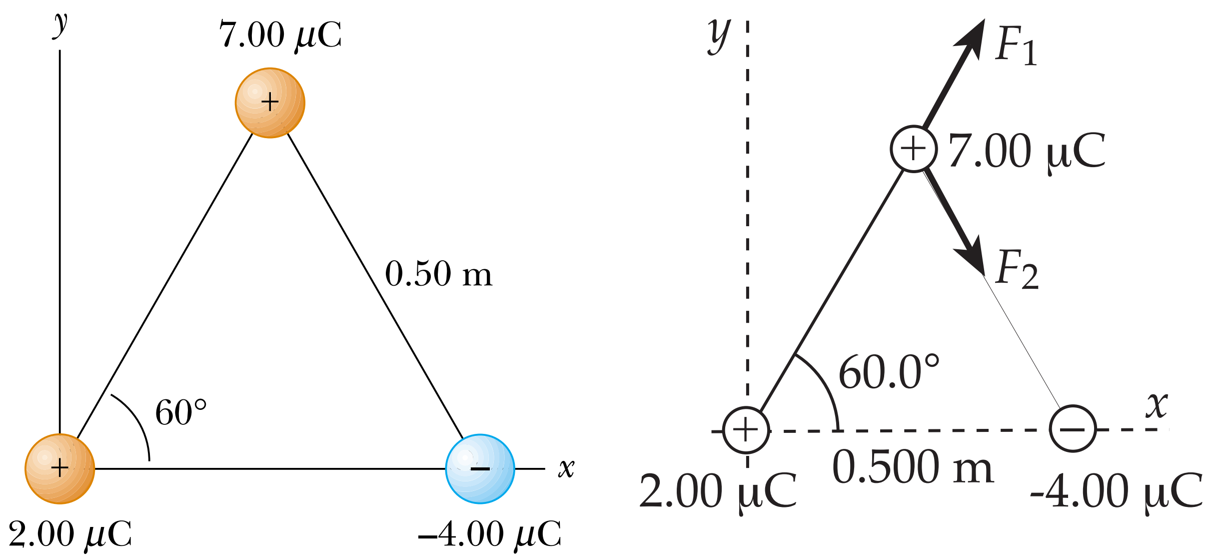

Figure 1: Problem 4.

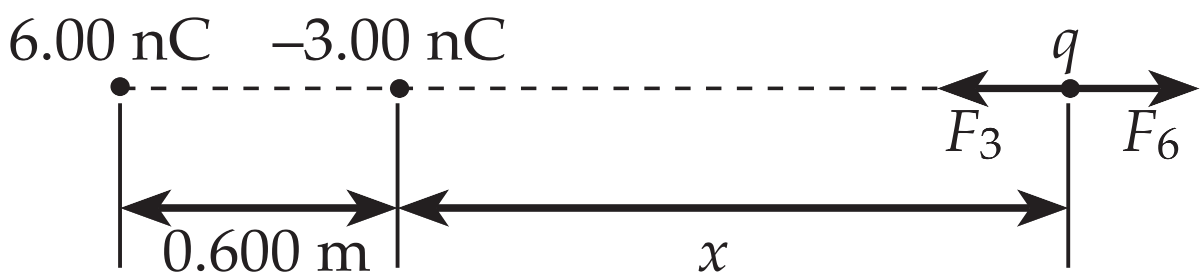

Figure 2: Problem 5.

Finally, we have charge 1 to deal with. It is a distance $\sqrt{3}$ away (see the Fig. 4). What is the y component of the force from charge 1? First, we can find the component of the force in the $x-y$ plane (see Fig. 4): $F_{1,x-y} = F_1 \cos\phi = F_1 \frac{\sqrt{2}}{\sqrt{3}}$. Now, we can find the component of the force along the y direction: $F_{1,y} = F_{1,x-y}\cos\theta = F_{1,x-y}\frac{1}{\sqrt{2}} = F_1 \frac{\sqrt{2}}{\sqrt{3}}\frac{1}{\sqrt{2}} = F_1\frac{1}{\sqrt{3}}$. Since we know charge 1 is a distance $s\sqrt{3}$ away, we can calculate the full force F_1 easily, and complete the expression for $F_{1,y}$, that is $F_{1,y} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{(s\sqrt{3})^2}\frac{1}{\sqrt{3}} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{s^2}\frac{1}{3\sqrt{3}}$. Now we have the y component for the force from every charge; the net force in the y direction is just the sum of all those: $F_{y,\text{net}} = F_{1,y}+F_{2,y}+F_{5,y}+F_{6,y} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{s^2}\left[1+\frac{1}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}\right] = \frac{1}{4\pi\epsilon_0}\frac{q^2}{s^2}\left[1+\frac{1}{\sqrt{2}}+\frac{1}{3\sqrt{3}}\right]$. Since the problem is symmetric in the x, y, and z directions, all three components must be equivalent. The force is then $\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{s^2}\left[1+\frac{1}{\sqrt{2}}+\frac{1}{3\sqrt{3}}\right](\hat{\imath}+\hat{\jmath}+\hat{k}) = 1.90\frac{1}{4\pi\epsilon_0}\frac{q^2}{s^2}(\hat{\imath}+\hat{\jmath}+\hat{k})$. *(ii)* $F = \sqrt{F_x^2+F_y^2+F_z^2} = 3.29\frac{1}{4\pi\epsilon_0}\frac{q^2}{s^2}$ away from the origin. *(iii)* There is zero contribution from the same face due to symmetry. The opposite face contributes $\frac{q\sin\phi}{\pi\epsilon_0 r^2}$ where $r = \sqrt{\frac{(\sqrt{2}s)^2}{4}+s^2} = \sqrt{1.5}\,s = 1.22s$ and $\sin\phi = s/r$, see Fig. 4. All in all, $E = \frac{qs}{\pi\epsilon_0 r^3} = 2.18\frac{1}{4\pi\epsilon_0}\frac{q}{s^2}$. *(iv)* The direction is $\hat{k}$.

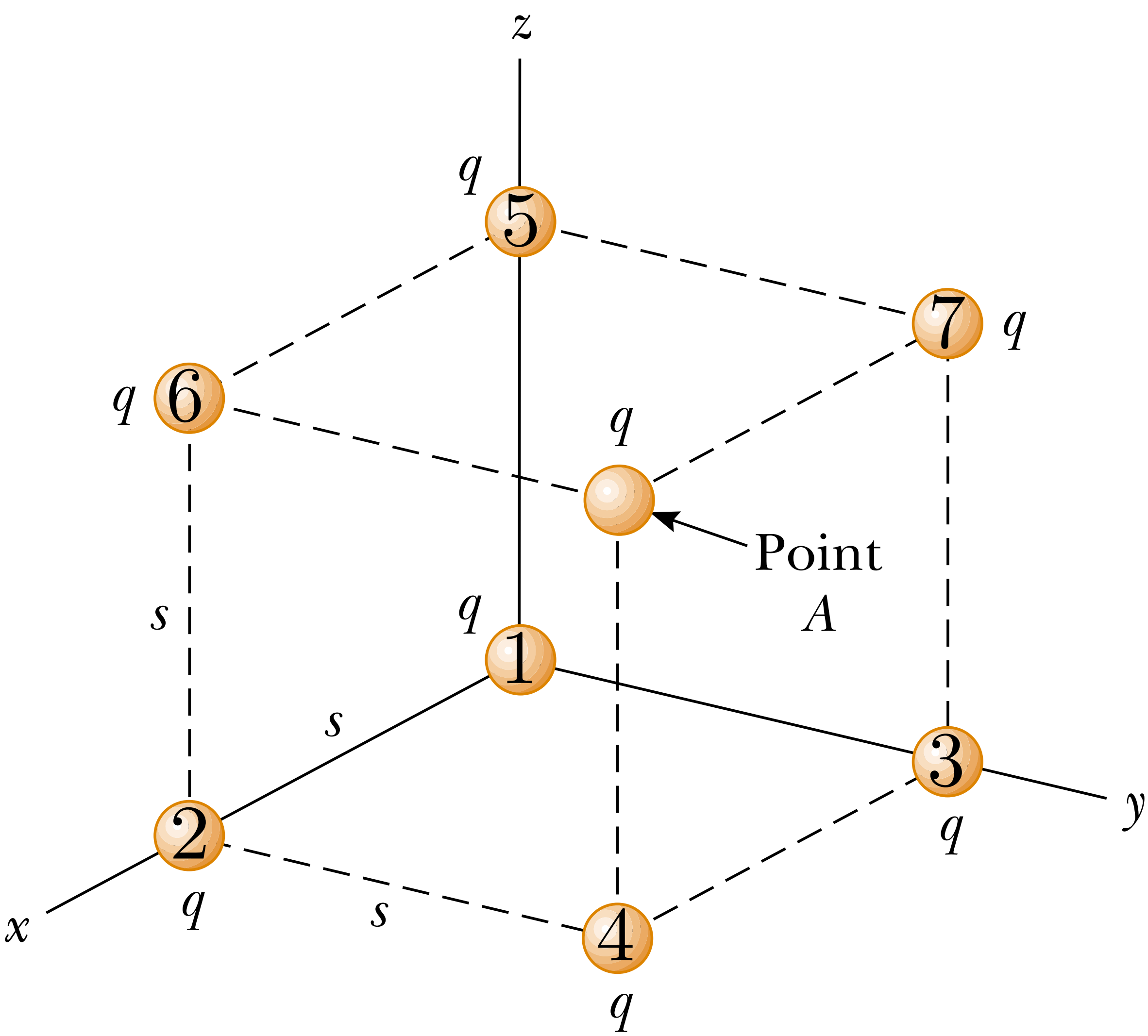

Figure 3: Problem 10.

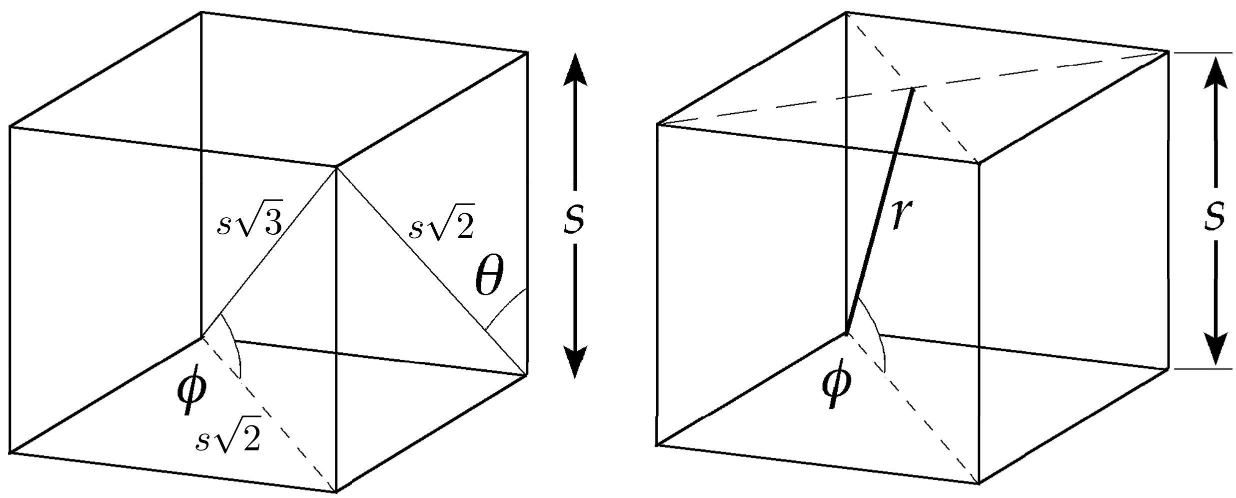

Figure 4: Problem 10, solution.